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Hausdorff Intuitionistic Fuzzy Filters

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ABSTRACT

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. By using intuitionistic fuzzy sets, we introduce and study the concept of intuitionistic fuzzy filters and define the concept of Hausdorffness on intuitionistic fuzzy filters, which can not be defined in crisp theory of filters, and study their properties for some extent.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12]. As generalizations of fuzzy sets, the concepts of intuitionistic fuzzy sets and interval valued fuzzy sets were introduced by Atanassov [1] and Gorzalczany [7], respectively. The theory of fuzzy filters has been studied in [9] [11] et al.

Recently, Ramakrishnan and Nayagam [12] introduced and studied the notion of interval valued fuzzy (IVF) filters and also the notion of Hausdorffness on IVF filters, which can not be defined in crisp theory of filters, and studied their properties.

In this paper, by using intuitionistic fuzzy sets, we introduce and study the concept of intuitionistic fuzzy filters and define the concept of Hausdorffness on intuitionistic fuzzy filters, and study their properties for some extent.

2. Preliminaries

First we shall present the fundamental definitions given by Atanassov [1]:

Definition 2.1. Let X be a non-empty fixed set. An intuitionistic fuzzy set (shortly, **IFS**), A is an object having the form

$$A=\{<x,\mu_A(x),\nu_A(x)>:x\in X\}$$

where the functions $\mu_A:X\to[0,1]$ and $\nu_A:X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\nu_A(x)\le 1$ for each $x\in X$.

An **IFS** $A=\{<x,\mu_A(x),\nu_A(x)>:x\in X\}$ in X can identified to an ordered pair $<\mu_A(x),\nu_A(x)>$ in $[0,1]^X\times[0,1]^X$ or to element in $[0,1]\times[0,1]^X$. Obviously, every fuzzy set $\{<\mu_A(x),x>:x\in X\}$ on X is an **IFS** of the form $A=\{<x,\mu_A(x), 1-\mu_A(x)>:x\in X\}$.

Definition 2.2. Let A and B be **IFS**s in X in the form $A=\{<x,\mu_A(x),\nu_A(x)>:x\in X\}$ and $B=\{<x,\mu_B(x),\nu_B(x)>:x\in X\}$. Then

(a) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$ for all $x\in X$;
(b) $A=B$ iff $A\subseteq B$ and $B\subseteq A$;
(c) $A^c=\{<x,\nu_A(x),\mu_A(x)>:x\in X\}$;
(d) $A\cap B=\{<x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)>: x\in X\}$;
(e) $A\cup B=\{<x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)>: x\in X\}$;
(f) If $\{A_i:i\in J\}$ is an arbitrary family of **IFS**s in X, then

$\cap A_i = \{<x, \wedge\mu_{A_i}(x), \vee\nu_{A_i}(x)> : x\in X\}$,

$\cup A_i = \{<x, \vee\mu_{A_i}(x), \wedge\nu_{A_i}(x)> : x\in X\}$.

(g) $0_\sim = \{<x,0,1> : x\in X\}$ and $1_\sim = \{<x,1,0> : x\in X\}$.

Definition 2.3. Let X and Y be two nonempty sets and $f: X\to Y$ be a function. Let $A = \{<x, \mu_A(x), \nu_A(x)> : x\in X\}$ be an **IFS** in X and $B = \{<y, \mu_B(y), \nu_B(y)> : y\in Y\}$ be an **IFS** in Y.

(a) The inverse image $f^{-1}(B)$ of B under f is the **IFS** in X defined by

$f^{-1}(B) = \{<x, f^{-1}(\mu_B)(x) f^{-1}(\nu_B)(x)> : x\in X\}$.

(b) The image $f(A)$ of A under f is the **IFS** in Y defined by

$f(A) = \{<x, f(\mu_A)(y), (1-f(1-\nu_A))(y)> : y\in Y\}$

where

$$f(\mu_A)(y) = \begin{cases} \sup_{x\in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y)\neq\Phi \\ 0, & \text{otherwise,} \end{cases}$$

$$(1-f(1-\nu_A))(y) = \begin{cases} \inf_{x\in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y)\neq\Phi \\ 1, & \text{otherwise.} \end{cases}$$

Notation 2.4. We shall use the symbol $f_-(\nu_A)$ for $1-f(1-\nu_A)$ for the sake of simplicity.

Now we list the properties of images and preimages, some of which we shall frequently use in Sections 3 and 4.

Theorem 2.5. Let A and A_i $(i\in J)$ be **IFSs** in X and B and B_i $(i\in J)$ be **IFSs** in Y and $f: X\to Y$ be a function. Then:

(a) If $A_1\subseteq A_2$, then $f(A_1)\subseteq f(A_2)$.

(b) If $B_1\subseteq B_2$, then $f^{-1}(B_1)\subseteq f^{-1}(B_2)$.

(c) $A\subseteq f^{-1}(f(A))$ (If f is injective, then $A=f^{-1}(f(A))$).

(d) $f(f^{-1}(B))\subseteq B$ (If f is surjective, then $f(f^{-1}(B))=B$).

(e) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i) f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$

(f) $f(\cup A_i) = \cup f(A_i) f(\cap A_i) \subseteq \cap f(A_i)$ (If f is injective, then $f(\cap A_i) = \cap f(A_i)$).

(g) $f^{-1}(1_\sim) = 1_\sim f^{-1}(0_\sim) = 0_\sim$.

(h) $f(0_\sim) = 0_\sim f(1_\sim) = 1_\sim$ if f is surjective.

(i) $f^{-1}(B)^c = f^{-1}(B^c)$, $f(A)^c\subseteq f(A^c)$ if f is surjective.

3. IF filters

Definition 3.1. A nonvoid family $\boldsymbol{F}$ of **IFSs** is called an *intuitionistic fuzzy filter* or ***IF*** filter if

(a) $0_\sim \notin \boldsymbol{F}$.

(b) If A, $B\in\boldsymbol{F}$, then $A\cap B\in\boldsymbol{F}$.

(c) If $A\in\boldsymbol{F}$ and $A\subseteq B$, then $B\in\boldsymbol{F}$.

Definition 3.2. (a) A nonvoid family $\boldsymbol{B}$ of **IFSs** is called an ***IF** filter base* if provided $\boldsymbol{B}$ does not contain $0_\sim$ and provided the intersection of any two element of $\boldsymbol{B}$ contains an element of $\boldsymbol{B}$. A family $\boldsymbol{S}$ is called a *subbase* of **IF** filter if it is nonvoid and the intersection of any finite number of elements of $\boldsymbol{S}$ is not $0_\sim$.

Remark 3.3. If $\boldsymbol{S}$ is a subbase of an **IF** filter, then the family $\boldsymbol{B}(\boldsymbol{S})$ consisting of all finite intersections of elements of $\boldsymbol{S}$ is an **IF** filter base. If $\boldsymbol{B}$ is an **IF** filter base, then the family $\boldsymbol{F}(\boldsymbol{B})$, consisting of all **IFSs** A such that $A\supseteq B$ for some $B\in\boldsymbol{B}$, is an **IF** filter. Furthermore, $\boldsymbol{B}(\boldsymbol{S})$ and $\boldsymbol{F}(\boldsymbol{B})$ are uniquely determined by $\boldsymbol{S}$ and $\boldsymbol{B}$, respectively.

Definition 3.4. The **IF** filter $\boldsymbol{F}(\boldsymbol{B})$ and $\boldsymbol{F}(\boldsymbol{B}(\boldsymbol{S}))$ (or, $\boldsymbol{F}(\boldsymbol{S})$) are called, respectively, the **IF** filter generated by $\boldsymbol{B}$ and the **IF** filter generated by $\boldsymbol{S}$. A family $\boldsymbol{B}$ is called a base of the **IF** filter $\boldsymbol{F}$ if $\boldsymbol{B}$ is a **IF** filter base and $\boldsymbol{F}=\boldsymbol{F}(\boldsymbol{B})$. Similarly, $\boldsymbol{S}$ is called a subbase of the **IFS** filter $\boldsymbol{F}$ if $\boldsymbol{S}$ is a subbase and $\boldsymbol{F}=\boldsymbol{F}(\boldsymbol{S})$.

Remark 3.5. (a) Let Φ be any indexed family of **IF** filters on X. Then

(i) $\cap_{F\in\Phi} F$ is also **IF** filter.

(ii) $\cap_{F\in\Phi} F$ is also **IF** filter if Φ is directed family of **IF** filters under inclusion $\subset$.

(b) Let $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ be two **IF** filter bases. Then $\boldsymbol{F}(\boldsymbol{B}_1)\subseteq\boldsymbol{F}(\boldsymbol{B}_2)$ if and only if for any $B\in\boldsymbol{B}_1$ there exists $A\in\boldsymbol{B}_2$ such that $A\subseteq B$.

Theorem 3.6. Let $\boldsymbol{F}$ be an **IF** filter on X and $Y\subseteq X$. Then $\boldsymbol{F}|Y$ is an **IF** filter on Y if $A|Y\neq 0_\sim$ for any $A\in\boldsymbol{F}$.

Theorem 3.7. Let $f: X\to Y$ be a function and $\boldsymbol{F}$ be an **IF** filter on X. Then $f(\boldsymbol{F})=\{f(F): F\in\boldsymbol{F}\}$ is an **IF** filter base on Y.

Theorem 3.8. Let $f: X \to Y$ be a surjection and G be an IF filter on Y. Then $f^{-1}(\mathbf{G}) = \{f^{-1}(G) : G \in \mathbf{G}\}$ is an IF filter base on X.

Definition 3.9. Let $(X_i, \mathbf{F}_i)$ be IF filters (i=1,2). A function $f: (X, \mathbf{F}_1) \to (Y, \mathbf{F}_2)$ is called IF *filter continuous* if for every $F \in \mathbf{F}_2$, $f^{-1}(F) \in \mathbf{F}_1$.

Example 3.10. Let $X=\{a,b,c\}$ and $Y=\{d,e,f\}$. Let A and B be IFS in X and Y respectively defined as follows:

$$A=\left\langle x, \left(\frac{a}{\alpha_1}, \frac{b}{\alpha_1}, \frac{c}{\beta_1}\right), \left(\frac{a}{\alpha_2}, \frac{b}{\alpha_2}, \frac{c}{\beta_2}\right)\right\rangle \quad \text{and}$$

$$B=\left\langle x, \left(\frac{d}{\alpha_1}, \frac{e}{\beta_1}, \frac{f}{\delta_1}\right), \left(\frac{d}{\alpha_2}, \frac{e}{\beta_2}, \frac{f}{\delta_2}\right)\right\rangle,$$

where $\alpha_1+\alpha_2 \le 1$, $\beta_1+\beta_2 \le 1$ and $\delta_1+\delta_2 \le 1$. Then clearly $\mathbf{B}_1=\{A\}$ and $\mathbf{B}_2=\{B\}$ are IF filter bases on X and Y respectively. Let $\mathbf{F}_1$ and $\mathbf{F}_2$ be the IF filter generated by $\mathbf{B}_1$ and $\mathbf{B}_2$ respectively. Let $f: X \to Y$ be a function defined by $f(a)=f(b)=d$ and $f(c)=e$. Then by definition $f^{-1}(\mathbf{B}_2)=\mathbf{B}_1$ and hence f is IF filter continuous.

However, constant function $f: (X, \mathbf{F}_1) \to (Y, \mathbf{F}_2)$ need not be IF filter continuous as shown by the following example.

Example 3.11. Let $X=\{a,b,c\}$ and $Y=\{d,e,f\}$. Let A and B be IFS in X and Y respectively defined as follows:

$$A=\left\langle x, \left(\frac{a}{\alpha_1}, \frac{b}{\alpha_1}, \frac{c}{\alpha_1}\right), \left(\frac{a}{\alpha_2}, \frac{b}{\alpha_2}, \frac{c}{\alpha_2}\right)\right\rangle \quad \text{and}$$

$$B=\left\langle x, \left(\frac{d}{\alpha_1}, \frac{e}{\beta_1}, \frac{f}{\delta_1}\right), \left(\frac{d}{\alpha_2}, \frac{e}{\beta_2}, \frac{f}{\delta_2}\right)\right\rangle \quad .$$

where $\alpha_1+\alpha_2 \le 1$, $\beta_1+\beta_2 \le 1$, $\delta_1+\delta_2 \le 1$, $\beta_1 < \alpha_1$ and $\beta_2 > \alpha_2$.

Clearly, $\mathbf{B}_1=\{A\}$ and $\mathbf{B}_2=\{B\}$ are IF filter bases on X and Y respectively. Let $\mathbf{F}_1$ and $\mathbf{F}_2$ be the IF filter generated by $\mathbf{B}_1$ and $\mathbf{B}_2$ respectively. Let $f: X \to Y$ be constant function defined by $f(x)=e$ for all $x \in X$.

Choose σ_i $(i=1,2)$ such that $\beta_1 < \sigma_1 < \alpha_1$ and $\beta_2 > \sigma_2 > \alpha_2$. Then clearly,

$$C=\left\langle x, \left(\frac{d}{\alpha_1}, \frac{e}{\sigma_1}, \frac{f}{\delta_1}\right), \left(\frac{d}{\alpha_2}, \frac{e}{\sigma_2}, \frac{f}{\delta_2}\right)\right\rangle \in \mathbf{F}_2 .$$

But

$$f^{-1}(C)=\left\langle x, \left(\frac{a}{\sigma_1}, \frac{b}{\sigma_1}, \frac{c}{\sigma_1}\right), \left(\frac{a}{\sigma_2}, \frac{b}{\sigma_2}, \frac{c}{\sigma_2}\right)\right\rangle \notin \mathbf{F}_1.$$

Hence f is not IF filter continuous.

The following statement is an immediate consequence of definitions.

Remark 3.12. (i) If $f: (X, \mathbf{F}) \to (Y, \mathbf{G})$ and $g: (Y, \mathbf{G}) \to (Z, \mathbf{H})$ are IF filter continuous, then the composition $f \circ g: (X, \mathbf{F}) \to (Z, \mathbf{H})$ is also IF filter continuous.
(ii) If $f: (X, \mathbf{F}) \to (X, \mathbf{F})$ is identity function, then f is IF filter continuous.
(iii) Let $f: (X, \mathbf{F}) \to (Y, \mathbf{G})$ be an IF filter continuous function. If $Z \subseteq X$ such that $F|Z \ne 0_\sim$ for any $F \in \mathbf{F}$, then the restriction $f|Z: (Z, \mathbf{F}|Z) \to (Y, \mathbf{G})$ is also IF filter continuous.

Theorem 3.13. A function $f: (X, \mathbf{F}) \to (Y, \mathbf{G})$ is IF filter continuous if and only if for every IFS point $x_{(\alpha,\beta)}$ in X and every $G \in \mathbf{G}$ such that $f(x_{(\alpha,\beta)}) \in G$, there exists a $F \in \mathbf{F}$ such that $x_{(\alpha,\beta)} \in F$ and $f(F) \subseteq G$.

Recall that the characteristic set of a fuzzy filter $\mathbf{F}$ with respect to a fuzzy set A is the set $C^A(\mathbf{F}) = \{\, a \in [0,1]$: for all $F \in \mathbf{F}$, there exists $x \in X$ such that $F(x) > A(x)+a\}$ and the characteristic value of $\mathbf{F}$ with respect to A is the supremum of $C^A(\mathbf{F})$ [9].
Now, we generalize these notions as follows:

Definition 3.14. Let $\mathbf{F}$ be an IF filter on X and A be an IFS in X. Then the *characteristic set of* $\mathbf{F}$ *with respect to* A is given by $C^A(\mathbf{F}) = \{(a,b) \in [0.1] \times [0,1]$: for all $F \in \mathbf{F}$, there exists $x \in X$ such that $\mu_F(x) > \mu_A(x)+a$, $\gamma_F(x)+b < \gamma_A(x)\}$ and $c^A(\mathbf{F}) = (\sup a, \inf b)$ is the *characteristic value of* $\mathbf{F}$ *with respect to* A, where the supremum and infremum are taken over all $(a,b) \in C^A(\mathbf{F})$, respectively.

Definition 3.15. Let $f: X \to Y$ be a function. An IFS A in X is called f-invariant if $f(x)=f(y) \Rightarrow A(x)=A(y)$.

Theorem 3.16. Let $f: (X, \mathbf{F}) \to (Y, \mathbf{G})$ be an IF filter continuous function. If IFS A in X is f-invariant, then $C^A(\mathbf{F}) \subseteq C^{f(A)}(\mathbf{G})$.

Definition 3.17. Let $\mathbf{F}$ and $\mathbf{G}$ be IF filters on X and Y, respectively. Then a function f :

$(X, F) \to (Y, G)$ is called **IF** *filter open* if for every $F \in F$, $f(F) \in G$ and f is called **IF** *filter homeomorphism* if it is both **IF** filter continuous **IF** filter open.

Theorem 3.18. If $f:(X,F) \to (Y,G)$ is injective **IF** filter open, then $C^{f(A)}(G) \subseteq C^{A}(F)$ for any **IFS** A in X.

The following corollary are immediate from Theorems 3.16 and 3.18.

Corollary 3.19. If (X,F) and (Y,G) are **IF** filter homeomorphic, then $C^{A}(F) = C^{f(A)}(G)$ for any **IFS** A in X.

Corollary 3.20. If $f:(X,F) \to (Y,G)$ is f-invariant **IF** filter continuous, then $c^{f(A)}(G) = c^{A}(F)$ for any **IFS** A in X.

4. Hausdorff IF filters

For two ordinary sets A and B, it is well-known that $A \cap B = \phi \Leftrightarrow A \subseteq B^c$.
This equivalent is no longer valid for fuzzy set. So Ramakrishnan and Nayagam [12] chose the notion of fuzzy disjointness that agrees with ordinary set theoretic disjointness in crisp case as follows: Two fuzzy set A and B in X are said to *intersect* if $\mu_A(x)+\mu_B(x)>1$ for some $x \in X$, and A and B are said to be *disjoint* if these sets do not intersect.

Now we extend these notions to **IFS**s as follows:

Definition 4.1. Two **IFS**s A and B in X are said to *intersect at* $x \in X$ if $\mu_A(x)+(1-\nu_B(x))>1$ or $\mu_B(x)+(1-\nu_A(x))>1$. Otherwise A and B do not intersect at x. A and B are said to be *disjoint* if these sets do not intersect anywhere.

Definition 4.2. An **IF** filter (X, F) is called *Hausdorff* if for any $x, y \in X$ with $x \neq y$, there exist $F_1, F_2 \in F$ such that $\nu_{F_1}(x) < \frac{1}{2}$, $\nu_{F_2}(y) < \frac{1}{2}$ and $\mu_{F_1}(z)+(1-\nu_{F_2}(z)) \le 1$ and $\mu_{F_2}(z)+(1-\nu_{F_1}(z)) \le 1$ for any $z \in X$.

Example 4.3. Let $X=\{a,b,c\}$ and B_i $(i=1,2,3,4)$ be **IFS**s in X defined as follows:

$$B_1 = \langle x, (\tfrac{a}{\alpha}, \tfrac{b}{\beta}, \tfrac{c}{\delta}), (\tfrac{a}{1-\alpha}, \tfrac{b}{1/4}, \tfrac{c}{1-\delta}) \rangle,$$
$$B_2 = \langle x, (\tfrac{a}{\beta}, \tfrac{b}{\delta}, \tfrac{c}{\alpha}), (\tfrac{a}{1-\beta}, \tfrac{b}{1-\delta}, \tfrac{c}{1-\alpha}) \rangle,$$
$$B_3 = \langle x, (\tfrac{a}{\alpha}, \tfrac{b}{\beta}, \tfrac{c}{\delta}), (\tfrac{a}{1/4}, \tfrac{b}{1-\beta}, \tfrac{c}{1-\delta}) \rangle,$$
$$B_4 = \langle x, (\tfrac{a}{\alpha}, \tfrac{b}{\beta}, \tfrac{c}{\delta}), (\tfrac{a}{1-\alpha}, \tfrac{b}{1-\beta}, \tfrac{c}{1/4}) \rangle,$$

where $\alpha, \beta, \delta \in (0,1/4)$. Let F be an **IF** filter generated by $B = \{B_1, B_2, B_3, B_4\}$. Then clearly (X, F) is a Hausdorff **IF** filter.

Recall that a sequence $\{x_n\}$ of fuzzy filter (X, F) is said to *converge filterly to* $x \in X$ if for every $F \in F$ such that $\mu_F(x) > \frac{1}{2}$, there exists $n_0 \in \mathbf{N}$ such that $\mu_F(x_n) > \frac{1}{2}$ for all $n \ge n_0$, equivalently $1-\nu_F(x_n) < \frac{1}{2}$ for all $n \ge n_0$.

Now we extend above definition to **IF** filter as follows:

Definition 4.4. Let (X, F) be an **IF** filter. A sequence $\{x_n\}$ of X is said to *converge* **IF** *filterly to* x (denoted by $\{x_n\} \to_{IF} x$), and x is called *an* **IF** *limit* of F, if for every $F \in F$ such that $\nu_F(x) < \frac{1}{2}$, there exists $n_0 \in \mathbf{N}$ such that $1-\nu_{F^c}(x_n) < \frac{1}{2}$ for all $n \ge n_0$, equivalently $\mu_F(x_n) > \frac{1}{2}$ for all $n \ge n_0$.

Theorem 4.5. Let $f:(X,F) \to (Y,G)$ be an **IF** continuous function and $\{x_n\}$ be a sequence in X. If $\{x_n\} \to_{IF} x$, then $\{f(x_n)\} \to_{IF} f(x)$.

Theorem 4.6. In Hausdorff **IF** filter (X, F), every **IF** filterly convergent sequence of points of X has exactly one **IF** limit.

Theorem 4.7. Let (X, F) be an Hausdorff **IF** filter and $Y \subseteq X$. If $A|Y \neq 0_{\sim}$ for any $A \in F$, then $(Y, F|Y)$ is also Hausdorff **IF** filter.

Theorem 4.8. Let $f:(X, F) \to (Y, G)$ be a bijective **IF** filter open function. If (X,F) is Hausdorff **IF** filter, then $(Y, \boldsymbol{G})$ is a Hausdorff **IF** filter.

Lemma 4.9. Let $f:(X, \boldsymbol{F}) \rightarrow Y$ be a surjection. Then $\boldsymbol{G}=\{G \in ([0,1] \times [0,1])^X \mid f^{-1}(G) \in \boldsymbol{F}\}$ is an **IF** filter on Y.

Definition 4.10. The **IF** filter defined in above lemma is called *Quotient* **IF** *filter determined by the surjective function f.*

Suppose $(X, \boldsymbol{F})$ and $(Y, \boldsymbol{H})$ are **IF** filters and $f:(X, \boldsymbol{F}) \rightarrow (Y, \boldsymbol{H})$ is surjection. The following theorem gives conditions on f that make $\boldsymbol{H}$ equal to the quotient **IF** filter $\boldsymbol{G}$ on Y determined by f.

Theorem 4.11. Let $(X,\boldsymbol{F})$ and $(Y,\boldsymbol{H})$ be **IF** filters, $f:(X, \boldsymbol{F}) \rightarrow (Y, \boldsymbol{H})$ be a surjective **IF** filter continuous function and let $\boldsymbol{G}$ be the quotient **IF** filter on Y determined by f. If f is **IF** filter open, then $\boldsymbol{G}=\boldsymbol{H}$.

Theorem 4.12. Let $f:(X, \boldsymbol{F}) \rightarrow Y$ be bijective function and $\boldsymbol{G}$ be the quotient **IF** filter on Y determined by f. If $(X, \boldsymbol{F})$ is a Hausdorff **IF** filter, then $(Y, \boldsymbol{G})$ is a Hausdorff **IF** filter.

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