A Study on the Color Membership Computation Method using Fuzzy Color Model

Dae-Won Kim[0] Kwang. H. Lee
Department of Computer Science, AITrc, KAIST
(dwkim, khlee)@if.kaist.ac.kr

퍼지 컬러 모델을 이용한 컬러의 소속 정도를 결정하는 방법에 관한 연구

김대원[0] 이광형
한국과학기술원 전산학과, 첨단정보기술연구센터

Abstract

In this paper we focused on the color representation problem based on fuzzy set theory. The main factor is the determination or computation of color membership function and color difference formula. The mathematical formula to calculate the color difference should generate a uniform color scaling, and due to this reason we adopted a CIELAB color space as a fundamental feature space. With the help of the CIELAB color space we created a new color model, referred to fuzzy color model, which can represent the ambiguous characteristics underlying colors. Based on the proposed color difference formula between fuzzy colors, we could obtain the membership computation method of an arbitrary color for a given color family.

1. Introduction

Color plays an important role in our daily lives. Most people have to deal with color and color problems occasionally, and people who work professionally with color must deal with them everyday. There are many strange things about color and color vision that most people do not notice. Even though color seems intuitive and simple it is not. It involves some of the most complicated things on the earth [1][2][3].

There have been lots of researches devoted to modeling the color system. RGB color space is the most simple and familiar one, and due to the basic and well-known properties it's used for most of color applications. Munsell color system is the most intuitive and useful to the artists and designers. It's developed to describe appropriate color naming scheme and provide a systematic way of color ordering. And from the late 1920s there has been an effort to improve visual evaluations of color and to develop instrumental methods for specifying and measuring color. To meet this demand, an international standard commission CIE was established, which defined many new color concept and uniform color spaces including CIELUV color space and CIELAB color space.

In this paper, we focused on the color representation problem based on fuzzy set theory. The main factor is the determination or computation of color membership function and color difference formula. The mathematical formula to calculate the color difference should generate a uniform color scaling, and for this reason we adopted a CIELAB color space as a fundamental feature space. With the help of the CIELAB color space we created a new color model, referred to fuzzy color model, which represent the ambiguous characteristics underlying colors. Besides the basic color distance in the CIELAB space, we defined a new color distance measure between fuzzy color families that is crucial to the determination of color membership function.

2. Color Representation

We recognize, exploit and get information from various color objects. We don't exactly know the color perception mechanism in our brain, but it works very well. The focus of this paper deals with how to model and represent color in computer as human beings do. In computer science, the color concept has been handled as a crisp value for a long time, but actually it is inherently ambiguous and fuzzy. And the crisp representation fails under certain situations according to the chosen color space.

Conventional color spaces including RGB, CMY, and HSV space do not coincident with human perception, and do not represent the color ambiguity. And these have non-uniform color difference scaling [4]. In addition to the above models, CIE standard community proposed lots of uniform color spaces, for example, CIELUV and CIELAB spaces, which are used as industrial color standards. The CIELAB is based on non-linear transformation of Munsell color space. They, however, do not represent the color ambiguity as well.

3. Fuzzy Color Model

3.1 Proposed approach

To create a color model we considered the following necessary properties: (1) proposed model must provide human perception-based color distance metric (2) proposed model must be uniform (3) one-point tuple description can't explain the exact color representation.

In order to satisfy the property 1 and 2, we take a CIELAB color space as a basic color system. The CIELAB color space is developed to provide the color difference formula like human perception distance with Munsell color description, and more, it is uniform space and covers the whole color gamut. To fulfill property 3, another color description or representation scheme must be developed. Traditional three components description, e.g., (R,G,B) or (L,a,b), is not enough. So we solve the problem based on fuzzy set theory. With fuzzy theory, color is described as a membership degree for a specific color. To do this, we proposed a fuzzy color model, its distance measure, and membership computation method. In addition to this fact, color modeling with fuzzy approach will give a more clear solution if we would think the inherent ambiguity and fuzziness of colors.

3.2 Relative fuzzy color model

The fuzzy color is described with three-dimensional ball-typed representation. To describe color concept in three-dimensional color space, sphere or ball-shaped model is preferred. When we look at a color or colored object, for example red color, some pair of red colors are difficult to distinguish, and beyond a certain boundary we can easily distinguish color pair.

To formulate this model, two numerical values should be specified: center value and JND value. (1) *center* is the center point of fuzzy color ball which is calculated from CIELAB color space and Munsell color wheel. We create thirteen fuzzy colors with Munsell color wheel and CIELAB color space. Ten colors from the major color family in Munsell color wheel, and three colors from L-axis including white, black, and gray color. To define color, the first thing to do is to assign the center value for a given color. The fuzzy colors are selected from major color family in Munsell color wheel: 5R, 5YR, ..., 5RP. So ten fuzzy colors are created and centers of those are computed from the LAB value which corresponds to the strongest color in major color family. (2) *JND (just noticeable distance)* value is used to calculate the relative color distance between fuzzy colors. Figure 1 depicts the fuzzy color ball and its membership function. In the proposed model, if a color point x is within JND distance it strongly belongs to that color and has membership degree 1.0. If color x is out of the range of JND, the membership degree is computed relatively by comparing with neighbor colors. The left and right shape of fuzzy membership function is determined based on distance result between fuzzy colors.

In this paper we should clearly distinguish three basic terminologies. (1) *color point* (or *color element*) is a tuple that describes basic color coordinates, e.g. (R,G,B) or (L,a,b) (2) fuzzy color family is a color grouping with similar color points, e.g. red family and blue family (3) for simplicity, we assume that color coordinates *Lab* notation is equal to $L^*a^*b^*$ notation.

In figure 1, there are fuzzy color C_i and color point x. The fuzzy color has its own center (ctr_i) and JND (jnd_i) values which build three dimensional ball-shaped representation. As mentioned earlier, if x is within the JND value of color C_i, it belongs to

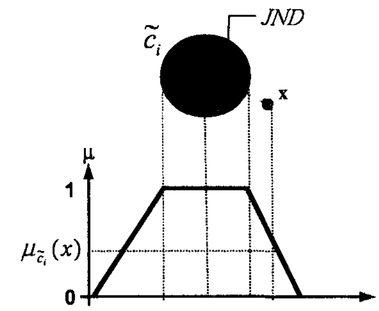

Figure 1. Relative fuzzy color model

color family C_i with membership degree 1.0. For color point x which is not in the area of color ball, the membership value is determined based on relative distance between neighbor fuzzy colors. This is discussed in later section. With this manner, we could process the color family identification for an arbitrary color point and compute the color membership degree for a given color family.

3.3 Distance measures of fuzzy color model

To effectively address the definition of fuzzy color ball and its membership computation method, we should describe two color distances: (1) distance between color points and (2) distance between fuzzy color family and color point.

3.3.1 Distance between color points

To successfully formulate the distance between fuzzy colors, a basic color distance measure must be established between color points.

Definition 1. *Let two color points be x and y, then the distance (ρ) between two color points is defined as*

$$\rho(x,y)=\sqrt{(x_L-y_L)^2+(x_a-y_a)^2+(x_b-y_b)^2}$$

where $x=(x_L,x_a,x_b)$ *and* $y=(y_L,y_a,y_b)$.

The above definition is trivially obtained from CIELAB color difference formula.

3.3.2 Distance between color family and color point

One of major concerns is to compute the distance between fuzzy color family and an arbitrary color element. With the distance measure between color points, we can define the distance between fuzzy color family and color point.

Definition 2. *Let fuzzy color family and color point be denoted by* C_i *and* x *respectively. Then the distance (δ) between fuzzy color family and color point is defined as*

$$\delta(C_i,x)=\begin{cases}\|\rho(C_i,x)\|-jnd_i\\ \sqrt{(ctr_L^i-x_L)^2+(ctr_a^i-x_a)^2+(ctr_b^i-x_b)^2}-jnd_i\end{cases}$$

where ctr^i *and* jnd_i *are the center and JND value of* C_i.

As can be seen in the above definition, the distance measure considers not only the center point but also the JND value.

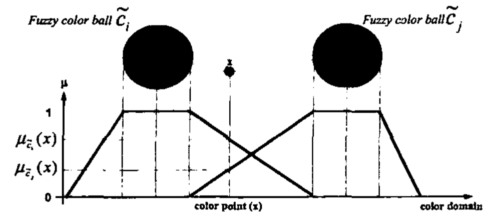

Figure 2. Color membership computation diagram

With the JND value, we can reflect the color's specific characteristics. Some colors have larger JND values, and another colors have smaller ones. So the adoption of JND values provides a more clear distance computation result.

4. Color Membership Computation Method

4.1 Definition of membership function

With the distance calculation method defined in the above section, we can establish a relative fuzzy color model.

Definition 3. *Let define $C_i = < ctr^i, jnd_i >$ as a three-dimensional fuzzy ball set with a membership function*

$$\mu_{\tilde{c}_i}(x) = \begin{cases} 1.0 & \text{if } \delta(\tilde{c}_i, x) \le JND \\ 0.0 & \text{if } \delta(\tilde{c}_j, x) \le JND, i \ne j \\ \left[\sum_{j=1}^{|\tilde{c}|} \frac{\delta(\tilde{c}_i, x)}{\delta(\tilde{c}_j, x)}\right]^{-1} & \text{otherwise} \end{cases}$$

where δ-function means the distance between fuzzy color family and color point

To compute a membership degree to a specific fuzzy color, we compute all distances between fuzzy colors. If a color x strongly belongs to a given color C_i, it is classified to that color with a membership degree 1.0. In another case, if the color x is within other color C_j, then it means color x has no relation to the fuzzy color family C_i, so the membership degree is 0.0. Except the above two cases, the color x is located in the middle of fuzzy color families. We compute the relative distance between color families and determine the membership value. And the important point is that the sum of membership values to the whole fuzzy color families must be equal to 1.0. With this constraint we can easily extend this concept to fuzzy cluster analysis which is one of the application field of fuzzy color model. Figure 2 depicts the membership computation situation between two colors.

The properties of the proposed fuzzy color model can be summarized as follows: (1) $\mu_i(x) = 1.0$ means that color point x is absolutely in the fuzzy color family C_i (2) $\mu_i(x) = 0.0$ means that color point x is absolutely in the fuzzy color family C_j $(i \ne j)$ (3) $0.0 < \mu_i(x) < 1.0$ means that color point x is partially related to fuzzy color families (4) Fuzzy color family C_i is the closest fuzzy color family to color point x s.t. $i = max_i(\mu_i(x))$ (5) The sum of all membership values of color point x to whole fuzzy color families should be 1.0.

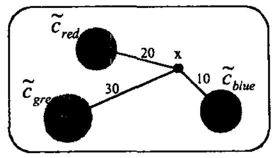

Figure 3. Numerical example of membership computation

4.2 Numerical example

In this section, we show a numerical example of membership computation. In figure 3, color element x doesn't belong to any fuzzy color families, and the distance to fuzzy color families is given: 20 to C_{red}, 30 to C_{green}, and 10 to C_{blue}. In this case, we should first compute the relative strengths between fuzzy colors, and based on the strength we compute the relative color membership values. The result memberships are calculated as:

$$\mu_{C_{red}}(x) = \left(\frac{\delta(C_{red}, x)}{\delta(C_{red}, x)} + \frac{\delta(C_{red}, x)}{\delta(C_{green}, x)} + \frac{\delta(C_{red}, x)}{\delta(C_{blue}, x)}\right)^{-1} = 0.27$$

$$\mu_{C_{green}}(x) = \left(\frac{\delta(C_{green}, x)}{\delta(C_{red}, x)} + \frac{\delta(C_{green}, x)}{\delta(C_{green}, x)} + \frac{\delta(C_{green}, x)}{\delta(C_{blue}, x)}\right)^{-1} = 0.18$$

$$\mu_{C_{blue}}(x) = \left(\frac{\delta(C_{blue}, x)}{\delta(C_{red}, x)} + \frac{\delta(C_{blue}, x)}{\delta(C_{green}, x)} + \frac{\delta(C_{blue}, x)}{\delta(C_{blue}, x)}\right)^{-1} = 0.55$$

5. Conclusion

In this paper we addressed the color representation method using fuzzy set theory. We proposed the relative fuzzy color model, and its attributes including color center and JND value. The key point of color modeling is to define the membership assignment of each color element. In order to accomplish this computation, we developed two color distance measures. Using the distance measures, we defined color membership function and showed a simple numerical example. As a further work we plan to apply these color model to color data clustering with fuzzy clustering algorithm.

Acknowledgement

This work was supported by the Korea Science and Engineering Foundation (KOSEF) through the Advanced Information Technology Research Center(AITrc).

6. References

[1] J.T Luke, The Munsell Color System: A Language For Color, Fairchild Publications, New York, 1996.

[2] P. Zelanski and M.P. Fisher, Color, Calmann and King Ltd, 1998.

[3] G. Wyszeck and W.S. Stiles, Color Science: Concepts and Methods: Quantitative Data and Formulae, Wisley-Interscience Publication, New York, 2000.

[4] D. W. Kim, K. H. Lee, "A fuzzy color modeling based on Munsell color system", 2^{nd} *International symposium on advanced intelligent systems*, 2001.

[5] G. Sharma and H.J. Trussell, Digital Color Imaging, *IEEE Trans. on Image Processing*, 6(7), 901-932, 1997.