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특이섭동 쌍일차 이산시 시스템에서의 안정화 제어에 관한 연구

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Stabilizing controller for singularly perturbed discrete time systems

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Abstract - In this paper, we present a stabilizing controller for the singularly perturbed discrete time bilinear systems. The proposed control method guarantees the robust stability for the resulting closed loop system with multi-input. We verify the proposed algorithm by a numerical example.

1. Introduction

Bilinear system is linear in control and linear in state but not jointly linear in state and control. It is important to understand its real properties or to guarantee the global stability or to improve the performance by applying the various control techniques to bilinear system rather than its linearized system

Singular perturbation theory and control techniques to solve the singularly perturbed systems have received much attention by many researchers. Recently, an excellent survey of the applications of the theory and control methods of singular perturbation and time scales and the importance features of the singularly perturbed systems have been reported in [1]. Also very efficient and high accurate optimal control methods for both continuous time and discrete time singularly perturbed linear systems are found in a recent book[2].

The stabilization problems for the bilinear systems have been widely studied in the past by many researchers, see for examples[3][4][5]. However, many researches are devoted to the continuous time bilinear systems. In the case of discrete time bilinear systems, a few research are reported. Recently, an approach to design the robust stabilizing nonlinear state feedback controller for the singularly perturbed discrete bilinear systems with single input was developed in [3].

This work is to extend the results of [3] and [6] to singularly perturbed discrete time multi-input bilinear systems.

2. Main result

2.1 Singularly perturbed discrete bilinear systems

We consider the singularly perturbed discrete bilinear systems described by

$$\begin{bmatrix} x_1(k+1) \\ \frac{1}{\varepsilon}x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}u(k) + \sum_{i=1}^{m} u_i \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{1}$$

where $x_1 \in R^{n_1}$ is a slow state vector, $x_2 \in R^{n_2}$ is a fast state vector, $u \in R^m$ is an input vector, ε is a small positive parameter, and A_{ij}, N_{ij}, B_i with $i,j=1,2$are constant matrices having appropriate dimension.

Let $z^T=[x_1^T \ x_2^T]$and $A=A_s+\varepsilon A_f$where

$$A_s=\begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}, \quad A_f=\begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}$$

$$B=\begin{bmatrix} B_1 \\ \varepsilon B_2 \end{bmatrix}, \quad M_i=\begin{bmatrix} M_{11}^i & M_{12}^i \\ \varepsilon M_{21}^i & \varepsilon M_{22}^i \end{bmatrix}$$

Then we can rewrite (1) as

$$z(k+1)=Az(k)+\sum_{i=1}^{n} z_i M_i u(k)+Bu(k) \tag{2}$$

The following Lyapunov function candidate is selected to derive the stabilizing control law.

$$V(z)=z^TPz \tag{3}$$

where P is a unique real symmetric positive-definite matrix satisfying the following discrete Lyapunov equation

$$(1+\gamma)^2 A_s^T P A_s - P = I_n \tag{4}$$

γ is a positive constant satisfying

$$\sqrt{(1+\gamma)}\,r(A)<1 \tag{5}$$

and $r(A)$ is a spectral radius of A.

Then the following theorem in [3] states that one can obtain the stabilizing control law for the singularly perturbed discrete time single-input bilinear systems by choosing C and x.

Theorem 1. *Consider singularly perturbed discrete time single-input bilinear systems*

$$z(k+1)=Az(k)+u(k)Nz(k)+Bu(k) \tag{6}$$

with the singular perturbation parameter ε satisfying the inequality

$$\varepsilon < \varepsilon^* = \left(\frac{\gamma}{(1+\gamma)^2 \| A_f^T P A_f \|} \right)^{\frac{1}{2}} \tag{7}$$

Then the following nonlinear state feedback control law globally and asymptotically stabilizes the equilibrium point of (6)

$$u(k) = \frac{-xCz(k)}{\sqrt{1+z^T(k)C^TCz(k)}} \tag{8}$$

where $C \in R^{1\times n}$ can be arbitrarily designed, γ is obtained from (4), and x satisfies the following inequality

$$x < x^* = \sqrt{a}\min\left(\left(\frac{\gamma}{(1+\gamma)^2 \|C^TB^TPBC\|} \right)^{\frac{1}{2}}, \left(\frac{\gamma^2}{(1+\gamma)^2 \|M^TPM\|} \right)^{\frac{1}{2}} \right) \tag{9}$$

where

$$\alpha = 1 - \frac{(1+\gamma)^2\varepsilon^2}{\gamma}\|A_f^TPA_f\| > 0 \qquad (10)$$

The proof of Theorem 1 can be found in [3]. extend Theorem 1 to the multi-input cases usin results of [6].

Theorem 2. *Consider singularly perturbed dis time multi-input bilinear systems (2) with a si perturbation parameter ε satisfying the inequali*

$$\varepsilon < \varepsilon^* = \left(\frac{\gamma}{(1+\gamma)^2\|A_f^TPA_f\|}\right)^{\frac{1}{2}} \qquad (11)$$

and arbitrarily designed $C \in R^{m\times n}$. Then the following nonlinear state feedback control law globally and asymptotically stabilizes the equilibrium point of (2).

$$u(k) = \frac{-\chi Cz(k)}{\sqrt{1+z^T(k)C^TCz(k)}} \qquad (12)$$

where γ is obtained through (5) and χ satisfie following inequality

$$\chi = \chi^* = \frac{\sqrt{\alpha}}{\sqrt{p+1}}\min\left(\left(\frac{\gamma}{(1+\gamma)^2\|C^TB^TPBC\|}\right)^{\frac{1}{2}}, \left(\frac{\gamma^2}{(1+\gamma)^3\|N_1\|^2\|P\|}\right)^{\frac{1}{2}}, \cdots, \left(\frac{\gamma^{n+1}}{(1+\gamma)^{n+1}\|N_n\|^2\|P\|}\right)^{\frac{1}{2}}\right) \qquad (13)$$

where $1 \le p \le n$ is a number of nonzero N_i matrices and α satisfies

$$\alpha = 1 - \frac{(1+\gamma)^2\varepsilon^2}{\gamma}\|A_f^TPA_f\| > 0 \qquad (14)$$

The following Lemmas are needed to prove the Theorem 2.

Lemma 1. *Consider the matrices A, B, and C, which have the same dimensions, and let C = A + B. For any positive constant γ and positive definite symmetric matrix D, the following relation holds:*

$$CDC^T \le (1+\gamma)ADA^T + (1+\gamma^{-1})BDB^T \qquad (15)$$

The proof of this lemma follows from [7].

Lemma 2. *Consider a vector $z \in R^{n\times 1}$ and mat $M_i \in R^{n\times m}$ and $N_i \in R^{n\times m}$ with $i = 1,2,\cdots,n$ Th the following relation holds:*

$$\sum_{i=1}^{n} z_iM_i = \sum_{i=1}^{n} Z_iN_i \qquad (16)$$

with unique N_i as follows:

$$N_i = \begin{bmatrix} n_{i1}^1 & n_{i2}^1 & \cdots & n_{im}^1 \\ n_{i1}^2 & n_{i2}^2 & \cdots & n_{im}^2 \\ & \vdots & & \\ n_{i1}^n & n_{i2}^n & \cdots & n_{im}^n \end{bmatrix} \qquad (17)$$

where n_{ij}^k of N_i is an (i,j)-th element of M_k matrix with $j=1,2,\cdots,m$ $k=1,2,\cdots,n$ And the i-th row of $Z_i \in R^{n\times n}$, $i=1,\cdots,n$ is z^T and the other rows of Z_i are zero row vectors.

The above lemma leads to the following result.

Corollary 1. *The norms of the equation (16) ha following relation.*

$$\|\sum_{i=1}^{n} z_iM_i\| \le \|z\|\sum_{i=1}^{n}\|N_i\| \qquad (18)$$

The proof of Lemma 2 and Corollary 1 can be found in [6].

In the following we prove the Theorem 2. By substituting the control law defined in (12) into (2), we can obtain the following closed-loop systems.

$$z(k+1) = \left(A_s + \varepsilon A_f - B\frac{\chi C}{\sqrt{1+z^T(k)C^TCz(k)}} - \{zM\}\frac{\chi C}{\sqrt{1+z^T(k)C^TCz(k)}}\right)z(k) \qquad (19)$$

Using the Lyapunov function defined in (3), the forward Lyapunov function can be written by substituting (19) and (2) into (3)

$$V(z(k+1)) = z^T(k)\eta^TP\eta z(k) \qquad (20)$$

where

$$\tau = \frac{1}{\sqrt{1+z^T(k)C^TCz(k)}} \qquad (21)$$

$$\eta = A_s + \varepsilon A_f - \tau\chi BC - \tau\chi\{zM\}C \qquad (22)$$

Using Lemma 1, the Lyapunov forward differenc given by

$$\begin{aligned} \Delta V = {} & z^T(k)(A_s + \varepsilon A_f - \tau\chi BC - \tau\chi\{zM\}C)^T \\ & P(A_s + \varepsilon A_f - \tau\chi BC - \tau\chi\{zM\}C)z(k) \\ & - z^T(k)Pz(k) \\ \le {} & z^T(k)\Big((1+\gamma)^2A_s^TPA_s + \frac{(1+\gamma)^2\varepsilon^2}{\gamma}A_f^TPA_f \\ & - P + (1+\gamma^{-1})\tau^2\chi^2((1+\gamma)C^TB^TPBC \\ & + (1+\gamma^{-1})C^T\{zM\}^TP\{zM\}C)\Big)z(k) \end{aligned} \qquad (24)$$

where $\{zM\} = \sum_{i=1}^{n} z_iM_i$. Using Lemma 1, 2 the term $C^T\{zM\}^TP\{zM\}C$ of (24) can be rewritten as

$$\begin{aligned} & C^T\left(\sum_{i=1}^{n} z_i(k)M_i\right)^T P\left(\sum_{i=1}^{n} z_i(k)M_i\right)C \\ = {} & C^T\left(\sum_{i=1}^{n} Z_i(k)N_i\right)^T P\left(\sum_{i=1}^{n} Z_i(k)N_i\right)C \\ \le {} & \sum_{i=1}^{n-1}\frac{(1+\gamma)^i}{\gamma^{i-1}}C^TN_i^TZ_i^TPZ_iN_iC \\ & + \frac{(1+\gamma)^{n-1}}{\gamma^{n-1}}C^TN_n^TZ_n^TPZ_nN_nC \end{aligned} \qquad (25)$$

Thus the inequality (24) becomes

$$\begin{aligned} \Delta V \le {} & z^T(k)\Big((1+\gamma)A_s^TPA_s + \frac{(1+\gamma)^2\varepsilon^2}{\gamma}A_f^TPA_f \\ & - P + \frac{(1+\gamma)^2\chi^2}{\gamma}\tau^2C^TB^TPBC \\ & + \frac{(1+\gamma)^2\chi^2}{\gamma^2}\tau^2\Big(\sum_{i=1}^{n-1}\frac{(1+\gamma)^i}{\gamma^{i-1}}C^TN_i^TZ_i^TPZ_iN_iC \\ & + \frac{(1+\gamma)^{n-1}}{\gamma^{n-1}}C^TN_n^TZ_n^TPZ_nN_nC\Big)\Big)z(k) \end{aligned}$$

Since $0 < \tau \le 1$, the above inequality can be rewritten by using Corollary 1.

$$\begin{aligned} \Delta V \le {} & z^T(k)\Big(-I_n + \frac{(1+\gamma)^2\varepsilon^2}{\gamma}\|A_f^TPA_f\| \\ & + \frac{(1+\gamma)^2\chi^2}{\gamma}\|C^TB^TPBC\| \\ & + \frac{(1+\gamma)^2\chi^2}{\gamma^2}\frac{1}{1+\|z^T(k)\|\|C^T\|\|C\|\|z(k)\|} \\ & \times\Big(\sum_{i=1}^{n-1}\frac{(1+\gamma)^i}{\gamma^{i-1}}N_a + \frac{(1+\gamma)^{n-1}}{\gamma^{n-1}}N_b\Big)\Big)z(k) \end{aligned} \qquad (26)$$

where

$$\begin{aligned} N_a &= \|C^T\|\|N_i^T\|\|Z_i^T\|\|P\|\|Z_i\|\|N_i\|\|C\| \\ N_b &= \|C^T\|\|N_n^T\|\|Z_n^T\|\|P\|\|Z_n\|\|N_n\|\|C\| \end{aligned} \qquad (27)$$

Since the following relation is always satisfic any z and C

$$\frac{\|C^T\|\|z^T(k)\|\|z(k)\|\|C\|}{1+\|z^T(k)\|\|C^T\|\|C\|\|z(k)\|}\le 1 \qquad (28)$$

we can rewrite (26) as follows:

$$\begin{aligned}\Delta V \le\ & z^T(k)\Big(-I_n+\frac{(1+\gamma)^2\varepsilon^2}{\gamma}\|A_f^TPA_f\| \\ & +\frac{(1+\gamma)^2x^2}{\gamma}\|C^TB^TPBC\| \\ & +\frac{(1+\gamma)^2x^2}{\gamma^2}\Big(\sum_{i=1}^{n-1}\frac{(1+\gamma)^i}{\gamma^{i-1}}\|N_i^T\|\|P\|\|N_i\| \\ & +\frac{(1+\gamma)^{n-1}}{\gamma^{n-1}}\|N_n^T\|\|P\|\|N_n\|\Big)\Big)z(k)\end{aligned} \qquad (29)$$

$$\begin{aligned}\le\ & z^T(k)\Big(-\alpha I_n+(n+1)\times\max\Big(\frac{(1+\gamma)^2}{\gamma}\|C^TB^TPBC\|, \\ & \frac{(1+\gamma)^3}{\gamma^2}\|N_1\|^2\|P\|,\frac{(1+\gamma)^4}{\gamma^3}\|N_2\|^2\|P\|, \\ & \cdots,\frac{(1+\gamma)^{n+1}}{\gamma^{n+1}}\|N_n\|^2\|P\|\Big)x^2I_n\Big)z(k)\end{aligned}$$

Thus the right hand side of (29) is negative if and only if

$$\begin{aligned}\max\Big(&\frac{(1+\gamma)^2}{\gamma}\|C^TB^TPBC\|, \\ &\frac{(1+\gamma)^3}{\gamma^2}\|N_1\|^2\|P\|, \\ &\cdots,\frac{(1+\gamma)^{n+1}}{\gamma^{n+1}}\|N_n\|^2\|P\|\Big)x^2<\frac{\alpha}{n+1}\end{aligned} \qquad (30)$$

Since N_i matrix obtained by Lemma 2 may be zero matrix, zero N_i matrices must be excluded in choosing the control law to prevent the denominator of any element included in the parenthesis of (30) from being zero.

2.2 A numerical example

The proposed control method is applied to discrete time multi-input bilinear systems described by

$$z(k+1)=Az(k)+\sum_{i=1}^{n}z_iM_iu(k)+Bu(k)$$

where

$$A=\begin{bmatrix}-0.6 & 1\\ -0.005 & 00.23\end{bmatrix},\ B=\begin{bmatrix}0.5 & 3\\ 0.01 & 0.025\end{bmatrix}$$

$$M_1=\begin{bmatrix}-2 & 1\\ -0.02 & 0\end{bmatrix},\ M_2=\begin{bmatrix}1 & 2\\ 0 & 0.03\end{bmatrix},\ C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\varepsilon=0.01$$

with the initial condition $x(0)=[1.0\ 1.5]^T$The simulation results are presented in Figure 1, 2 that show the trajectories of the state and input, respectively. These figures show that all states asymptotically converge to the equilibrium point.

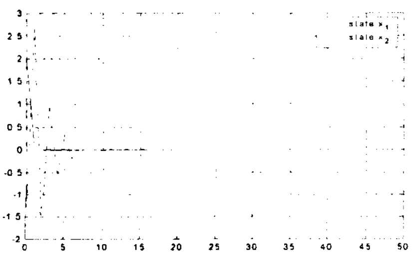

Fig. 1 State trajectories

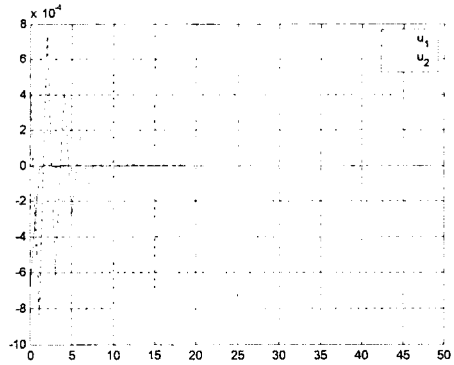

Fig. 2 Input trajectories

3. Conclusion

In this paper we have presented a robust stabilizing controller for singularly perturbed multi input discrete time bilinear systems by using the Lyapunov method. The resulting control method globally and robustly stabilizes the discrete time multi-input singularly perturbed bilinear systems.

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