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Analysis of Adiabatic Shearbands via High Resolution Scheme

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고분해능 스킴을 이용한 단열 전단띠 해석

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Key Words : Shearband(전단띠), high resolution scheme(고분해능 스킴)

Abstract

Development of adiabatic shear bands in thermoviscoplastic materials is analyzed via high resolution scheme. Presented here are our initial results, which are for one dimensional elasto-viscoplastic materials. As the mesh-sizes are getting small, the convergence result of plastic strain rate is obtained using elasto-viscoplastic materials. The further study cases will be reported at the presentation in the framework of the one and the two dimensional shearbanding, respectively. They will be compared with finite element solutions, and the advantage of the scheme will be discussed.

Nomenclature

σ : stress

x : coordinate

t : time

ρ : density

v : velocity

c : specific heat

θ : temperature

k : conductivity

$\dot{\varepsilon}^{vp}$: viscoplastic strain rate

μ : shear modulus

L : length of specimen

β : softening coefficient

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1. Introduction

Adiabatic shear banding is a major damage mechanism in ductile materials undergoing rapid deformation subjected to impact loading. This phenomenon is explained by thermal softening and the lack of time to diffuse away the heat caused by plastic or viscoplastic deformation. In the course of abrupt dynamic deformations, the thermal softening in a local region becomes greater than the hardening, and instability then gives rise to initiation of shear band. As the critical time is approached, the increasingly localized narrow band leads to a sudden drop of the stress sustained by the material, which is called stress collapse. Then the material outside the band undergoes elastic unloading and plastic deformation is localized on the band[1,2,4]. The width of this shear band is often extremely small compared with the body scale, and then analysis of shear band development manifests itself as a typical multi-scale problem, which involves two different scales of the shear band thickness and the body dimension under consideration. As a consequence, numerical computation of shear band development via finite element [4] or meshfree methods [3] is very challenging, particularly in terms of capturing the high strain gradient and the thickness of the band. This is due to the limitation of resolution capacity of such methodology and to numerical instability. In this work, we apply the high resolution scheme for analyzing adiabatic shearbanding, and ultimately we will compare the solutions with those from FEM to examine the effectiveness of the scheme in terms of accuracy and solution time. Preliminary results are described for one dimensional rigid viscoplastic material in this paper, and further results on elastic-viscoplastic materials will be reported at the Conference together with pertinent remarks on the overall efficacy of the high resolution scheme applied to the shear band problems.

2. Governing Equation

Consider the one-dimensional shearing of infinitely long plane strain block, as depicted in Fig. 1. We assume that the top and the bottom of the block are subjected to opposite uniform horizontal velocities such that it undergoes shear deformation. Elasto-viscoplastic material behavior is assumed and the material properties are taken to be constant for simplicity. Then the equation of motion and the energy equation for deformations of thermoviscoplastic materials are written as

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} \tag{1}$$

$$\rho c \frac{d\theta}{dt} = \frac{\partial}{\partial x}\left(k \frac{\partial \theta}{\partial x}\right) + \sigma \cdot \dot{\varepsilon}^{vp} \tag{2}$$

$$\frac{\partial \sigma}{\partial t} - \mu \frac{\partial v}{\partial x} = -2\mu \cdot \dot{\varepsilon}^{vp} \tag{3}$$

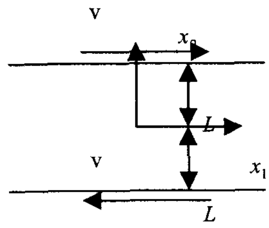

Fig. 1 One-dimensional shear band problem

For application of high resolution scheme, these equations are recast into the following conservative form:

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho v \\ \rho c \theta \\ \sigma \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} -\sigma \\ 0 \\ -\mu v \end{bmatrix} = \frac{\partial}{\partial x}\begin{bmatrix} 0 \\ k\theta_x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma \cdot \dot{\varepsilon}^{vp} \\ -2\mu \cdot \dot{\varepsilon}^{vp} \end{bmatrix} \tag{4}$$

For the high resolution scheme to be employed, we choose the central scheme proposed by Kurganov and Tadmor[5]. The major advantage of the central schemes is that they do not need any Riemann solver unlike the Godunov type upwinding approaches. Kurganov and Tadmor's central scheme [5] is the recent modification of Nessyahu-Tadmor scheme[6], which may be thought of as the extension of the first order Lax-Friedrichs scheme[7]. One of its main ingredients is that it admits a particularly simple semi-discrete formulation, which can be numerically implemented in a straightforward manner with the aid of the ODE integrators like Runge-Kutta methods. Furthermore, no characteristic information is needed for implementation other than the local wave speed, which is given as nothing but the elastic wave speed for an elastic-viscoplastic material.

Note that equation (4) now takes the standard convection-diffusion type of the form:

$$\frac{\partial}{\partial t}\mathbf{u}(x,t)+\frac{\partial}{\partial x}\mathbf{f}(\mathbf{u}(x,t))= \frac{\partial}{\partial x}\mathbf{Q}(\mathbf{u}(x,t),\mathbf{u}_x(x,t))+\mathbf{R}(\mathbf{u}(x,t)) \quad (5)$$

where,

$$\mathbf{u}=\begin{bmatrix}\rho v\\ \rho c\theta\\ \sigma\end{bmatrix},\ \mathbf{f}=\begin{bmatrix}-\sigma\\ 0\\ -\mu v\end{bmatrix},\ \mathbf{Q}=\begin{bmatrix}0\\ k\theta_x\\ 0\end{bmatrix},\ \mathbf{R}=\begin{bmatrix}0\\ \sigma\cdot\dot{\varepsilon}^{vp}\\ -2\mu\cdot\dot{\varepsilon}^{vp}\end{bmatrix}$$

For elasto-thermoviscoplastic materials, the flow stress σ depends on the strain hardening and the strain rate hardening, and temperature. We neglect the strain hardening for simplicity and take the following form,

$$\sigma=\frac{1}{\sqrt{3}}\left(\frac{\dot{\varepsilon}^{vp}}{\dot{\varepsilon}_0^{p}}\right)^m\sigma_0(1-\beta(\theta-\theta_0)) \quad (6)$$

which depicts the power-law rate hardening and a linear thermal softening.

For this type of equation, the semi-discrete form of Kurganov-Tadmor scheme is written as (see [5] for details)

$$\frac{d}{dt}\mathbf{u}_j(t)= -\frac{\mathbf{H}_{j+1/2}(t)-\mathbf{H}_{j-1/2}(t)}{\Delta x}+\frac{\mathbf{P}_{j+1/2}(t)-\mathbf{P}_{j-1/2}(t)}{\Delta x}+\mathbf{R}_j(t) \quad (7)$$

where

$$\mathbf{H}_{j+1/2}(t)=\frac{\mathbf{f}(\mathbf{u}^+_{j+1/2}(t))+\mathbf{f}(\mathbf{u}^-_{j+1/2}(t))}{2}-\frac{\mathbf{a}_{j+1/2}(t)}{2}\left[\mathbf{u}^+_{j+1/2}(t)-\mathbf{u}^-_{j+1/2}(t)\right]$$

$$\mathbf{P}_{j+1/2}(t)=\frac{1}{2}\left(\mathbf{Q}\left(\mathbf{u}_j(t),\frac{\mathbf{u}_{j+1}(t)-\mathbf{u}_j(t)}{\Delta x}\right)+\mathbf{Q}\left(\mathbf{u}_{j+1}(t),\frac{\mathbf{u}_{j+1}(t)-\mathbf{u}_j(t)}{\Delta x}\right)\right)$$

$$\mathbf{u}^+_{j+1/2}(t)=\mathbf{u}_{j+1}(t)-\frac{\Delta x}{2}(\mathbf{u}_x)_{j+1}(t)$$

$$\mathbf{u}^-_{j+1/2}(t)=\mathbf{u}_j(t)+\frac{\Delta x}{2}(\mathbf{u}_x)_j(t)$$

$$(\mathbf{u}_x)_j=\operatorname{minmod}\left(\alpha\frac{\mathbf{u}_j-\mathbf{u}_{j-1}}{\Delta x},\ \frac{\mathbf{u}_{j+1}-\mathbf{u}_{j-1}}{2\Delta x},\ \alpha\frac{\mathbf{u}_{j+1}-\mathbf{u}_j}{\Delta x}\right),\quad 1\le\alpha\le 2$$

$$\mathbf{a}_{j+1/2}(t)=\max\left\{\lambda\left(\frac{\partial\mathbf{f}}{\partial\mathbf{u}}(\mathbf{u}^-_{j+1/2})\right),\ \lambda\left(\frac{\partial\mathbf{f}}{\partial\mathbf{u}}(\mathbf{u}^+_{j+1/2})\right)\right\}$$

$$\lambda(\mathbf{A})=\max|\text{eigenvalue of } \mathbf{A}|$$

3. Numerical Result

We utilize the material properties given in Table 1 for numerical computation:

Table 1 Material properties

ρ	$7833\ kg/m^3$
c	$465\ J/kgK$
k	$54\ W/mK$
$\dot{\varepsilon}_0^p$	$0.001/s$
m	0.01
σ_0	$1250\ MPa$
β	0.0016
θ_0	$293\ K$
μ	$100GPa$

For the initiation of shear band, the following temperature perturbation is employed.

$$\theta_{imperfection}=\begin{cases}\theta_\varepsilon\left(1-\frac{x}{L}\right)e^{-5x/L}, & \text{if } x<L\\ 0, & \text{otherwise}\end{cases} \quad (8)$$

where the perturbation temperature θ_ε is $2\,K$ and L is $0.5mm$.

At $x = 0$, which is the center of shear band, the plastic strain rate is plotted in Fig. 2. In the figure, as the mesh-sizes are getting small, the converge results of plastic strain rate are obtained.

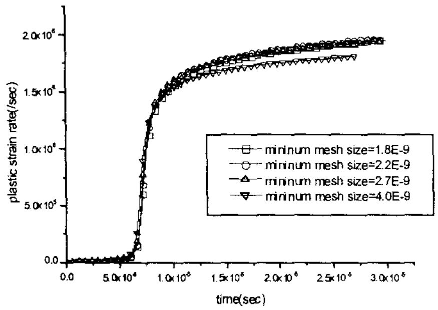

Fig. 2 Plastic strain rate at the shear band

4. Result

In this study, via Kurganov-Tadmor high resolution scheme, convergent "post-behavior" result of 1-D elastic-viscoplastic adiabatic shearband problem has been obtained. And we can get comparison results with FEM to seek advantages of using high resolution scheme in further study.

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