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Interface element method (IEM) for a partitioned system with non-matching interfaces

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일치하지 않는 경계를 갖는 분리된 시스템을 위한 계면 요소법

김 현 규 †

Key Words : Interface element method, Non-matching interface, Finite elements, Moving least square, Partitioned domains, Global-local analysis

Abstract

A novel method for non-matching interfaces on the boundaries of the finite elements in partitioned domains is presented by introducing interface elements in this paper. The interface element method (IEM) satisfies the continuity conditions exactly through interfaces without recourse to the Lagrange multiplier technique. The moving least square (MLS) approximation in the present study is implemented to construct the shape functions of the interface elements. Alignment of the boundaries of sub-domains in the MLS approximation and integration domains provides a consistent numerical integration due to one form of rational functions in an integration domain. The compatibility of displacements on the boundaries of the finite elements and the interface elements is always preserved in this method, and the completeness of the shape functions of the interface elements guarantees the convergence of numerical solutions. The numerical examples show that the interface element method is a useful tool for the analysis of a partitioned system and for a global-local analysis.

1. INTRODUCTION

The finite element methods have been widely used to solve boundary value problems, and a great deal of efforts has been concentrated on making this method useful in many engineering fields. Nevertheless, there are drawbacks in dealing with a large structure partitioned into several substructures, and with a local region in a global system. Meshes in a partitioned system may be constructed independently by different works, and mesh refinements may be imposed selectively on some local regions. Non-matching meshes on the boundaries of partitioned domains prevent a model from assembling domains into a global system without any modification of meshes along the interfaces. In particular, many steps of successive refinements with compatible connections are required to obtain a reasonable resolution in a local region imbedded in a global system.

A substantial amount of researches related to the hybrid FEM [1, 2, 3] has been endeavored by using Lagrange multipliers to impose the continuity constraints on the interfaces. An intermediate space between two boundaries provides a reasonable decoupling of non-matched interfaces, which is the mortar finite elements [4, 5, 6] to glue partitioned domains. However, these methods based on the Lagrange multiplier technique satisfy the

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continuity conditions in a variational sense, and require a complex formulation and additional degrees of freedom.

We propose a novel method named the interface element method (IEM) [7] to join partitioned domains by taking moving least square (MLS) approximations. The IEM satisfies the continuity condition on the interfaces and the compatibility condition on the boundaries of finite elements and interface elements. In the present method, additional degrees of freedom in the hybrid finite element methods are not required to enforce the continuity condition between partitioned domains, thereby introducing the interface elements.

2. INTERFACE ELEMENT METHOD (IEM)

In this section, the interface element method (IEM) is described in connection with some properties of the interface elements. Figure 1a shows partitioned domains Ω_1 and Ω_2 discretized into finite elements by different works, which produce a non-matching interface. The IEM glues these partitioned domains by introducing the interface elements defined on the finite elements bordering on the non-matching interfaces, as shown in Figure 1b.

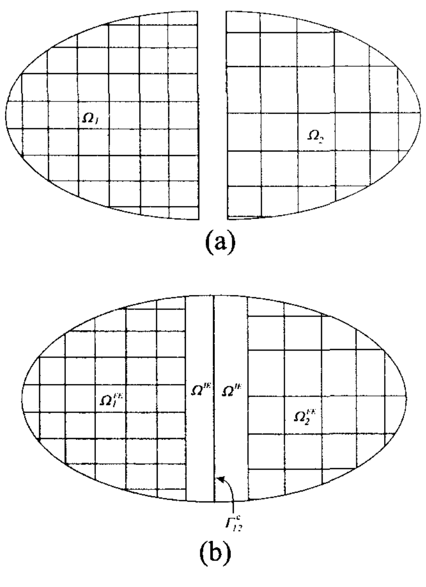

Figure 1. Interface element method for partitioned domains with non-matching interface: (a) partitioned domains Ω_1 and Ω_2 ; (b) interface element domain Ω^{IE} defined on the finite elements bordering on the non-matching interface.

2.1 Definitions

Let a set Ω_i be the domain i obtained by partitioning an open domain $\Omega \subset \mathbb{R}^3$. Then, the interface $\Gamma_{ij}^c\,(i<j)$ is defined by

$$\Gamma_{ij}^c = \partial\Omega_i \cap \partial\Omega_j , \quad i<j \tag{1}$$

We denote the interface set Γ^c to be the union of all interfaces Γ_{ij}^c. Layers of the finite elements bordering on the interfaces Γ^c in domains Ω_i are the interface element domains Ω^{IE} as illustrated in Figure 2. The interface element domains Ω^{IE} are discretized into the interface elements Λ :

$$\overline{\Omega}^{IE} = \bigcup_{\Lambda \in \Im_h^{IE}} \Lambda \tag{2}$$

where $\overline{\Omega}^{IE}$ is the closure of Ω^{IE}, and $\Im_h^{IE}$ is the triangulation established on the interface element domains Ω^{IE}, i.e., the set Ω^{IE} is subdivided into a finite number of subsets called the interface elements. Let Ω^{FE} denote the domains discretized into the finite elements K.

$$\overline{\Omega} = \overline{\Omega}^{FE} \cup \overline{\Omega}^{IE} \tag{3}$$

where

$$\overline{\Omega}^{FE} = \bigcup_{K \in \Im_h^{FE}} K$$

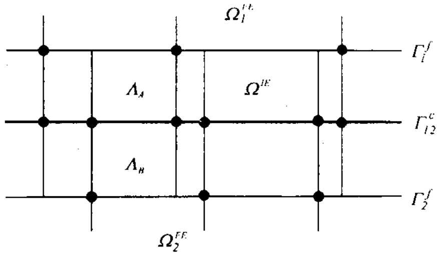

Figure 2. Interface elements Λ_A and Λ_B in the interface element domain Ω^{IE} between the finite element domains Ω_1^{FE} and Ω_2^{FE}.

2.2 *Construction of the interface elements*

The shape functions of interface elements are constructed by the moving least square (MLS) approximations [8] which have been developed for curve and surface fitting of random data. In order to construct the shape functions on the interface element domains Ω^{IE}, we need to define sub-domains or influence domains of nodes in the MLS approximations. In this paper, we only consider the procedure for constructing the shape functions of interface elements in a model partitioned into two domains Ω_1 and Ω_2 with bilinear quadrilateral finite elements.

To define the sub-domains of nodes on the interface Γ_{12}^c, we extend the finite elements bordering on the interface Γ_{12}^c in the domain Ω_1 to the finite elements bordering on the interface Γ_{12}^c in the domain Ω_2. Conversely, the finite elements in the domain Ω_2 are extended to the finite elements bordering on the interface Γ_{12}^c in the domain Ω_1. Figure 2 illustrates the extensions of the finite elements in the interface element domains Ω^{IE}. Then the pseudo nodes are defined at the intersections between the extended lines and the boundaries Γ^f of finite elements and interface elements. The pseudo nodes are denoted by open circles in Figure 3. The interface elements are constructed by the real and pseudo nodes in rectangular regions, as shown in Figure 3.

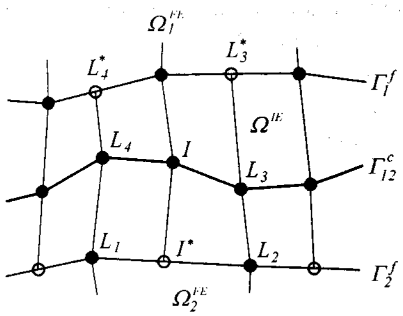

Figure 3. Pseudo nodes I^*, L_3^* and L_4^* defined by extending the finite elements bordering on the interface Γ_{12}^c.

The sub-domains of the nodes in the interface element domains Ω^{IE} are defined on the basis of the finite elements bordering on the interface Γ_{12}^c.

The choice of weight functions in the MLS approximations is crucial in the form of the shape functions of interface elements. In the present study, the weight functions $w(\boldsymbol{x})$ in sub-domains are constructed by

$$w(\boldsymbol{x}) = 1 - 3r(\boldsymbol{x})^2 + 2r(\boldsymbol{x})^3 \quad \text{for } \boldsymbol{x} \in \Omega^{IE} \tag{4}$$

with

$$r(\boldsymbol{x}) = 1 - s(\boldsymbol{x}) \ \text{ and } \ s(\boldsymbol{x}) = \sum_{k=1}^{4} N^k(\boldsymbol{x})\,\overline{w}^k \tag{5}$$

where k indicates the real and pseudo nodes in an interface element, and $N^k(\boldsymbol{x})$ are the bilinear finite element shape functions defined on an interface element. The nodal values $\overline{w}^k = \overline{w}(\boldsymbol{x}_k)$ at the nodes k should be defined on the real and pseudo nodes in the interface elements. Since the weight functions of nodes on the interfaces have zero values at the boundaries of sub-domains, the values of $\overline{w}_I^k$ at the real and pseudo nodes on the boundaries Γ_1^f and Γ_2^f are zero.

Another important requirement in the construction of the shape functions of interface elements is the compatibility condition on the boundaries Γ^f of finite elements and interface elements. It is easy to show that any forms of weight functions defined on the intervals between neighboring nodes in one-dimension produces the finite element shape functions in the MLS approximations with the linear basis. Consequently, the interface elements are compatible with the finite elements on the boundaries Γ^f.

The shape functions of interface element can be expressed as

$$v_h\big|_\Lambda(\boldsymbol{x}) = \sum_{K=1}^{N_\Lambda} \phi^K(\boldsymbol{x})\hat{v}^K \quad \text{for } \boldsymbol{x} \in \Lambda \tag{6}$$

where $\hat{v}^K$ are the values of local degrees of freedom, $\phi^K(\boldsymbol{x})$ are the shape functions of interface elements, and N_Λ is the number of local degrees of freedom associated with the interface elements Λ.

The shape functions of interface elements should include a basis which is related to the completeness in terms of the ability to represent rigid body motions and strain fields. In the IEM, the shape functions of interface elements reproduce a linear combination $f(\boldsymbol{x})$ of the basis exactly:

$$\sum_{K=1}^{N_\Lambda} \phi^K(\boldsymbol{x}) f(\boldsymbol{x}^K) = f(\boldsymbol{x}) \quad \text{for } \boldsymbol{x} \in \Lambda \tag{7}$$

3. NUMERICAL EXAMPLES

Several problems in two-dimensional linear elasticity are solved to illustrate the effectiveness of the present method. The numerical results of the IEM as applied to problems in two-dimensional elasto-statics, specifically a cantilever beam and a plate with a circular hole, are now discussed.

3.1 *A cantilever beam*

We consider a cantilever beam problem. We use regularly distributed nodes for a model to examine the present method. Three different meshes $h = 1.0$, $h = 0.5$ and $h = 0.25$ are used in the domain Ω_2. The Young modulus and the Poisson's ratio are $E = 1.0 \times 10^6$ and $\nu = 0.25$, respectively. The plane stress condition is imposed, and 5×5 integration points are used in an interface element. The traction $P = 1.0 \times 10^4$ is applied at the right-end of the beam. Fig. 4 shows the distributions of the stress σ_{11} for these three cases. The distributions of the stress σ_{11} show a reasonable connection through the non-matching interfaces. The convergences for displacement and energy in the domain Ω_2 are plotted in Fig. 5.

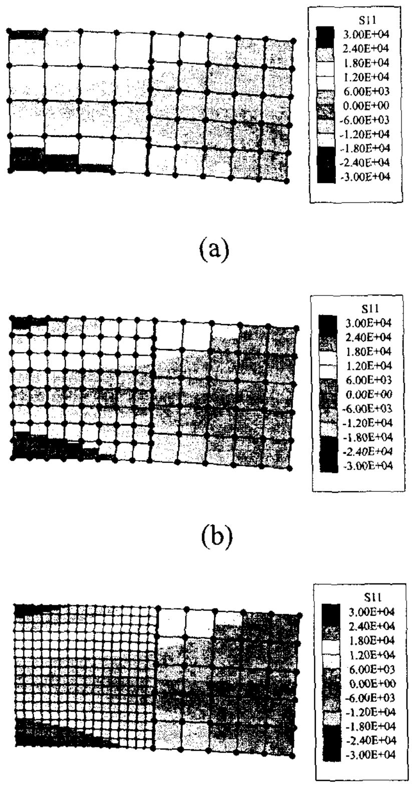

Figure 4. Distribution of the stress σ_{11} for a cantilever beam problem: (a) $h = 1.0$, (b) $h = 0.5$, (c) $h = 0.25$.

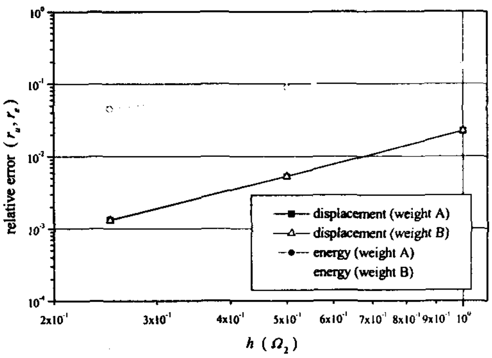

Figure 5. Convergences on displacement and energy norms in Ω_2 of a cantilever beam problem.

3.2 *A plate with a hole*

Next, we consider an infinite plate with a circular hole of radius a. The plate is subjected to a uniform tension, $\sigma_0 = 1.0 \times 10^5$ at infinity. The traction boundary conditions, as given by the exact solutions, are imposed on the outer boundary at $r = 5.0$. The non-matching interface is located at $r = 2.0$ or $b_1 = 1.0$. Due to symmetry, only a part, $0.0 \le r \le 5.0$, of the upper right quadrant of the plate is modeled under the plane stress condition. The Young modulus and the Poisson's ratio are $E = 1.0 \times 10^6$ and $\nu = 0.25$, respectively. We take 5×5 integration points in an interface element. The mesh in the domain Ω_2 is refined in order to examine the validity of the IEM. Fig. 6 shows the distributions of the stress σ_{11} for $h = 0.5$, $h = 0.25$ and $h = 0.125$ in the radial direction in the domain Ω_2. The maximum stress σ_{11} at the integration point in an element located on the top of the hole with these three meshes in the domain Ω_2 is plotted in Fig. 7. In this figure, the stress σ_{11} approaches the exact value ($\sigma_{11} = 3\sigma_0$) of the stress concentration by taking local refinements.

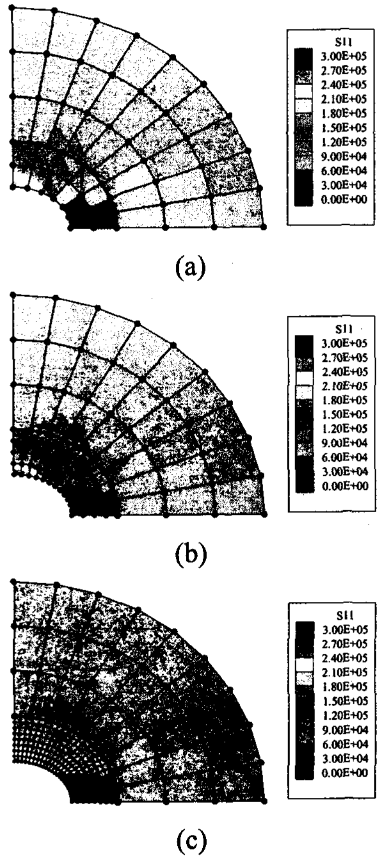

Figure 6. Distribution of the stress σ_{11} for a plate with a hole problem: (a) $h = 0.5$, (b) $h = 0.25$, (c) $h = 0.125$.

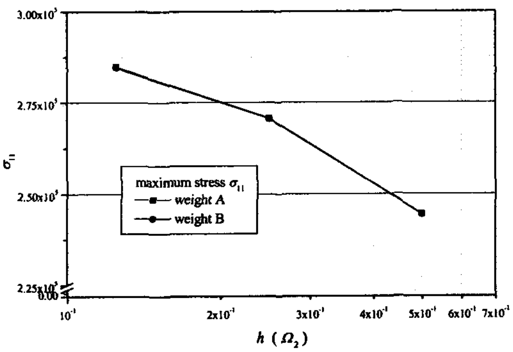

Figure 7. The stress maximum σ_{11} at the integration points in the finite element on the top of hole in the analysis of a plate with a hole.

4.CONCLUSIONS

We presented the interface element method (IEM) to join partitioned domains with non-matching interfaces on the boundaries of the bilinear quadrilateral finite elements in partitioned domains. The MLS shape functions of the interface elements satisfy the completeness and the continuity conditions on the interface element domains, which reproduce any m th order polynomial exactly. Moreover, the compatibility on the boundaries of the finite elements and the interface elements provides a reasonable connection of partitioned domains. The alignment of the boundaries of the sub-domains and integration domains alleviates a difficulty in numerical integrations. The IEM may be a very useful tool for analyzing a partitioned system, and for a global-local analysis.

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