

An investigation of the Reynolds Number dependence of the Axisymmetric Jet Mixing Layer using the Proper Orthogonal Decomposition

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Key Words: POD, Proper orthogonal decomposition, Hot-wire probe, Axisymmetric jet.

Abstract

The Proper Orthogonal Decomposition (POD) technique was applied to investigate the effects of Reynolds number and the characteristics of the organized motions or coherent structures as a function of downstream position from $x/D=2$ to 6 in a turbulent axisymmetric shear layer at Reynolds numbers of 78,400, 117,600, and 156,800. Data were collected simultaneously using the 138 hot-wire probe used by Citriniti and George (2000). The POD was then applied to a double Fourier transform in time and azimuthal direction of the double velocity correlation tensor. The lowest azimuthal mode for all POD modes, which dominated the dynamics at $x/D = 3$ in the previous experiments, dies off rapidly downstream. This is consistent with a trend toward homogeneity in the downstream evolution, and suggests that some residual value may control the growth rate of the far jet. On the other hand, for the higher azimuthal modes, the peak shifts to lower mode numbers and actually increases with downstream distance. These mixing layer data, normalized by similarity variables for the mixing layer, collapse at all downstream positions and are nearly independent of Reynolds numbers.

1. Introduction

The Proper Orthogonal Decomposition (POD) technique is applied to find the effects of Reynolds number and the characteristics of the organized motions or coherent structures as a function of downstream position in a turbulent axisymmetric shear layer [7]. In this experiment measurements were made at positions of $x/D = 2$ to 6 spaced 0.5 x/D apart along downstream direction for the three different jet Reynolds number of 78,400, 117,600,

and 156,800. Data were taken using the 138-wire probe originally used by Citriniti and George [1], but only at a single downstream position and a single Reynolds number. Lumley [8] suggested that the large scale, or coherent, structures occur in an identifiable manner in a given ensemble of random vector fields, and that this structure makes a significant contribution to the distribution of the total kinetic energy in the flow field. Lumley further suggested that the lowest order eigenfunction, i.e. the most energetic eddies, can be identified with the largest scale structure in the flow. Glauser [4] modified Lumley's hypothesis and suggested that the POD merely represented an optimal basis for examining the life-cycle of the coherent structures, with different eigenfunctions representing it at various times in its life cycle. Glauser [4] was the first to apply the POD to the axisymmetric jet mixing layer.

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A subsequent investigation by Citriniti and George [1] obtained the dynamics of the large structures from instantaneous realization of the streamwise velocity field using 138 simultaneously-sampled hot-wire anemometer probes. They showed that only a few azimuthal Fourier modes are necessary to represent the evolution of the large scale structure from the turbulent field. Furthermore, the velocity reconstructions using the POD provided evidence for both azimuthally coherent structures that exist near the potential core and counter-rotating, streamwise vortex pairs(or ribs) in the region between successive azimuthally coherent structures as well as coexisting for short periods with them. They also observed that the most spatially correlated structure in the flow ejects fluid in the streamwise direction in a volcano-like eruption, and attributed this to the attempted leap-frogging of vortex rings in the flow as suggested by Grinstein et al. [5]. The Glauser and Citriniti experiments were purposely performed at Reynolds numbers that were presumed sufficiently high to insure near Reynolds number independence of the large scale structures. This presumption was recently challenged by Holmes et al. [6] who suggested that more complicated modal structures might evolve with increasing Reynolds number. Therefore one of the goals of this investigation was to quantify how the energy distribution among the various modes varied as a function of Reynolds number and to determine when or if ever the asymptotic trends become Reynolds number. Also, since both the Glauser and Citriniti experiments were performed at only a single downstream position, another goal was to investigate whether the modal character of the flow changed with downstream position.

2. Experimental Configuration

For this experiment, the jet exit velocity was 12, 18, and 24 m/s which translates to a Reynolds number based on exit diameter of 78,400, 117,600, and 156,800 respectively for the 9.8 cm diameter nozzle. The streamwise velocity was captured simultaneously at all 138 positions using the 138 hot-wire probe array (Figure 1). To obtain the data simultaneously

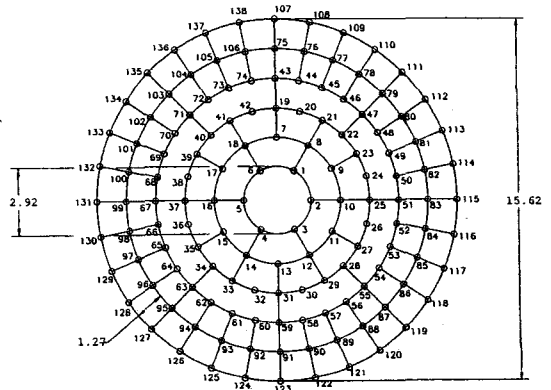


Figure 1. The schematic of the 138 Hot-wire probe array

at all 138 positions, a sample/hold amplifier was used in each anemometer board separately. The sampling frequency, 4,001.6 Hz, was sufficient to satisfy the temporal Nyquist criterion, and very long wires ($l = 1\text{ cm}$) were used to avoid spatial aliasing. The record length of each block of data was 4096 samples giving a bandwidth of 0.98 Hz and a length of 1.02 sec. In all, 388 such blocks were used in the statistical analysis, which reduced the variability of the cross-spectra to less than 5 %.

3. Results and Conclusions

In brief, the results show that of the three speculations cited above about how the flow would behave downstream or under different conditions, two of the three are wrong. Contrary to our expectations, the distributions of normalized POD mode energy have strong dependence on $x=D$. Even more surprising, mode-0 behaves in a manner entirely different than the higher modes. Also, the results are very nearly independent of Reynolds number, contrary to the suggestion of Holmes et al. [6], but consistent with the presumptions of the original investigators.

The lowest azimuthal mode for all POD Modes (not shown), which dominated the dynamics at $x/D = 3$ in the Glauser and Citriniti experiments, dies off rapidly downstream. This is consistent with an approach toward homogeneity in the downstream evolution, and suggests that perhaps some residual value may control (or reflect) the growth rate of the far jet. If

so, this could provide an important clue as to why and how equilibrium similarity governs the far jet, and also why the jet growth rate may reflect the upstream conditions.

On the other hand, for the higher azimuthal modes, the peaks shift to lower mode numbers and actually increase with downstream distance. Figure 2 shows a normalized plot of how the eigenvalues from the first POD mode, $\lambda^{(1)}$ vary as a function of mode azimuthal number for ($m \geq 1$), downstream distance and Reynolds number. Clearly they do not vary at all with this normalization. What is most surprising is that these mixing layer data are normalized by similarity variables for the far jet; namely, λ/x versus $m \cdot x/D$. Thus the further the distance downstream, the lower the number of the peak.

The presentation will also show dynamic reconstructions of the velocity field at each location (in the manner of Citriniti and George). Most evident by contrast is the diminution with distance downstream of the "volcano"-like events described by them, and an evolution into a more "propeller"-like rotating pattern where the number of "blades" diminishes downstream. It will be argued that this behavior is similar to that predicted from inviscid instability theory.

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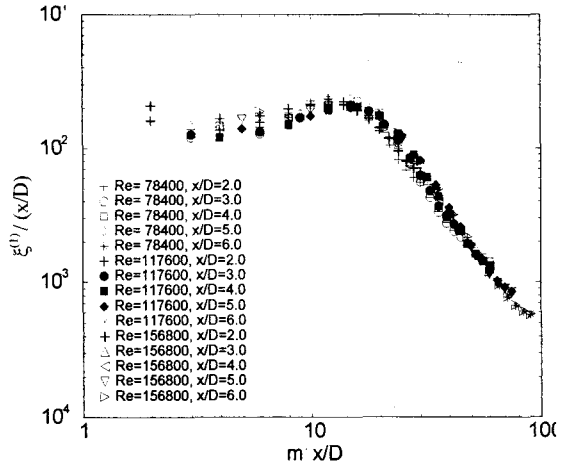


Figure 2 The first POD-mode energy distribution normalized in shear layer variables, ($m \cdot x/D$), at $Re_D=78,400, 117,600, \text{ and } 156,800$.

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