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Additive MACA의 행동에 관한 연구

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Behavior of additive MACA

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요약

직전자수가 2개인 nongroup CA 중 전이행렬의 최소다항식이 $x^d(x+1)$인 선형 MACA의 행동을 분석하고 그 tree를 구성하는 알고리즘을 제안한다. 또한 선형 MACA의 0-tree에서 0이 아닌 상태를 여원으로 갖는 또 하나의 MACA의 행동을 밝혔다.

1. Introduction

An analysis of the state-transition behavior of group cellular automata(abbreviately, CA) was studied by many researchers([1], [5], [7], [9], [11]). The characteristic matrix of group CA is nonsingular. But the characteristic matrix of nongroup CA is singular. Although the study of nonsingular linear machines has received considerable attention from researchers, the study of the class of machines with singular characteristic matrix has not received due attention. However some properties of nonsingular CA have been employed in several applications([6], [8], [10]). In this paper, we present a detailed analysis of the behavior of complemented CA derived from a linear CA by replacing the XORs with XNORs at some (or all) of the cells. Also, we give the specific features displayed in the state-transition behavior of the complemented MACA C' resulting from inversion of the next-state logic of some (or all) of the cells of multiple-attractor CA(abbreviately, MACA) C. We call C' the CA corresponding to C. Especially we investigate the behavior of the complemented MACA which the complement vector F is taken as a nonzero state in the 0-tree of a linear MACA.

2. Linear Nongroup CA

Definiton 2.1[3]. A state with a self-loop in the state-transition diagram of a nongroup CA are referred to as an *attractor*.

The tree rooted at a cyclic state α is called the α-tree.

Definiton 2.2[3]. The *nongroup CA* for which the state-transition diagram consists of a set of disjoint components forming (inverted) tree-like structures rooted at attractors are referred to as multiple-attractor CA(*MACA*).

Definiton 2.3[3]. The *depth* of a CA is defined to be the minimum number of clock cycles to reach the cyclic state from any nonreachable state in the state-transition diagram of the CA.

Since the 0-tree and another tree rooted at a nonzero cyclic state have very interesting relationships, the study of the 0-tree is necessary and very important.

Theorem 2.4[6]. The number of predecessors of a reachable state and the number of predecessors of the state 0 in a linear nongroup CA are equal.

Definiton 2.5[4]. A state X at level $l(l \leq$ depth) of the α-tree is a state lying on that tree and it evolves to the state α exactly after l-cycles(l is the smallest possible integer for which $T^l X = \alpha$).

Definiton 2.6[4]. A state Y of an n-cell CA is an $r-$*predecessor* $(1 \leq r \leq 2^n - 1)$ of a state X if $T^r Y = X$, where T is the characteristic matrix of the CA.

3. Behavior of complemented MACA derived from a linear MACA

In this section, we present the behavior of complemented MACA derived from a linear MACA. Especially we investigate the behavior of complemented MACA from a MACA C which the complement vector F is taken in the 0-tree as a nonzero state of C .

Theorem 3.1. Let $\overline{T}^p$ denote p times application of the complemented CA operator $\overline{T}$. Then

$$\overline{T}^p[F(x)]$$
$$= [T^p \oplus T^{p-1} \oplus \cdots \oplus T^2 \oplus T \oplus I][F(x)]$$

Theorem 3.2. Let C be a linear MACA with depth d and F be a state at the level $i(0 < i \leq d)$ of the 0-tree in C as a complement vector. Then $\overline{T}^{i-1}F$ is an attractor in the complemented MACA C′ corresponding to C .

Theorem 3.3. Suppose that there exists at least one attractor in the complemented CA C′ corresponding to an n-cell linear MACA C with k attractors. Then the number of attractors in C′is the same as that in the original linear one.

Theorem 3.4. In the state-transition diagram of the complemented MACA C′ corresponding to a linear MACA C , the sum of different predecessors of any reachable state is the nonzero 1-predecessor of the state 0 of C .

Theorem 3.5. Let the dimension of the null space of the state-transition matrix T of a linear MACA C be 1. Let F be a state at level $l(> 0)$ in the 0-tree of C and C′ be the corresponding complemented MACA. Then the states of 0-tree of C are rearranged in the state-transition diagram of C′ as the following:

(a) All states at levels higher than l of C will remain unaltered.

(b) The states at level l of C get rearranged at level up to $(l-1)$ of C′.

(c) The states at levels up to $(l-1)$ of C get rearranged at level l of C′.

(d) F lies on level $(l-1)$ of C′.

4. Construction of a tree from a given 0-tree in a linear MACA

In this section we construct an α-tree of a MACA C with two predecessor if we knew a 0-basic path of the 0-tree and a nonzero attractor α of C.

Theorem 4.1. [2] Let C be a linear single attractor CA having two predecessor. If the states of the state-transition diagram of C are labeled such that $S_{l,k}$ be the $(k+1)$-th state in the l-th level of the 0-tree in C, then the following hold:

$$S_{l,k} = S_{l,0} \oplus \sum_{i=1}^{l-1} b_i S_{i,0}$$

where $b_{l-1} b_{l-2} \cdots b_1$ is the binary representation of k and the maximum value of k is $2^{l-1}-1$.

Definiton 4.2. Let C be a linear MACA with two predecessor and the depth of C be d. Let β be a nonreachable state of the α-tree of C. Then we call the path $\beta \rightarrow T\beta \rightarrow \cdots \rightarrow \alpha$ a α-basic path of the α-tree of C.

Theorem 4.3. Let C be a linear MACA having two predecessor(depth = d) and T be the characteristic matrix of C.
If $S_{d,0} \rightarrow S_{d-1,0} \rightarrow \cdots \rightarrow S_{1,0} \rightarrow 0$ is a 0-basic path of the 0-tree of C, then

$$(S_{d,0} \oplus \alpha) \rightarrow (S_{d-1,0} \oplus \alpha) \rightarrow \cdots \rightarrow (S_{1,0} \oplus \alpha) \rightarrow \alpha$$

is a α-basic path of the α-tree of C.

Theorem 4.4. Let C be a linear MACA having two predecessor. If the states of the state-transition diagram of C are labeled such that $S^{\alpha}_{l,k}$ (resp. $S_{l,k}$) be the $(k+1)$-th state in the l-th level of the α-tree(resp. 0-tree) in C, then the following hold:

$$S^{\alpha}_{l,k} = S^{\alpha}_{l,0} \oplus \sum_{i=1}^{l-1} b_i S_{i,0}$$

where $b_{l-1} b_{l-2} \cdots b_1$ is the binary representation of k and the maximum value of k is $2^{l-1}-1$.

(Algorithm for construction of a tree of a linear MACA C)

Step 1. Find x such that $(T \oplus I)x = 0$.

Step 2. Find the maximum value of k such that $x^k \mid m(x)$ where $m(x)$ is the minimal polynomial of T.

Step 3. Find any y such that $T^k y = 0$ and $T^{k-1} y \neq 0$ where k is in Step 2.

Step 4. Find a 0-basic path $y \rightarrow Ty \rightarrow \cdots \rightarrow 0$.

Step 5. Construction of the 0-tree using
$$S_{l,n} = S_{l,0} \oplus \sum_{i=1}^{l-1} b_i S_{i,0}$$

Step 6. Find all basic paths using
$$S^{\alpha}_{l,0} = S_{l,0} \oplus \alpha$$

Step 7. Find all trees in C using
$$S^{\alpha}_{l,n} = S^{\alpha}_{l,0} \oplus \sum_{i=1}^{l-1} b_i S_{i,0}$$

4. Conclusion

In this paper, we present a detailed analysis of the behavior of complemented CA derived from a linear CA. Also, we give the specific features displayed in the state-transition behavior of the complemented MACA C' of a linear MACA C. Moreover we present an algorithm for construction of a tree of C and investigate the behavior of C' which the complement vector is taken as a nonzero state in the 0-tree of C.

In future we will investigate the behavior of the complemented MACA which the complement vector taken as a nonzero state in any tree of C.

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