Analysis of non-uniformly perforated muffler in concentric resonator type

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Abstract

In the muffler industry, the main purpose of the research works is to determine a way to increase the TL (Transmission Loss) properties of the muffler, without deteriorating the back pressure influence. In order to obtain better results, several works have been done by changing the geometrical characteristics of the muffler or the type of the muffler. This work will focus on the perforated muffler components with concentric chamber, to investigate the effect of a non-uniform porosity along the inner perforated tube of the muffler on the TL. It is noted that varying the perforation ratio affects the peaks frequencies of the TL, especially for $2\pi < kL < 4\pi$ (in the case of L = 200 mm for the concentric resonator). The magnitudes of the TL, for this range of frequencies, vary noticeably by changing the porosity distribution.

1. Introduction

Several works have been done to explain and to analyze the characteristics of perforated-muffler elements such as the analysis done by Sullivan and Crocker [1]. But their method showed some weaknesses, such as the effect of the non rigid boundary conditions. Sullivan [2] followed this method by introducing a segmentation procedure for modeling all types of perforated element mufflers. This method requires a discretization of a uniformly perforated elements, so that one needs to increase the number of segments to obtain a better prediction.

Later, a decoupling approach was introduced by Jayaraman and Yam [3] to get closed-form solutions. This method solved the problems and the limitations cited previously but could be applied only for the case of the mean-flow Mach numbers in the ducts being equal, which is not true in the case of actual muffler systems.

The decoupling approach has been then held by Thawani and Jayaraman to concentric resonator mufflers [4] but limited to the case of zero mean flow.

Later, Munjal, Rao and Sahasrabudhe described a generalized decoupling approach [5], which can be applied to actual (unequal) Mach numbers in the adjoining tubes. This method tallied well with the Jayaraman and Yam's expressions for the hypothetical case of equal Mach numbers. For the present study, this latter decoupling approach held by Munjal, Rao and Sahasrabudhe will be used to describe the transmission properties of the non-uniform porosity mufflers.

In this study, only the grazing flow conditions will be considered. The cross-flow conditions can be considered by changing the definition of the four-pole variables. Furthermore, the present study will be limited to zero mean flow velocity, in order to focus on the effects of the porosity variations.

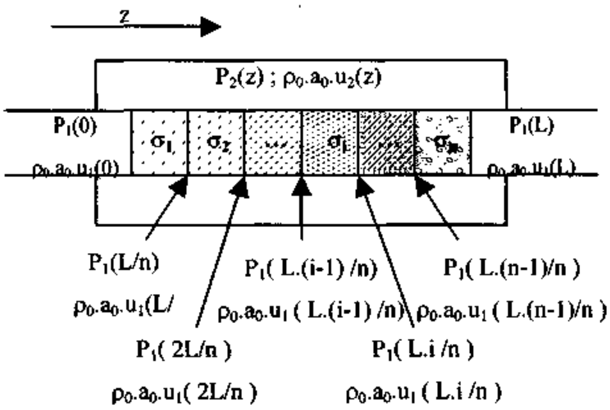

Fig. 1. Definition of geometrical and acoustic notations

2. Porosity variation along the perforated tube

To obtain a non-uniform porosity along a perforated tube, one can change several parameters such as the diameter or the number of holes along the tube. In this study, only the number of holes will be varying, whereas the diameter will be remained constant.

Five different non-uniform porosity tubes will be considered here, denoted as tube 1 to 5 (Fig. 2). Consider that the porosity is uniform on each section of the tube but varying with z. The determination of the porosity is hence easy because it can be considered uniform for each portion (Fig. 1).

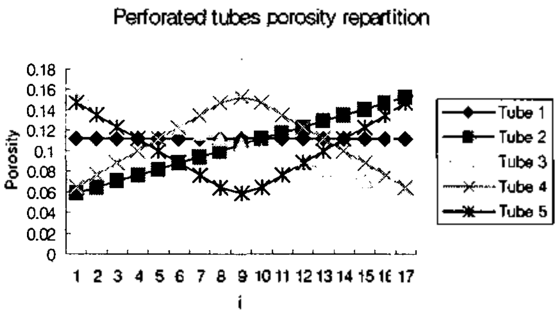

Fig. 2. Distribution of porosity along the center tube.

3. Impedance model

Several impedance models will be used to calculate the transmission loss of the muffler. However, the use Lee and Ih's impedance model [6] appears to be the closest empirical one to comply with the measured transmission loss for such mufflers:

$$Z = R_n + i X_n \tag{1}$$

Here, M_0 is the Mach number of the mean flow,

$$R_n = a_0(1 + a_1|f - f_0|)(1 + a_2 M_0)(1 + a_3 a)(1 + a_4 t)/\sigma \tag{2a}$$

and

$$X_n = (b_1 + b_2 f + b_3 f^2)/\sigma \tag{2b}$$

with

$a_0 = 8.20\text{e-}4$, $a_1 = 8.23\text{e-}3$
$a_2 = 12.9$, $a_3 = 44.8$,
$a_4 = -165$,

$$b_n = \gamma_{c_n 1} + \gamma_{c_n 2} M_0 + \gamma_{c_n 3} M_0^2 \qquad (n = 1, 2, 3)$$

$$\gamma_{c_1 1} = 5.14e-13, \qquad \gamma_{c_1 2} = -0.392,$$

$$\gamma_{c_1 3} = 2.46.$$

4. Transfer matrix

For uniform porosity perforates, the transfer matrix can be written as follows:

$$\begin{bmatrix} p_1(0) \\ p_2(0) \\ \rho_0 a_0 u_1(0) \\ \rho_0 a_0 u_2(0) \end{bmatrix} = [T] \begin{bmatrix} p_1(l) \\ p_2(l) \\ \rho_0 a_0 u_1(l) \\ \rho_0 a_0 u_2(l) \end{bmatrix} \tag{3}$$

This relation can be applied to each partition of different porosity:

$$\begin{bmatrix} p_1(0) \\ p_2(0) \\ \rho_0 a_0 u_1(0) \\ \rho_0 a_0 u_2(0) \end{bmatrix} = [T_1] \begin{bmatrix} p_1(\frac{1}{N}) \\ p_2(\frac{1}{N}) \\ \rho_0 a_0 u_1(\frac{1}{N}) \\ \rho_0 a_0 u_2(\frac{1}{N}) \end{bmatrix} \tag{4}$$

In a similar way, one can obtain

$$\begin{bmatrix} p_1(\frac{(i-1).l}{N}) \\ p_2(\frac{(i-1).l}{N}) \\ \rho_0 a_0 u_1(\frac{(i-1).l}{N}) \\ \rho_0 a_0 u_2(\frac{(i-1).l}{N}) \end{bmatrix} = [T_i] \begin{bmatrix} p_1(\frac{i.l}{N}) \\ p_2(\frac{i.l}{N}) \\ \rho_0 a_0 u_1(\frac{i.l}{N}) \\ \rho_0 a_0 u_2(\frac{i.l}{N}) \end{bmatrix}$$

$$\begin{bmatrix} p_1(\frac{(N-1).l}{N}) \\ p_2(\frac{(N-1).l}{N}) \\ \rho_0 a_0 u_1(\frac{(N-1).l}{N}) \\ \rho_0 a_0 u_2(\frac{(N-1).l}{N}) \end{bmatrix} = [T_N] \begin{bmatrix} p_1(l) \\ p_2(l) \\ \rho_0 a_0 u_1(l) \\ \rho_0 a_0 u_2(l) \end{bmatrix} \tag{5}$$

The final expression of the transfer matrix becomes

$$\begin{bmatrix} p_1(0) \\ p_2(0) \\ \rho_0 a_0 u_1(0) \\ \rho_0 a_0 u_2(0) \end{bmatrix} = [T_1]..[T_i]..[T_N] \begin{bmatrix} p_1(l) \\ p_2(l) \\ \rho_0 a_0 u_1(l) \\ \rho_0 a_0 u_2(l) \end{bmatrix}, \tag{6a}$$

or

$$[T] = [T_1]..[T_i]..[T_N]. \tag{6b}$$

One can finally obtain, from the foregoing expression, the transfer matrix or four pole parameters, applied to the concentric resonator configuration, defined by

$$\begin{bmatrix} p_1(0) \\ \rho_0 a_0 u_1(0) \end{bmatrix} = \begin{bmatrix} T_a & T_b \\ T_c & T_d \end{bmatrix} \begin{bmatrix} p_1(l) \\ \rho_0 a_0 u_1(l) \end{bmatrix}, \qquad (7)$$

where

$$T_a = T_{11} + A_1 A_2,\ T_b = T_{13} + B_1 A_2,$$
$$T_c = T_{31} + A_1 B_2,\ T_d = T_{33} + B_1 B_2,$$

$$A_1 = \frac{(X_1 T_{21} - T_{41})}{F_1},\ B_1 = \frac{(X_1 T_{23} - T_{43})}{F_1},$$

$$A_2 = T_{12} + X_2 T_{14},\ B_2 = T_{32} + X_2 T_{34},$$
$$F_1 = T_{42} + X_2 T_{44} - X_1 (T_{22} + X_2 T_{24}),$$
$$X_1 = -j\tan(k_0 l_a),\ X_2 = +j\tan(k_0 l_b).$$

5. Comparison of transmission Loss

Using the transfer matrix, the transmission loss can be obtained and compared to the transmission loss measured experimentally using three-microphone method [7]. The transmission loss of a uniformly perforated muffler (Fig. 4) will be first calculated so as to determine the effects of a non-uniform repartition of holes (Figs. 5-8).

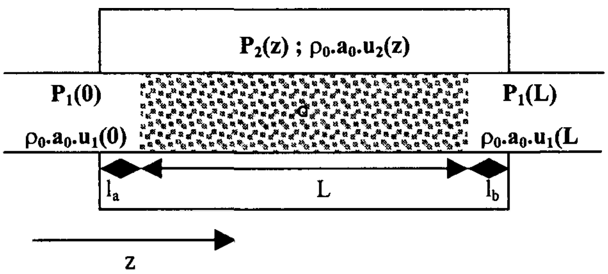

Fig. 3. Definition and dimension of test resonator.
Length of perforate, L = 170 mm; Diameter of inner tube, 35 mm; Diameter of outer tube, 110 mm; Diameter of holes, 4 mm; Thickness of holes, 2 mm; l_a = l_b = 15 mm; Thickness of the outer tube, 5 mm; gas temperature, T = 26°C; flow Mach number, M=0.

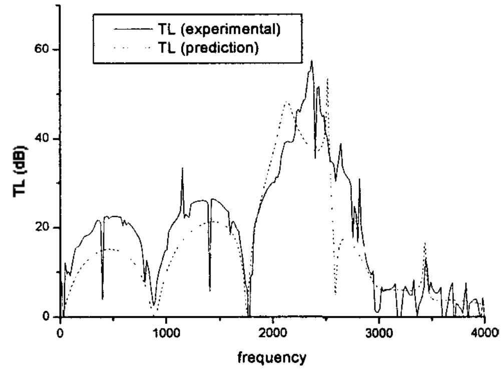

Fig. 4. TL for the tube 1 without mean flow

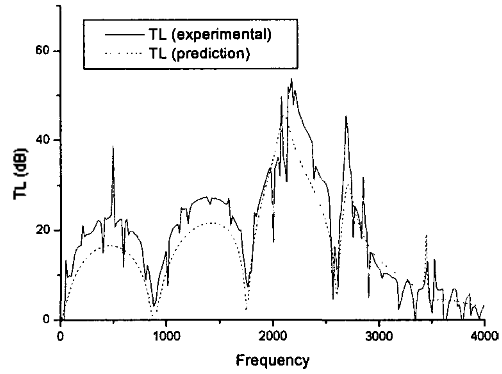

Fig. 5. TL for the tube 2 without mean flow.

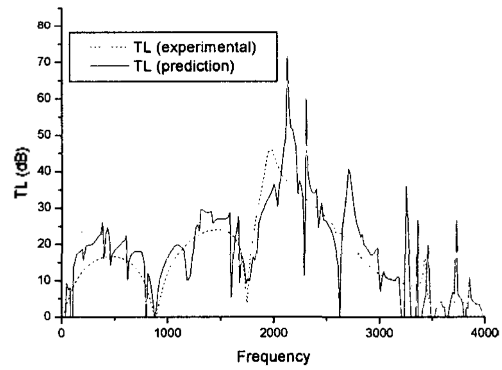

Fig. 6. TL for the tube 3 without mean flow.

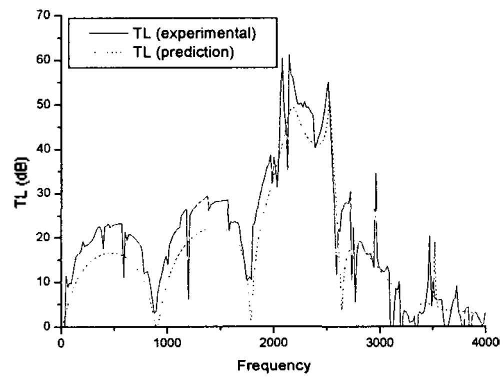

Fig. 7. TL for the tube 4 without mean flow.

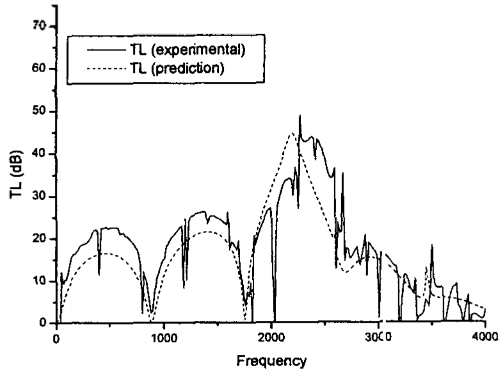

Fig. 8. TL for the tube 5 without mean flow.

These results show some reversed peaks, which can be observed at some specific frequencies. These distortions of the measured transmission loss are caused by the modes $(f_n = n \times c/2l)$ of the tube at the upstream part of the muffler, of which the length corresponds to l = 70 cm, and also by the deteriorated coherence of the signal between the reference microphone and the measurement microphones.

3. Conclusions

This study shows that the variation of the porosity along the concentric resonator as a muffler component affects the transmission loss, but only for the specific range of frequencies located between $2\pi < kL < 4\pi$, in comparison with the uniform porosity model. For $0 < kL < 2\pi$, the obtained transmission loss is only due to the resonator. But for higher frequencies, by changing the repartition of holes enables to change the frequencies and the number of transmission loss maxima. Consequently, the average porosity for the whole resonator center pipe cannot be applied to the prediction of the transmission loss characteristics of such muffler. Thus, one may use a non-uniform porosity model to obtain desired transmission loss properties, but more diversified works would be helpful to describe more precisely how the different porosity variations affect the transmission loss (increasing porosity along the tube, decreasing porosity, etc.).

Further works focusing on the back pressure resulting from these non-uniform porosity mufflers can be pursued in order to define what different models should be employed in the actual system, e.g., an internal combustion engine exhaust, without decreasing the engine performances.

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