Acoustic Characteristics of Sand Sediment Slab with Water- and Air-filled Pores

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Acoustic pressure transmission coefficient and phase velocity are measured as the functions of water porosity and air porosity in sand sediment slabs with water- and air-filled pores. Pores in the sand sediment slab are modeled as the structure of circular cylindrical tube shape filled with water and air. The first kind (fast) wave and second kind (slow) wave, identified by Biot, in the solid and fluid mixed medium are affected by the presence of water and air pores. Acoustic characteristics of such porous medium in water are also theoretically investigated in terms of the modified Biot-Attenborough (MBA) model, which uses the separate treatment of viscosity effect and thermal effect in non-rigid porous medium with water- and air-filled pores. The information on the fast waves introduces new concepts of the generalized tortuosity factor and dynamic shape factor.

I. INTRODUCTION

Acoustic wave propagation in porous medium with water- and air-filled pores attracted attention for a long time for various reasons. However, the mechanism of acoustic wave interactions with such a medium is not well understood. In this paper, a theoretical and experimental study is tried toward the complete understanding of acoustic properties for porous medium with water- and air-filled pores. Pores in the sand sediment slab are modeled as the structure of circular cylindrical tube shape filled with water and air. Acoustic wave propagation in porous medium with water- and air-filled pores is modeled by the sound wave propagation in porous medium containing circular capillary pores. The first and second waves, which were identified by Biot [1], are not separately observed in the measurement of sediment slab without cylindrical pores, which has the water porosity of about 0.43, the grain size less than 0.5 mm, and the density 2616 kg/m^3. However, the first and second waves are separately observed in the measurement of sediment slab with cylindrical pores. The modified Biot-Attenborough (MBA) model [2] to the porous material, which uses the separate treatment of viscous effect and thermal effect [3] for the acoustic wave propagation in the non-rigid porous medium with circular cylindrical pores, is adopted to investigate the sound wave velocity and transmission coefficient in circular capillary water- and air-filled pores. The theoretical and experimental data such as phase velocity and transmission coefficient are compared as a step toward the validation of the model.

II. REVIEW OF THE MODIFIED BIOT-ATTENBOROUGH FOR POROUS MEDIUM

The MBA model for non-rigid medium with circular cylindrical pores [2] treats viscosity and thermal effects separately. Three phenomenological parameters for the boundary, velocity, and impedance, s_1, s_2, and s_3, are introduced to take into account non-rigid frame. The concepts of the generalized tortuosity factor and dynamic shape factor are introduced.

From the continuity equation $-\rho_0 \frac{\partial \langle v \rangle}{\partial x} = \frac{\partial \rho}{\partial t}$ and the equation of motion $\frac{\partial p}{\partial x} = \rho_c(\omega) \frac{\partial \langle v \rangle}{\partial t}$, the complex density may be written by

$$\rho_c(\omega) = \rho_0[1 - 2(\lambda\sqrt{i})^{-1})T(\lambda\sqrt{i})]^{-1}$$

where $T(\lambda\sqrt{i}) = J_1(\lambda\sqrt{i})/J_0(\lambda\sqrt{i})$ with J_0 and J_1 are the zeroth and first order of cylindrical Bessel functions respectively. ρ_0 is the equilibrium fluid density, p is the pressure, and $\langle v \rangle$ is the average particle velocity over the pore cross section. A dimensionless parameter, which is related to the size of the viscous boundary layer at the pore wall, is introduced by $\lambda = as_1(\omega/\nu)^{1/2}$ where a is the pore radius and ν is the kinematic viscosity of fluid.

The thermal equation produces the complex compressibility of fluid $C(\omega)$:

$$C_c(\omega) = (\gamma p_0)^{-1}[1 + 2(\gamma - 1)(N_{Pr}^{1/2}\lambda\sqrt{i})^{-1})T(N_{Pr}^{1/2}\lambda\sqrt{i})]$$

where N_{Pr} is the Prandtl number.

The propagation in a single pore is extended to bulk cylindrical pores for non-rigid medium so as for theoretical results to satisfy experimental results phenomenologically. The average particle velocity $\langle v \rangle$ of a single pore is related to the average velocity u over the unit cross section of the porous medium: $\langle v \rangle \simeq q[\frac{C_c(w) z_b \omega}{k_b}]u$ where q is the tortuosity of the pore, z_b is the characteristic impedance of the bulk medium, and k_b is the propagation constant of the bulk medium. From the equation of continuity $-q[\frac{C_c(w) z_b \omega}{k_b}]\frac{\partial u}{\partial x} = \frac{1}{\rho_0}\frac{d\rho}{dp}\frac{\partial p}{\partial t}$ and the equation of motion $-\frac{\partial p}{\partial x} = \rho_b(\omega)\frac{\partial u}{\partial t}$, the wave equation becomes $\frac{\partial^2 p}{\partial x^2} = \frac{1}{c_b^2}\frac{\partial^2 p}{\partial t^2}$ where $c_b^2 = \frac{\omega^2}{k_b^2}$.

A complex propagation constant k_b in the case of non-rigid pore frame is parameterized from the wave equation:

$$k_b^2 = q^2[k_m^2 k_c^2/((1-\Omega)^{s_2} k_c^2 + \Omega^{s_2} k_m^2)]$$

where Ω is the porosity and $k_c^2 = \omega^2 C_c(\omega)\rho_c(\omega)$. The form of the propagation constant is based on a specific parameterization of the phase velocity since $Re[k_b] = \omega/c_b$:

$$c_b^2 = [(1-\Omega)^{s_2} c_m^2 + \Omega^{s_2} c_c^2]/q^2$$

where c_m is the longitudinal sound velocity for pure frame material and c_c is the sound velocity for pore fluid.

The characteristic impedance of the medium is given by $z_b = R + iX = q^2\omega\rho_b(\omega)/k_b$. The effective impedance of bulk porous medium is parameterized by

$$z_b = q^2[-s_3\Omega^2 + (\rho_c c_c - \rho_m c_m + s_3)\Omega + \rho_m c_m]$$

where ρ_m and c_m are respectively pure matter density and sound velocity of pore frame and ρ_c and c_c are respectively the pure fluid density and sound velocity of pore.

III. APPLICATION OF THE MBA MODEL TO SEDIMENT WITH WATER- AND AIR-FILLED PORES

An extended MBA model can be applied to sediment with water- and air-filled pores. The comparision between the fast and slow wave and the generalization of the tortuosity factor and dynamic shape factor are discussed.

A. Effective Propagation Constant and Impedance

The following parameterizations for the propagation constant and impedance are suggested.

The propagation constant for the total wave in the presence of water- and air-filled pores may take into account the air effect in porous medium:

$$k_{bm}^2(\Omega,\beta) = [(1-\beta)k_{bw}^2(\Omega) + (1-\Omega)k_{ba}^2(\beta)]/(2-\Omega-\beta),$$

where k_{bw} is the propagation constant in medium with water pores and k_{ba} is the propagation constant in medium with air pores for the total wave. The effective impedance of bubbly porous medium for the total wave is given by

$$z_{bm}(\Omega,\beta) = [(1-\beta)z_{bw}(\Omega) + (1-\Omega)z_{ba}(\beta)]/(2-\Omega-\beta),$$

where z_{bw} is the impedance in medium with water pores and z_{ba} is the impedance in medium with air pores for the total wave.

The propagation constant $k_{bm}^2(\Omega,\beta)$ and the impedance $z_{bm}(\Omega,\beta)$ respectively become $k_{bw}^2(\Omega)$ and $z_{bw}(\Omega)$ in the limit of porous medium without air tube porosity and respectively become $k_{ba}^2(\beta)$ and $z_{ba}(\beta)$ in the limit of porous medium without water tube porosity.

The above result can be applied to the case in which a plain acoustic wave encounters porous medium delimited by a plane normal to the incident wave. Applying the boundary conditions, the transmission and reflection coefficients become

$$T = \frac{2}{(1+z_1/z_3)\cos(k_{bm}d) + i(z_{bm}/z_3 + z_1/z_{bm})\sin(k_{bm}d)},$$

$$R = \frac{(1-z_1/z_3)\cos(k_{bm}d) + i(z_{bm}/z_3 - z_1/z_{bm})\sin(k_{bm}d)}{(1+z_1/z_3)\cos(k_{bm}d) + i(z_{bm}/z_3 + z_1/z_{bm})\sin(k_{bm}d)}$$

where the characteristic impedances are $z_1 = \rho_1 c_1$, $z_{bm} = \rho_{bm} c_{bm}$, and $z_3 = \rho_3 c_3$ and the thickness of intermediate medium is d.

B. Comparison between Fast Wave and Slow Wave

The concept of the tortuosity factor and dynamic shape factor for the slow wave may be generalized for the fast wave. The tortuosity factor q represents the change of pore direction compared with the normal direction. It has the particular value $q = 1/\cos\theta$ for the slow wave of a medium containing parallel cylindrical pores inclined at angle θ to the surface normal: $q > 1$ for the slow wave. The phase velocity becomes slower and attenuation becomes higher as the angle θ becomes bigger. The tortuosity factor q can be generalized for the fast wave. In the case of the fast wave, $q = \cos\theta$ is suggested for a medium containing parallel cylindrical pores inclined at angle θ to the surface normal: $q < 1$ for the fast wave. The phase velocity becomes faster and attenuation becomes lower as the angle θ becomes bigger.

The dynamic shape factor n is related to the shape of the pore [3,2]: $\lambda = \frac{a}{n}s_1(\omega/\nu)^{1/2}$ where $0.5 \le n \le 1$. It has the extreme values $n = 1$ for a circular cross section and $n = 0.5$ for a parallel-side slit for the slow wave. It may be generalized to have values $n \ge 1$ for the different cross sections in the case of the fast wave.

The fast and slow wave have contrast in several physical characteristics. The tortuosity factor q is bigger than 1 for the slow wave but is smaller than 1 for the fast wave. It becomes bigger for the fast wave and smaller for the slow wave as the porosity increases. In the case of a pore shape other than circular pore, the dynamic shape factor n is smaller than 1 for the slow wave [3] but is bigger than 1 for the fast wave. It becomes bigger for the fast wave and smaller for the slow wave as the porosity increases. The phase velocity becomes slower for the fast wave and faster for the slow wave as the porosity increases. The transmission coefficient becomes lower for the fast wave and higher for the slow wave as the porosity increases. The phase velocity and transmission coefficient of the fast wave decrease as the porosity increases while those of the slow wave increase as the porosity increases. This can be interpreted as the tortuosity increase of the fast wave and the tortuosity decrease of the slow wave as the function of porosity. The attenuation constant becomes higher for the fast wave and lower for the slow wave as the porosity increases.

IV. THEORETICAL AND EXPERIMENTAL RESULTS

The transmission coefficient and phase velocity of sand sediment slab are measured in water bath and theoretically calculated in terms of the MBA model.

The transmission coefficient and phase velocity of sediment slab are measured in terms of the pulse technique with a half wavelength in the broad band frequencies with center frequencies 1 MHz and 2.25 MHz. The standard deviations of the measurement data are within 5 percent. The sand sediment container with the size 42 mm $\times$ 100 mm $\times$ 100 mm is shown in Figure 1. Pores in the sediment slab are modeled as the structure of circular cylindrical tube shape filled with water and air. The pore direction is normal to the slab surface, the slab thickness is 42 mm, and the pore radius is 1.5 mm. The measured porosity of the pure water-saturated sediment without circular cylindrical pores is 0.43. Five types of sediment slabs hold total pore porosity values 0, 0.05, 0.11, 0.18, and 0.26 only for circular cylindrical pores. Nominal air porosity depends on the numbers of cylindrical pore tubes blocked by styrofoam balls and water porosity depends on the numbers of cylindrical pore tubes unblocked. Therefore, the total pore porosity in figures of this paper stands for water porosity plus air porosity only for circular cylindrical pores without accounting for the sediment porosity itself. In the case of water pores, actual total water porosities of the specimens are respectively 0.43, 0.46, 0.49, 0.53, and 0.58. In the case of air pores, actual water/air porosities of the specimens are respectively 0.43/0., 0.41/0.05, 0.38/0.11, 0.35/0.18, and 0.32/0.26.

Figure 2 shows 1 MHz transmitted signals through sediment slab with water-filled cylindrical pores as a function of time for the 1.5 mm radius. The first part of a receiving signal mainly corresponds to the signal through sediment slab and the second part does to the signals through pores. The fast and slow waves are separated due to the difference of travel time in sediment and water.

Experimental phase velocity of sediment slab is given as a function of water porosity in Figure 3. The phase velocity of the fast wave decreases as water porosity increases but the phase velocity of the slow wave increases as water porosity increases. The phase velocities of the fast and slow waves are almost same as air porosity increases. For the same porosity and pore size, the overall phase velocity does not change significantly as frequency increases around 1 MHz frequency.

Experimental transmission coefficient of sediment slabs is given as a function of air porosity for total wave in Figure 4. Figure 5 represents the 3D-plots of the theoretical transmission coefficient as the functions of water porosity and air porosity. Theoretical calculation uses the impedance parameter $s_3 = 5 \times 10^8\ Pa \cdot s/m$ for water pores and $s_3 = 5 \times 10^7\ Pa \cdot s/m$ for air pores. The attenuation coefficient for pure sediment material $\alpha = \alpha_0 f^n = 0.00003 f$ with the frequency f is used.

For the same porosity and pore size, the overall transmission coefficient decreases as frequency increases due to higher attenuation at higher frequency. Figures 6 show the experimental transmission coefficient as a function of frequency for water- and air-filled sediment slab. The difference of the transmission coefficient at the air and water porosity $(0,0)$ and the transmission coefficient at the air and water porosity $(0, 0.26)$ is due to the transmission contribution through pure sand sediment.

The transmission coefficient of the fast wave decreases as the water porosity increases at the constant air porosity. However, the transmission coefficient of the slow wave increases as the water porosity increases at the constant air porosity. The overall transmission coefficient in the region of low porosity is roughly dominated by the fast wave, but the overall transmission coefficient in the region of high porosity is dominated by the slow wave.

The transmission coefficient of the fast wave decreases as the air porosity increases at the constant water porosity. The transmission coefficient of the slow wave also decreases as the air porosity increases at the constant water porosity and becomes almost zero at the small water porosity. The overall transmission coefficient at low air and water porosity or at high air and low water porosity is dominated by the fast wave, but the overall transmission coefficient at high water and low air porosity is dominated by the slow wave.

Both phase velocities and transmission coefficients of the first kind wave and second kind wave in the solid and fluid mixed medium are sensitive to both air and water porosity. The transmission coefficient at an air porosity is lower than that at the same water porosity, but the phase velocity is almost same. The transmission coefficient depends on frequency but the phase velocity does not depend on frequency.

V. CONCLUSIONS

Transmission coefficient and phase velocity are measured as functions of water and air porosity in sediment slabs with water- and air-filled pores. Both phase velocities and transmission coefficients of the first kind (fast) and second kind (slow) waves in the solid and fluid mixed medium are sensitive to both air and water porosity. The phase velocity and transmission coefficient of the fast wave decrease as the porosity increases while those of the slow wave increase as the porosity increases. The transmission coefficient at an air porosity is lower than that at the same water porosity, but the phase velocity is almost same. The transmission coefficient depends on frequency but the phase velocity does not depend on frequency.

Acoustic characteristics of such porous medium in water are also theoretically investigated in terms of the modified Biot-Attenborough (MBA) model, which uses the separate treatment of viscosity effect and thermal effect in non-rigid porous medium with water- and air-filled pores. The concept of the tortuosity factor and dynamic shape factor for the slow wave may be generalized for the fast wave. Based on the measurements of the phase velocity and transmission coefficient, the parameterizations for the propagation constant and impedance are suggested as the functions of water and air porosity.

This study may be extended to investigate acoustic wave propagation in bubbly fluid-like sediment and in osteoporosis diagnosis.

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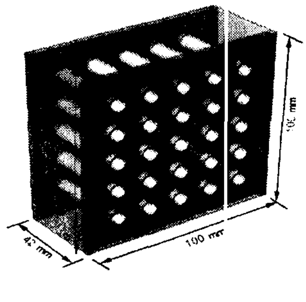

Fig. 1. Sand sediment container with water- and air- filled cylindrical pores of the radius 1.5 mm.

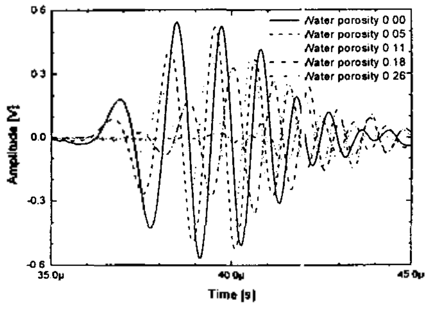

Fig. 2. 1 MHz transmitted signal as a function of time through sediment with water-filled pores.

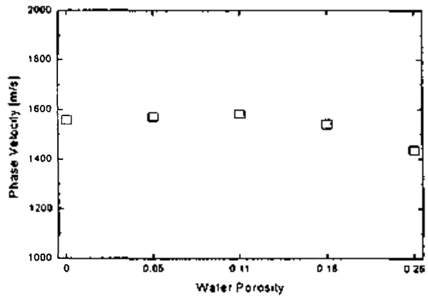

Fig. 3. Experimental phase velocity as a function of water porosity at the center frequency 1 MHz for water-filled sediment slab.

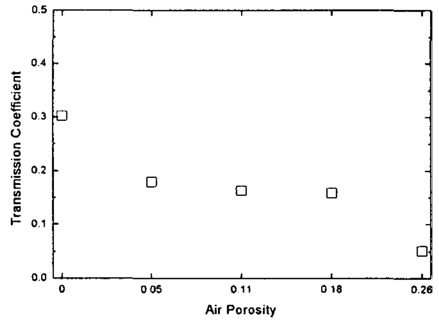

Fig. 4. Experimental transmission coefficient as a function of air porosity at the center frequency 1 MHz for air-filled sediment slab.

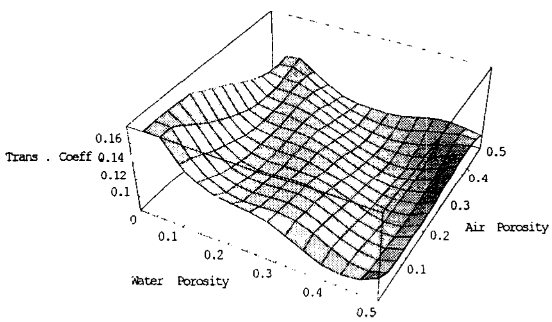

Fig.5.Theoretical 3D-plot of the transmission coefficient as the functions of water porosity and air porosity for the frequency 1 MHz.

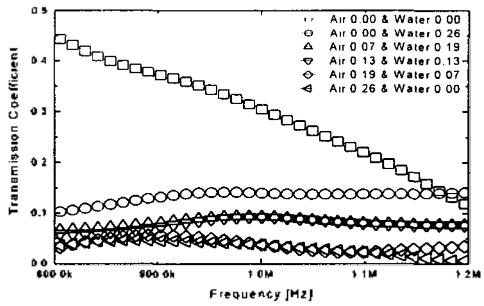

Fig. 6. Experimental transmission coefficient as a function of frequency for water- and air-filled sediment slab.