

## 전자기 과도 현상 해석을 위한 Z 영역에서의 주파수 의존 교류시스템 등가

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### Z-Domain Frequency Dependent AC System Equivalent for Electromagnetic Transient Studies

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**Abstract** - Modern power systems are very complex and to model them completely is impractical for electromagnetic transient studies. Therefore areas outside the immediate area of interest must be represented by some form of Frequency Dependent Network Equivalent (FDNE). In this paper a method for developing FDNE is presented and demonstrated. The FDNE is generated by Linearized Least Square fitting the frequency response of a z-domain formulation. The advantage of this approach is that a direct implementation occurs, which does not incur errors due to discretization inherent with implementing a fitting function in the s-domain. The developed FDNE is accurate and efficient.

## 1. INTRODUCTION

The analysis of power system transients can be accurately and efficiently performed using time domain techniques, however, detailed representation of a large complex power system will entail a prohibitive amount of computation. There is a need to represent only small portion of the power system in detail and to model the remainder of the system (external network) by an efficient equivalent. Conventional equivalents based on the fundamental frequency short circuit level are inadequate for representing the external networks behaviour when simulating transients, due to the presence of other frequency components. The basis of frequency dependent modelling is the relationship between the time and frequency domain. The time domain response is the convolution of the systems impulse response with the input excitation. In the frequency domain this convolution is a multiplication. If the frequency response is correctly represented then the correct time domain will follow. What is required is an equivalent that faithfully represents the external networks behaviour over a range of frequencies. The required frequency range is dependent on the phenomena under investigation, and hence the likely frequencies involved.

The use of Frequency Dependent Equivalents dates back to the late sixties with the pioneering work of Hingorani etc. Although many approaches have been adopted, most early research effort has been on modelling the

external system by an appropriate network of lumped R,L and C components whose values are chosen so the equivalent network will have the same frequency response as the external network. Using RLC components allows implementation in existing transient programs with minimal change, however it restricts the possible frequency response that can be represented.

This paper presents the formulation for developing FDNE using z-domain fitting of the frequency response and illustrates its use. This FDNE have been applied to the New Zealand lower South Island AC power system. The electromagnetic transient package PSCAD/EMTDC is used to assess the transient response of the 1-2 port FDNE developed.

The study results have indicated the robustness and accuracy of for developing 1/2 port FDNE for electromagnetic transient.

## 2. Z-DOMAIN FDNE

In general, there are three stages to using frequency dependent network equivalents (FDNE) in the Z-domain, they are: (1) Calculation of system response (Impedance or admittance) which the equivalent must mimic (2) Fitting of Model parameters (Identification Process) (3) Implementation of frequency dependent equivalent in transient simulation program.

The frequency dependent ac system equivalents cannot model nonlinearities, therefore all components exhibiting significant nonlinear behaviour must be removed from the ac system being equivalenced. This will increase the number of ports in the equivalent as the nonlinear component will be connected to the new port.

### 2.1 System Frequency Response

The starting point for creation of a frequency dependent equivalent is the calculation of the external system's behaviour by determining the driving point and transfer impedance (or admittance) matrices at the boundary busbar(s), over the frequency range of interest. The equivalent must mimic this behaviour. If experimental data is available this can be used

however very rarely is there sufficient data for model generation. This leaves either time or frequency domain identification techniques. In frequency domain identification it is frequency domain data that is used to identify the model parameters, however this frequency domain data can be obtained from time-domain or frequency domain simulation, as depicted in Fig. 1.

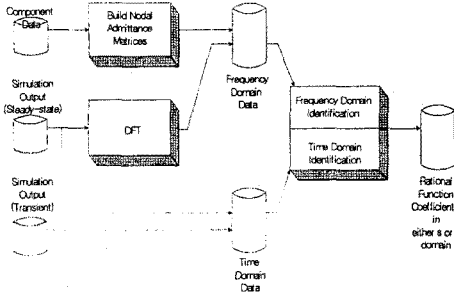


Fig. 1 Curve Fitting Options

## 2.2 Fitting of Model

One of the advantage of the z-domain fitting over s-domain fitting is that it gives a more accurate fit for a given order of rational function. This is because all the poles and zeroes are constrained to be in the frequency of interest (i.e. below the Nyquist frequency). Another advantage of z-domain fitting is that the accuracy of the simulation is clearly seen. Moreover FDNE developed by z-domain fitting does not suffer from numerical oscillations that are prevalent when a detailed representation is used. The z-domain is a representation of a discrete system and hence there is no error in its implementation on a digital computer. The fitting errors alone are present. Stability of the fit is essential, without it the system cannot be simulated. Testing the stability of the fit is easily achieved after performing the fit, however the illusive goal is to incorporate stability criteria as part of the fitting process. When simulating multi-terminal equivalents, such as three phase system with mutual coupling between phases, an admittance matrix rather scalar admittance must be fitted as functions of frequency. Although the fitting of each element in the matrix may be stable, inaccuracies in the fit can result in the complete system having instabilities at some frequencies. Again there is a need not to fit each element independently, but in such a way as to ensure the system of fitted terms are stable.

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + K + a_m z^{-m}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + K + b_m z^{-m}} \quad (1)$$

Evaluating the frequency response of the rational function and equating it to the required values gives:

$$H(j\omega) = \frac{\sum_{k=0}^m (a_k z^{-kj\omega\Delta t})}{1 + \sum_{k=1}^m (b_k z^{-kj\omega\Delta t})} \quad (2)$$

where  $b_0 = 1$

Using  $H(j\omega) = c(j\omega) + jd(j\omega)$  and Splitting into Real and Imaginary components (using  $e^{-kj\omega\Delta t} = \cos(k\omega\Delta t) - j\sin(k\omega\Delta t)$ ) gives:

$$\begin{aligned} -c(j\omega) &= \sum_{k=1}^m (b_k \cdot (c(j\omega) \cos(k\omega\Delta t) + d(j\omega) \sin(k\omega\Delta t))) \\ &\quad - a_k \cos(k\omega\Delta t) - a_0 \\ -d(j\omega) &= \sum_{k=1}^m ((b_k \cdot (d(j\omega) \cos(k\omega\Delta t) \\ &\quad - c(j\omega) \sin(k\omega\Delta t)) - a_k \sin(k\omega\Delta t)) \end{aligned} \quad (3)$$

In matrix form the set of equations to be solved is:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} C \\ D \end{pmatrix} \quad (4)$$

As the number of sample points exceeds the number of unknown coefficients singular value decomposition is used to solve this equation.

## 2.2 Implementation of Model

Given a rational function in Z, i.e.

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \Lambda a_m z^{-m}}{1 + b_1 z^{-1} + b_2 z^{-2} + \Lambda b_m z^{-m}} = \frac{I(z)}{V(z)} \quad (5)$$

Multiplying both sides by the denominators gives:

$$\begin{aligned} (1 + b_1 z^{-1} + b_2 z^{-2} + \Lambda b_m z^{-m})I(z) &= \\ (a_0 + a_1 z^{-1} + a_2 z^{-2} + \Lambda a_m z^{-m})V(z) & \\ \text{Rearranging gives} & \\ I(z) = a_0 V(z) + (a_1 z^{-1} + a_2 z^{-2} + \Lambda a_m z^{-m})V(z) + & \\ (b_1 z^{-1} + b_2 z^{-2} + \Lambda b_m z^{-m})i(z) & \end{aligned} \quad (6)$$

$$= G_{equiv} + I_{History}$$

Transforming back to discrete time:

$$\begin{aligned} i(n\Delta t) &= a_0 v(n\Delta t) + a_1 v(n\Delta t - \Delta t) + a_2 v(n\Delta t - 2\Delta t) + \\ &\quad K a_m v(n\Delta t - m\Delta t) + b_1 i(n\Delta t - \Delta t) + b_2 i(n\Delta t - 2\Delta t) + \\ &\quad K b_m i(n\Delta t - m\Delta t) \end{aligned} \quad (7)$$

$$i(n\Delta t) = G_{equiv} \cdot v(n\Delta t) + I_{History}$$

where  $G_{equiv} = a_0$

$$\begin{aligned} I_{History} &= a_1 v(n\Delta t - \Delta t) + a_2 v(n\Delta t - 2\Delta t) + K a_m v(n\Delta t - m\Delta t) + \\ &\quad + b_1 i(n\Delta t - \Delta t) + b_2 i(n\Delta t - 2\Delta t) + K b_m i(n\Delta t - m\Delta t) \end{aligned} \quad (8)$$

This is often referred to as an ARMA (autoregressive moving average) model. The coefficients represent the moving average (i.e.  $b_i = 0$ ). This is an all-zero or finite impulse response model. The coefficients represent the moving average (i.e.  $a_i = 0$ ). This is an all-pole or infinite impulse response model. Hence any

rational function in z-domain is easily implemented with no error as it is simply a Norton equivalent with the conductance being  $a_0$  and current source  $I_{History}$ .

### 3. ILLUSTRATIVE EXAMPLE

To illustrate the use of FDNE developed using z-domain fitting of the frequency response the New Zealand lower South Island AC system will be given in Fig. 2.

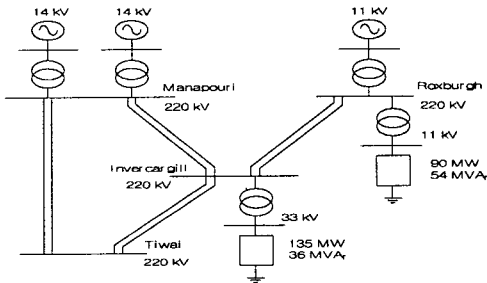


Fig. 2. Configuration of the New Zealand lower South Island AC system.

#### 3.1 1-port FDNE

The simulation package PSCAD/EMTDC was used to demonstrate the techniques and a custom component written to implement the z-domain rational function. Most of the complexity is in the deviation of the rational function coefficients which is performed by a utility program. The techniques can equally be applied to any electromagnetic transient program, as only the ability to represent a z-domain rational function is required. Due to lack of space only the salient points will be given.

Fig. 3,4,5 display the comparison of voltage between 1-port FDNE's and explicit representation of the New Zealand lower South Island AC system with linearload, fault and nonlinearload. The FDNE's is 12th order models and perform reasonably well. The FDNE gives smaller steady-state error and better accurate transient representation. The 1-port FDNE developed using z-domain matching is very good.

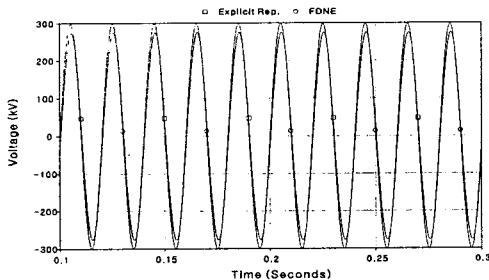


Fig. 3. Comparison of transient result for explicit represent and 1-port FDNE with linear load.

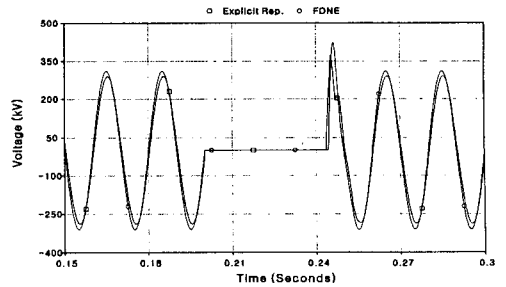


Fig. 4. Comparison of transient result for explicit represent and 1-port FDNE with Fault.

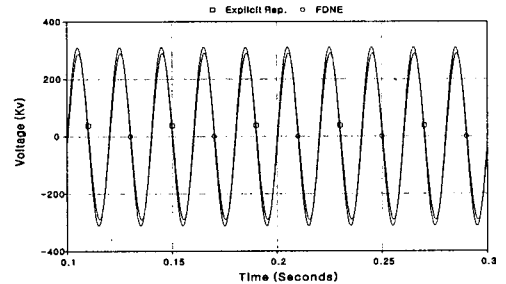


Fig. 5. Comparison of transient result for explicit represent and 1-port FDNE with nonlinear load.

#### 3.2 2-port FDNE

Figure 6,7,8 display the comparison of terminal voltage between 2-port FDNE's and explicit representation of the New Zealand lower South Island AC system with linearload, fault and nonlinearload. The FDNE's is 8th order models and perform reasonably well.

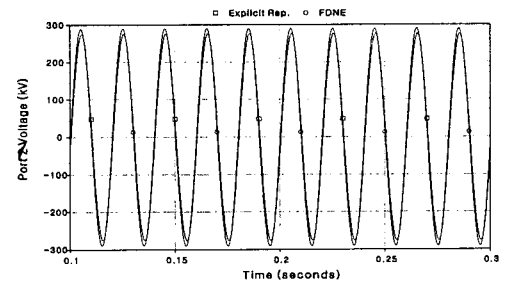
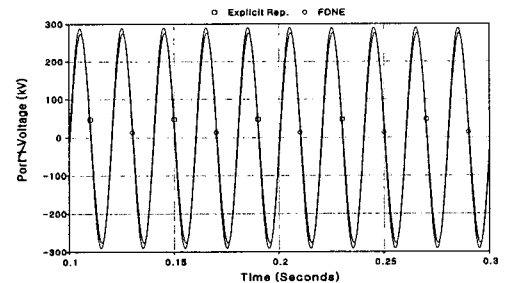


Fig. 6. Comparison of transient result for explicit represent and 2-port FDNE with linear load.

## 4. CONCLUSION

This paper has presented the use of z-domain fitting of a frequency response for developing frequency dependent network equivalents (FDNE) and the various issues have been discussed. The advantages are:

- 1) accuracy due to pole/zero placement in the simulation frequency spectrum
- 2) ease of implementation and that no error involved in its implementation.

Ensuring stability is also more difficult in the z-domain with no ad hoc methods available other than reducing the order and re-fitting. Incorporating stability criteria as part of the fitting process still needs to be achieved.

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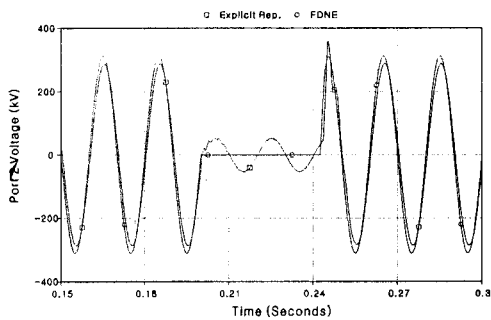
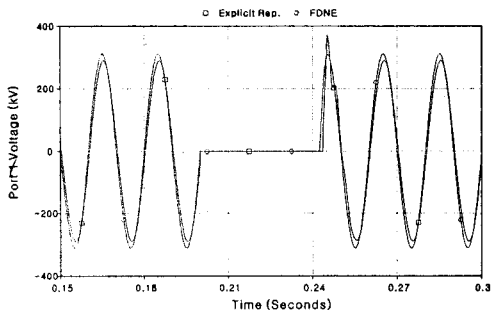


Fig. 7. Comparison of transient result for explicit represent and 2-port FDNE with fault.

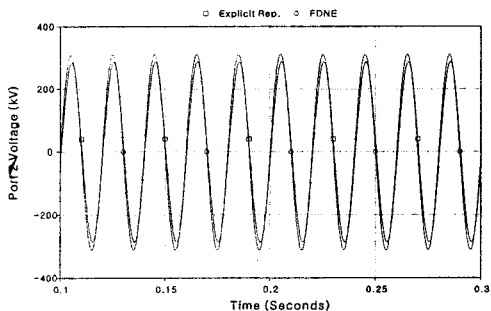
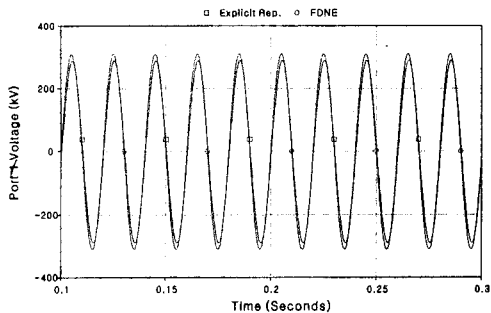


Fig. 8. Comparison of transient result for explicit represent and 1-port FDNE with nonlinear load.