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등각사상법과 유한요소법을 이용한 2단계 최적설계법

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A Novel Optimization Procedure Utilizing the Conformal Transformation Method

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Abstract - A large number of methods for the design optimization have been proposed in recent years. However, it is not easy to apply these methods to practical use because of many iterations. So, in the design optimization, physical and engineering investigation of the given model are very important, which results in an overall increase in the optimization speed.

This paper describes a novel optimization procedure utilizing the conformal transformation method. This approach consists of two phases and has the advantage of grasping the physical phenomena of the model easily. Some numerical results that demonstrate the validity of the proposed method are also presented.

1. INTRODUCTION

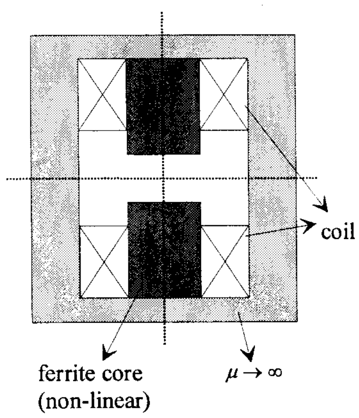
Electrical machinery makes use of the electromagnetic phenomena. So, to get hold of its characteristics, we analyze an electromagnetic field by the numerical analysis methods. In recent years, the application of optimization methods coupled with magnetic field analysis is receiving ever increasing attention, and a large number and variety of methods have been proposed [1]-[5]. These methods, however, are not always easy to be applied to practical use because of many iterations and a relatively large amount of CPU-time.

In this paper, with this background, we propose a novel optimization approach, which consists of two phases and perform an optimization procedure by combining magnetic field analysis with optimization methods. First, we apply the 2D theoretical analysis to electromagnetic field calculations and determine the shape of the investigated model broadly. The reason for the adoption of the theoretical method in the first phase is that we can grasp easily the physical phenomena of the subject thus examines the later developments. Next, we regard the shape obtained in the first phase as the initial one and perform the optimal design in more detail by the numerical analysis.

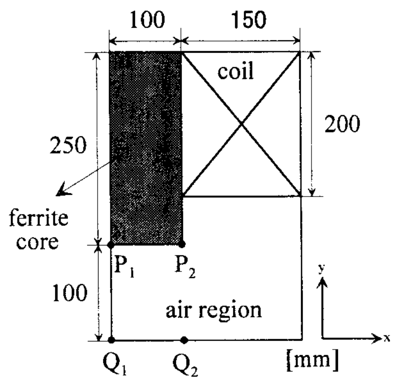
To show the validity and advantage of the proposed approach, we investigate an example whose goal is to make the magnetic flux density uniform on an observation line. This model is composed of a pair of ferrite cores and exciting coils. As the first phase, assuming the ferrite cores have the permeability of infinity, we apply the conformal transformation method as the theoretical analysis and calculate the magnetic flux density analytically [6]. By this method, we alter the shape of the ferrite cores to make the magnetic flux density uniform in the air-gap. As the second phase, we perform the optimal design in more detail by combining the finite element method (FEM) with the Simulated Annealing method (SA) subject to the ferrite cores with non-linear permeability [7]-[8]. This approach results in an overall increase in the speed of optimization.

2. INVESTIGATED MODEL

Figure 1(a) shows the investigated model. This model is composed of a pair of ferrite cores and exciting coils surrounding iron cores with the permeability of infinity. Utilizing the symmetry, we analyze the quadrant of the model as shown in Fig. 1(b).



(a) full view



(b) partial view
Fig. 1. Investigated Model

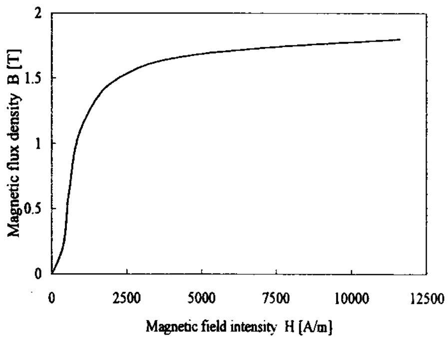


Fig. 2. B-H Characteristic Curve of the Material

The ferrite cores have the permeability of non-linear (carbon steels S45C) and are energized by direct current of 4.5×10^4 [AT]. The B-H characteristic curve of the material is shown in Fig. 2. The design variables are the air-gap length $P\sim Q$ and the goal of the optimization is to make the magnetic flux density $B_y=1.0$ [T] on the line $Q_1\sim Q_2$.

3. APPLICATION OF THE CONFORMAL TRANSFORMATION METHOD

As the first phase of our optimizing process, we apply the conformal transformation method to the investigated model. In order to make use of this method, we consider the surface of a pair of ferrite cores to be composed of planes whose normal direction is parallel and vertical to the x-axis as shown in Fig. 3. When we assume a pair of ferrite cores have the permeability of infinity, the magnetic field intensity at the X_P on the x-axis, which the point we are interested in, is influenced by planes whose normal direction is vertical (①) and parallel (②) to the x-axis. Therefore, we investigate each effect (① ,②) by using the conformal transformation method.

3. 1 The Influence of the Plan whose Normal Direction is Vertical to the X-axis

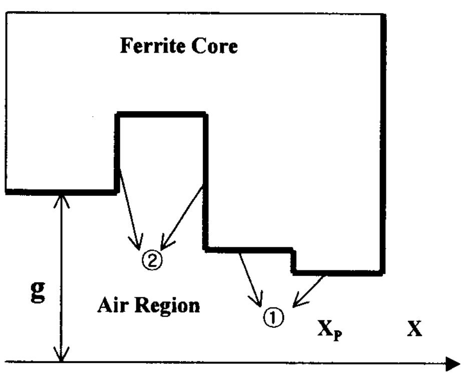


Fig. 3. Plans whose Normal Direction is Parallel and Vertical to the X-axis

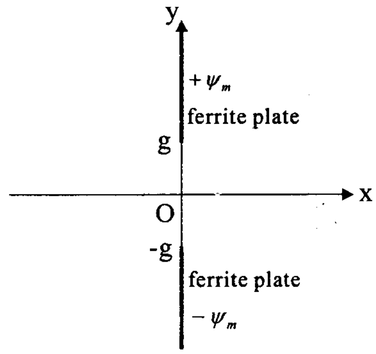


Fig. 4. Two Semi-infinite Ferrite Plates

Here, we consider two semi-infinite ferrite plates which are space 2g apart, and the magnitude of the magnetic scalar potential on the y-axis is Ψ_m as shown in Fig. 4. First we begin our study by considering the properties of a regular function of the type,

$$z=f(w)=x(u,v)+iy(u,v) \tag{1}$$

which defines a complex variable $z=x+iy$ as some function of another complex variable $w=u+iv$. The magnetic scalar potential at any point is divided from the imaginary part, v of (2).

$$z=a\ \sinh(bu+ibv) \tag{2}$$

where, $a=g$ and $b=\frac{\pi}{2}\frac{1}{\Psi_m}$

Expanding (2) and so we have

$$x=a\ \sinh(bu)\cos(bv)$$
$$y=a\ \cosh(bu)\sin(bv) \tag{3}$$

Squaring and adding these equations eliminates v to give,

$$\left(\frac{x}{a\sinh bu}\right)^2+\left(\frac{y}{a\cosh bu}\right)^2=1 \tag{4}$$

and squaring and substituting them eliminates u, so that

$$\left(\frac{y}{a\sin bv}\right)^2-\left(\frac{x}{a\cos bv}\right)^2=1 \tag{5}$$

Any straight line parallel to the $v-$axis has the equation $u=$ constant. For a constant value of u, (4) represents an ellipse in the $z-$plan, so that any straight line parallel to the $v-$axis is transformed by the equation $z=a\ \sinh(bw)$ into an ellipse in the $z-$plan. Any straight line parallel to the $u-$axis has the equation $v=$ constant, and form (5) it is seen that such a line is transformed into a hyperbola in the $z-$plan. This fact shows in Fig. 5. To express the aspect as shown in Fig. 4, it necessary to satisfy Fig. 4. Two Semi-infinite Ferrite Plates the following conditions from (5).

$$a \sin bv \rightarrow a$$
$$a \cos bv \rightarrow 0$$

From above condition, we obtain the following solution.

$$bv = \pm \frac{\pi}{2} \quad (6)$$

As the field in Fig. 4 is symmetrical with the $x-$axis, we show the upper aspect of one half as shown in Fig. 6 (a). We calculate the magnetic field intensity on the $x-$axis. From (3), we have

$$\begin{aligned} \partial x &= ab \cosh(bu)\cos(bv)\partial u - ab \sinh(bu)\sin(bv)\partial v \\ \partial y &= ab \sinh(bu)\sin(bv)\partial u + ab \cosh(bu)\cos(bv)\partial v \end{aligned} \quad (7)$$

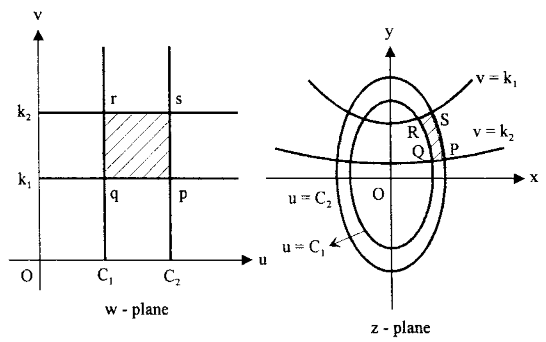
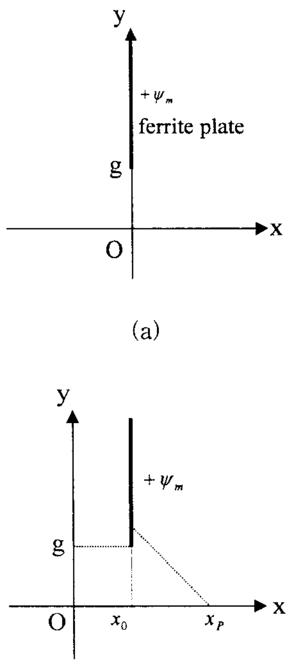


Fig. 5. Conformal Transformation



(b)
Fig. 6. One Half of the Field

By substituting $bv = \frac{\pi}{2}$ into (3), (7), we obtain the characteristics on the $x-$axis as follows:

$$v = o \quad (8)$$

$$\frac{\partial v}{\partial y} = \frac{1}{ab\cosh(bu)} \quad (9)$$

$$\frac{\partial u}{\partial x} = \frac{1}{ab\cosh(bu)} \quad (10)$$

where (9) represents the derivative of the magnetic scalar potential v with respect to y and is equal to the negative of magnetic field intensity, (10). Naturally, the following expression is held.

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} \quad (11)$$

Referring to (9) and (10), $\cosh(bu)$ can be written as follows.

$$\begin{aligned} \cosh(bu) &= \sqrt{1 + \sinh^2(bu)} \\ &= \sqrt{1 + \left(\frac{x}{a}\right)^2} = \frac{1}{a}\sqrt{a^2 + x^2} \end{aligned} \quad (12)$$

Therefore, the magnetic field intensity on the $x-$axis, H_y is obtained as follows.

$$\begin{aligned} H_y &= -\left(\frac{\partial v}{\partial y}\right) = -\left(\frac{\partial u}{\partial x}\right) \\ &= -\frac{1}{b\sqrt{a^2 + x^2}} = -\frac{2}{\pi}\frac{\Psi_m}{b\sqrt{a^2 + x^2}} \end{aligned} \quad (13)$$

When the ferrite plate is located as shown in Fig. 6 (b), (13) can be written as follows.

$$H_y = -\frac{2}{\pi}\frac{\Psi_m}{\sqrt{g^2 + (x_0 - x_P)^2}} \quad (14)$$

Now. we take infinitesimal length of the ferrite plate and consider its on the x_P as shown in Fig. 7. Hence, (14) can be expressed as follows.

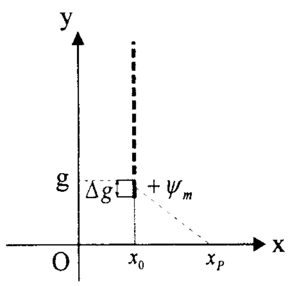


Fig. 7. Infinitesimal Length at (x_0, g)

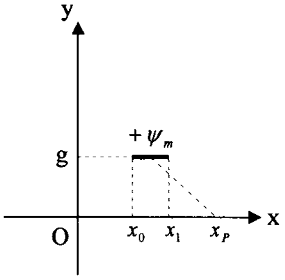


Fig. 8. Infinitesimal Part over the Interval $[x_0, x_1]$

Then as shown in Fig. 8, when the infinitesimal part is arranged over the interval $[x_0, x_1]$, the magnetic field intensity is calculated as follows.

$$H_{P_1} = \int_{x_0}^{x_1}\left(-\frac{\partial H_y}{\partial g}\right)$$

$$= -\frac{2\Psi_m}{\pi g}\left[\frac{x_1 - x_P}{\{g^2 + (x_1 - x_P)^2\}^{\frac{1}{2}}} - \frac{x_0 - x_P}{\{g^2 + (x_0 - x_P)^2\}^{\frac{1}{2}}}\right] \quad (16)$$

3. 2 The Influence of the Plan whose Normal Direction is parallel to the X-axis

Now, we consider the transformation,

$$z = aw + c\varepsilon^{bw} + c \quad (17)$$

where $a = \frac{g}{\Psi_m}$, $b = \frac{\pi}{\Psi_m}$ and $c = \frac{g}{\pi}$.

Expanding (17) and so we have

$$x = au + c\varepsilon^{bu}\cos(bv) + c \quad (18)$$

$$y = av + c\varepsilon^{bu}\sin(bv) \quad (19)$$

According with the previous study, we take the upper aspect of one half. Next, we calculate the magnetic field intensity on the x-axis. From (18) and (19), we have

$$\partial x = \{a + bc\varepsilon^{bu}\cos(bv)\}\partial u - bc\varepsilon^{bu}\sin(bv)\partial v$$

$$\partial y = bc\varepsilon^{bu}\sin(bv)\partial u + \{a + bc\varepsilon^{bu}\cos(bv)\}\partial v \quad (20)$$

By substituting $y=0$ into (18), (19) and (20), we obtain the characteristics on the x-axis as follows:

$$v = 0 \quad (21)$$

$$\frac{\partial v}{\partial y} = \frac{\Psi_m}{g}\frac{1}{1+\varepsilon^{bu}} \quad (22)$$

$$\frac{\partial u}{\partial x} = \frac{\Psi_m}{g}\frac{1}{1+\varepsilon^{bu}} \quad (23)$$

As stated above, the following expression is held naturally from (22) and (23).

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} \quad (24)$$

Therefore, the magnetic field intensity H_y on the x-axis is obtained as follows.

$$H_y = -\left(\frac{\partial v}{\partial y}\right) = -\frac{\Psi_m}{g}\frac{1}{1+\varepsilon^{bu}} \quad (25)$$

where

$$\varepsilon^{bu} = \varepsilon^{b\left(\frac{x}{2a} - \frac{c}{a}\right)} = \varepsilon^{\left(\frac{x}{2c} - 1\right)} \quad (26)$$

Now, we take infinitesimal length of the ferrite plate and consider its influence on the x_P as shown in Fig. 9. Hence, (25) can be expressed as follows.

$$-\frac{\partial H_y}{\partial x} = \frac{\Psi_m}{g}\frac{\partial\left(\frac{1}{1+\varepsilon^{bu}}\right)}{\partial u}$$

$$\approx -\frac{\Psi_m}{g^2}\frac{\varepsilon^{\left(\frac{1}{2c}x_P - 1\right)}}{\left\{1+\varepsilon^{\left(\frac{1}{2c}x_P - 1\right)}\right\}^3} \quad (27)$$

Then, as shown in Fig. 10, when the infinitesimal part is arranged over the interval $[g_0, g_1]$, the magnetic field intensity is calculated as follows.

$$H_{P_2} = \int_{g_0}^{g_1}\left(-\frac{\partial H_y}{\partial x}\right)dy$$

$$= \frac{\Psi_m \pi \varepsilon^{\left(\frac{1}{2c}(x_P - x_0) - 1\right)}}{\left\{1+\varepsilon^{\left(\frac{1}{2c}(x_P - x_0) - 1\right)}\right\}^3}\left(\frac{1}{g_1} - \frac{1}{g_0}\right) \quad (28)$$

Therefore, the total magnetic field intensity at the x_P is obtained from (16) and (28) as follows'

$$H_P = \sum H_{P_1} + \sum H_{P_2} \quad (29)$$

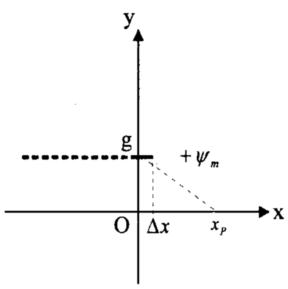


Fig. 9. Infinitesimal Length at $(0, g)$

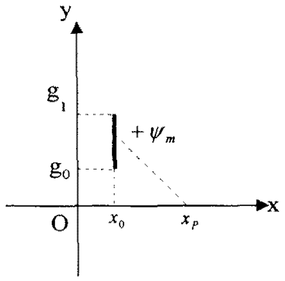


Fig. 10. Infinitesimal Part over the Interval [g_0, g_1]

The proposed optimization approach consists of two phases. The first phase is to set up a broadly optimal shape by the conformal transformation method. In this paper we deal with only a simple application, i.e. the salient magnet pole. However, it is applicable to a more complex application that is already developed by a ordinary text book [6]. For example, we can treat the slots and teeth configuration in electrical machine.

4. NUMERICAL EXAMPLE

The proposed optimization approach consists of two phases. The first phase is to set up a broadly optimal shape by the conformal transformation method. In this paper we deal with only a simple application, i.e. the salient magnet pole. However, it is applicable to a more complex application that is already developed by a ordinary text book [6]. For example, we can treat the slots and teeth configuration in electrical machine. The proposed optimization approach is verified by a concrete numerical example (Fig. 1). To make the magnetic flux density on the line $Q_1 \sim Q_2$ equal to 1.0[T], we alter the shape of the ferrite core ($P_1 \sim P_2$) by the proposed approach, which consists of two phases.

First, assuming the permeability of infinity, we utilize the conformal transformation method as the theoretical analysis and calculate the magnetic flux density by applying (29). By using the simplest algorithm, we determine the proper initial shape. At each iteration we modify only one of the design variables by the constant step-size so as to decrease the maximum error value $|B-1.0|_{\max}$ · $|B-1.0|_{\max}$ decreases with an increase in iteration and the magnetic flux density is converged to the target value with less than 3 percent. As the result of the first phase, the shape obtained is shown in Fig. 11. From the physical point of view, it is obvious the optimal shape is monotone decreasing with respect to the $x-$axis and this corresponds with the result by the conformal transformation method.

Next, making use of the above result, we determine the optimal design in more detail by combining the FEM with the SA under the condition that the ferrite cores have the permeability of non-linear. The numbers of nodes and elements are 1132 and 2128 respectively. The seven design variables which represent the air-gap length $P \sim Q$ are shown in Fig. 12. As the result of the second phase, Fig. 13 and Fig. 14 show the optimal shape and the magnetic flux density on the line $Q_1 \sim Q_2$. The magnetic flux density of this approach is converged to the target value with less than 2 percent.

To confirm the validity and advantage of the proposed approach, we have analyzed this model by applying the conventional approach which does not utilize the first phase. By this approach, we obtain the optimal shape and the magnetic flux density as shown in Fig. 15 and Fig. 16 respectively. The magnetic flux density of the conventional approach is converged to the target value with less than 3 percent. However, to achieve the optimal shape, this approach needs about 8800 iterations whereas the proposed approach about 2000 iterations. These results are summarized in Table 1. Furthermore, as the conventional method does not grasp physical phenomena, the shape obtained is hard to manufacture. Therefore, we show the validity and advantage of the proposed approach.

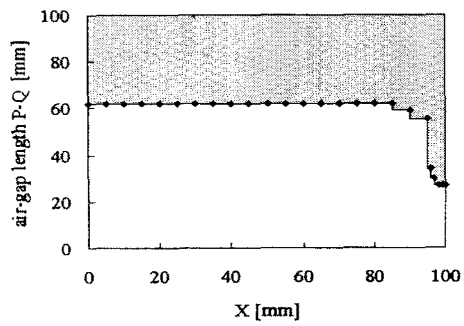


Fig. 11. Proper Initial Shape

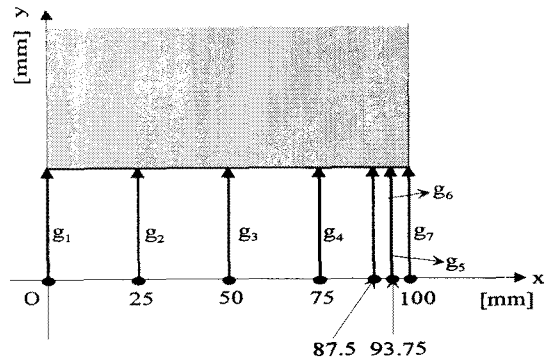


Fig. 12. Design Variables

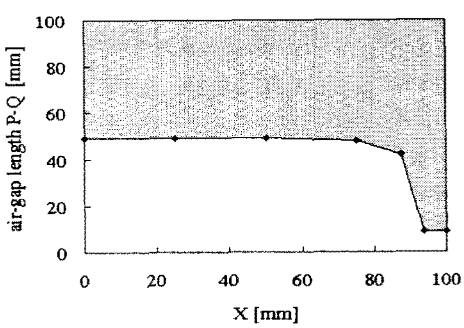


Fig. 13. Final Optimal Shape (Proposed Approach)

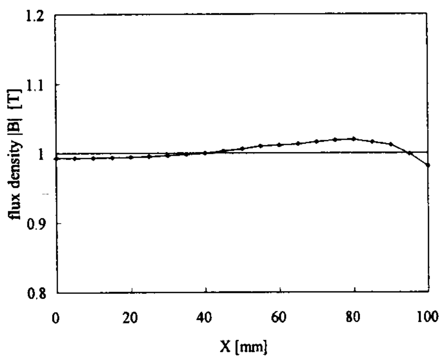


Fig. 14. The Magnetic Flux Density by the Proposed Approach

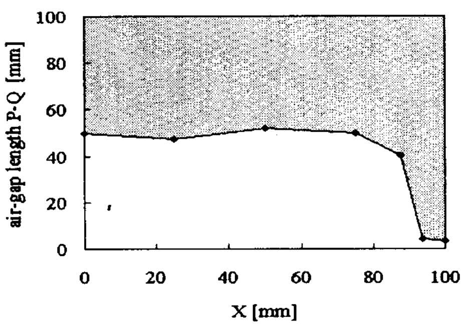


Fig. 15. Final Optimal Shape (Conventional Approach)

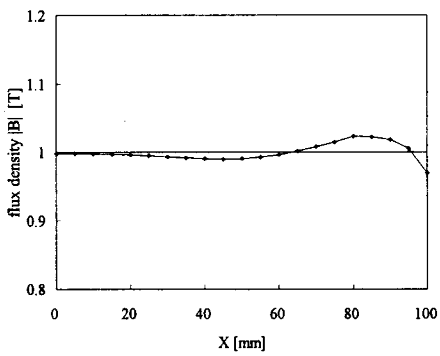


Fig. 14. The Magnetic Flux Density by the Conventional Approach

Table 1. CPU-time and Iteration

	Conventional Approach	Proposed Approach
CPU-time [%]	100	21.4
Iteration	8800	2000

5. CONCLUSION

In this paper, we propose a novel optimization procedure which consists of two phases for the purpose of increasing the overall optimization speed. This approach, in which the conformal transformation method is utilized in the first phase, has the advantage of grasping physical phenomena of the model and determining the proper initial shape. As the second phase, we regard the shape obtained in the first phase as the initial one and perform the optimal design in more detail by the numerical analysis. To show the validity and advantage of this method, a concrete numerical example is presented. As a result, the optimal shape is obtained much faster in the case of the conventional method.

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