A FILTERING CONDITION AND STOCHASTIC ADAPTIVE CONTROL USING NEURAL NETWORK FOR MINIMUM-PHASE STOCHASTIC NONLINEAR SYSTEM

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ABSTRACT

In this paper, some geometric condition for a stochastic nonlinear system and an adaptive control method for minimum-phase stochastic nonlinear system using neural network are provided. The state feedback linearization is widely used technique for excluding nonlinear terms in nonlinear system. However, in the stochastic environment, even if the minimum phase linear system derived by the feedback linearization is not sufficient to be controlled robustly. In the viewpoint of that, it is necessary to make an additional condition for observation of nonlinear stochastic system, called perfect filtering condition. In addition, on the above stochastic nonlinear observation condition, I propose an adaptive control law using neural network. Computer simulation shows that the stochastic nonlinear system satisfying perfect filtering condition is controllable and the proposed neural adaptive controller is more efficient than the conventional adaptive controller

1. INTRODUCTION

In the past two decades, differential geometry has provide to be an effective means of analysis and design of nonlinear control system as it was in the past for a linear algebra in relation to linear system. Through the study of many researchers in the nonlinear control theory, the geometric approach has been shown to be very efficient in solving various synthesis problems of linear and nonlinear systems as noninteracting control problems and disturbance decoupling problems which has constant DC disturbance or linear combinatorial noise with wide range frequency [6].

However, in the nonlinear system including white noise, the disturbance decoupling problem is not the same to the conventional problem. The reason is that the nonlinear system including white noise is not a system with simple The white noise can represents an uncountably large value even if the probability of that is very low. Hence, essentially, the nonlinear system with a white noise is not a bounded input system in that the disturbance generated by a white noise looks like another input. Consequently, the control of the stochastic system needs very robust control law such that LQ control and various kinds of Kalman filters[8].

Adaptive control is other choice of a nonlinear system control. Over the last 3 decades, adaptive control theory has evolved as a powerful methodology for designing nonlinear feedback controllers for system with parametric uncertainty[2]. However, in the nonlinear control system, adaptive control was not seriously considered until recently. The reason is that adaptive control law is basically based on the linear system, thus, it was very difficult to apply to the nonlinear system. Recently, as the techniques in adaptive control of nonlinear systems were facilitated by advances in geometric nonlinear control theory, in particular, feedback linearization method[6], new adaptive control strategies such as the backstepping procedure, tuning functions, has been developed.

Neural network techniques have been found to be particularly useful for controlling highly uncertain, nonlinear, and complex systems. The feasibility of applying neural network architecture for modeling unknown functions in dynamic environments has been demonstrated by several studies. Most of these studies are based on gradient techniques for deriving parameter adaptive law for system identification. While such schemes are perform well in many cases, in general, there are no systematic analytical methods of ensuring the stability, robustness, and performance properties of the overall systems.

In an attempt to overcome these problems, there have been recent studies of neural network learning algorithms based on Lyapunov's stability theory. The advantage of these training method is that the adaptive law is derived based on the Lyapunov synthesis law and therefore guarantee the stability of the system.

Even though the adaptive control law of nonlinear system control with neural network is derived by Lyapunov's theory, there exist a critical problem. Since most neural network includes the nonlinear functions such as hyperbolic tangent, Gaussian distribution function and various kinds of penalty functions, it is very difficult to develop a certain canonical control law for general case of nonlinear systems. In many cases, therefore, derived control law is well worked in a stability guaranteed region or for some particular nonlinear system.

In addition, it is necessary to develop a novel state space model of neural network. In the nonlinear system control using the feedback linearization method, the nonlinear system is described as the linear state space model called as state space model of neural network, it may be easy to develop the control law for nonlinear system control using neural network .

Furthermore, the feedback linearization for stochastic nonlinear system control needs some additional filtering condition. In attempt to control the stochastic nonlinear system, it is very reasonable viewpoint that states describing system dynamics have to be estimated correctly in the environment of white noise. However, in marked contrast to the case of linear system, the state transition probability has a purely nonlinear transition property governed by estimation (Lie) algebra [3]. Therefore, an innovation process generated by the difference of a system output and estimated output is not defined globally, thus, the conventional Kalman-Bucy filter or extended Kalman filter cannot work correctly in whole R^n.

Consequently, the introduction of an additional filtering condition is prerequisite to a stochastic nonlinear system with the feedback linearization, and using the filtering condition, the state estimation of nonlinear system with feedback linearization can be transformed the conventional linear stochastic system[4].

This paper organized as follows. Section 2 discuss the basic theory of the paper such that feedback linearization, neural network, the filtering condition of stochastic nonlinear system with feedback linearization. In section 3, the adaptive control law for neural network controller of the nonlinear system with linearization input is provided. Finally, section 4, 5 represents the computer simulation of the proposed control algorithm and conclusion, respectively.

2. FUNDAMENTAL THEORY

2.1. Feedback Linearization

Consider the following single input single output system[6][1]

$$\begin{aligned} \dot{x} &= f(x)+g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where x is time dependent system state vector such that $x \in R^n$, f, g, h is C^∞ functionals mapping R^n into R^n, R into R^n and R^n into R, respectively and y is system output such that $y \in R$.

Suppose that there exist the relative degree r which is $r \in Z(0,n]$ [6] such that the row vectors of differential

$$dh(x^o), dL_f h(x^o), \ldots, dL_f^{r-1}h(x^o) \tag{2}$$

are linearly independent, where $L_f h(x)$ is the Lie derivation for $h(x)$ along with the vector field $f(x)$. Then, the feedback linearization input u is obtained as follows [6]:

$$u = \frac{1}{L_g L_f^{r-1}h(x)}\left(-L_f^r h(x)+v\right) \tag{3}$$

where v is reference input.

The state feedback u transform the given nonlinear system (1) to the following linear system with the coordinate transform $z = \Phi(x)$.

$$\begin{aligned} \dot{z} &= Az+Bv \\ \dot{z}_i &= q_i(z) \qquad \forall i \in Z[r+1,n] \\ y &= Cz \end{aligned} \tag{4}$$

where A, B, C are matrices such that $A \in R^{r\times r}$, $B \in R^r$, $C^T \in R^r$ and are detailed as follows

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \cdots & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ 1 \end{pmatrix} \tag{5}$$

$$C = (1, 0, \ldots, 0)$$

If the relative degree r equals to n, then the zero dynamics $\dot{z}_i = q_i(z) \forall i \in Z[r+1,n]$ is diminished. It is similar to non zero system in linear system, thus the zero dynamics is absolutely stable. The nonlinear system with an absolutely stable zero dynamics is called as minimum phase system[1]. The discussion in the paper is only about the minimum phase system.

2.2. The Filtering Condition for Feedback Linearization

Let $(\Omega, \mathcal{F}, P)$ be a probability space and $\{\mathcal{F}_t, t \geq 0\}$ be a filtration defined on it. A process $\{W_t, t \geq 0\}$ is called an $\mathcal{F}_t$ measurable Wiener process satisfying that $W_0 = 0$ w.p.1. $\mathcal{F}(W_t - W_s : t \geq s)$ is independent of $\{\mathcal{F}_t, \forall t \geq 0\}$, the increments $w_t - w_s$ are normally distribution with mean 0 and variance $\sigma^2 > 0, \forall t \geq s \geq 0$, and the sample path of W_t are in $C[0,\infty)$ where $C[0,T)$ denotes the space of those functions from $[0,T]$ into $R[?]$. In addition, suppose that the following stochastic nonlinear system [4]

$$\begin{aligned} dX_t &= (f(X_t)+g(X_t)u)\,dt + b(X_t)dW_t \\ Y_t &= h(X_t) \end{aligned} \tag{6}$$

where $X_t \in R^n$ and $Y_t \in R$ are both $\mathcal{F}_t$ measurable Ito stochastic process and sample path continuous, f is an R^n valued vector field such that f is second differentiable continuous and has compact support i.e. $f \in C_0^2(R^n)$.

In attempt to obtain the filtering condition, define the following infinitesimal operator

Definition 1 *Let X_t be the Ito process defined by 6 and $f \in C_0^2(R^n)$. Then f in the domain of the infinitesimal operator $\mathcal{L}_{f,b}$ i.e. $f \in \mathcal{D}_{\mathcal{L}_{f,b}}$ such that*

$$\mathcal{L}_{f,b}\Psi(x) = L_f\Psi(x) + \frac{1}{2}b(x)\cdot H(\Psi(x))b(x) \tag{7}$$

where $H(\Psi(x))$ is a Hessian of $\Psi(x)$. In addition, define an operator $H_b\Psi(x)$ such that

$$H_b\Psi(x)b(x)\cdot H(\Psi(x))b(x) \tag{8}$$

Furthermore, to use the stochastic differential geometry, introduce the following definition[7].

Definition 2 *The space $\mathcal{M}$ is the family of all continuous locally square integrable martingales M_t relative to $\mathcal{F}_t$ such that $M_0 = 0$ a.s.*

The space $\mathcal{A}$ is the the family of all continuous $\mathcal{F}_t$ adapted process A_t such that $A_0 = 0$ and $t \mapsto A_t$ is of bounded variation on every finite interval a.s.

The space $\mathcal{B}$ is the the family of all predictable $\mathcal{F}_t$ process Φ_t such that, w.p.1, $t \mapsto \Phi_t$ is of bounded on each bounede interval a.s.

The space $\mathcal{O}$ is the totality of quasimartingales such as $\mathcal{M} \oplus \mathcal{A}$ defined on $\mathcal{F}_t$

Since all Ito process X_t is quasimartingale, it is reasonable to let $d\mathcal{O} = \{dX_t : X_t \in \mathcal{O}\}$, $d\mathcal{M} = \{dM_t : M_t \in \mathcal{M}\}$, and $d\mathcal{A} = \{dA_t : A_t \in \mathcal{A}\}$. Moreover, the following lemma can be introduced.

Lemma 1 *The following stochastic differential space properties are hold*

$$d\mathcal{O}\cdot d\mathcal{O} \subset d\mathcal{A},\; d\mathcal{A}\cdot d\mathcal{O} = 0,\; d\mathcal{O}\cdot d\mathcal{O}\cdot d\mathcal{O} = 0 \tag{9}$$

Through the Definition 1 to Lemma 1, the following proposition is obtained [4]

Proposition 1 *In the minimum phase nonlinear system, if the relative degree r satisfies the following filter condition*

$$\begin{aligned} L_b\mathcal{L}_{f,b}^i h(x^0) &= 0\,\forall x \in B\{x^o,\varepsilon\},\ \forall 0 \leq i \leq r-1 \\ E_{\mathcal{F}_{D_{d^r h(X_t)}}} L_b\mathcal{L}_{f,b}^{r-1}h(x^0) &= 0 \end{aligned} \tag{10}$$

and the relative degree r satisfies the following

$$L_g\mathcal{L}_{f,b}^i h(x^0) = 0\,\forall x \in B\{x^o,\varepsilon\},\ \forall 0 \leq i \leq r-1 \tag{11}$$

then the coordinate transform function $\dot{Z}_t = \dot{\Phi}(X_t) = [d^{(k)}h]$ is derived such that

$$\begin{aligned} d^k h(X_t) &= \mathcal{L}_{f,b}^k h(x)dt & \forall k < r \\ d^r h(X_t) &= v + L_b\mathcal{L}_{f,b}^{r-1}h(x)dW_t & k = r \end{aligned} \tag{12}$$

with the following pathwise state feedback

$$u = \frac{1}{L_g\mathcal{L}_{f,b}^{r-1}h(X_t)}\left(-\mathcal{L}_{f,b}^r h(X_t)+v\right) \tag{13}$$

proof : In case of $i = 0$, $i < r$, since $L_g h(X_t) = 0$ and $L_b \mathcal{L}^0_{f,b} h(X_t) = 0$ by assumption, we obtain the following equation.

$$\begin{aligned} dh(X_t) &= \left(L_f h(X_t) + L_g h(X_t) + \tfrac{1}{2} H_b h(X_t)\right) dt \\ &+ L_b h(X_t) dW_t \\ &= \mathcal{L}_{f,b} h(X_t) dt \end{aligned} \tag{14}$$

In addition, for $i = 1$, the equation for $d^2 h(X_t)$ is

$$\begin{aligned} d^2 h(X_t) &= \left(L_f \mathcal{L}_{f,b} h(X_t) + \tfrac{1}{2} H_b \mathcal{L}_{f,b} h(X_t)\right. \\ &+ L_g \mathcal{L}_{f,b} h(X_t) dt + L_b \mathcal{L}_{f,b} h(X_t) dW_t \\ &= \left(\left(L_f \left(L_f h(X_t) + \tfrac{1}{2} H_b h(X_t)\right)\right)\right. \\ &+ \tfrac{1}{2} H_b \left(L_f h(X_t) + \tfrac{1}{2} H_b h(X_t)\right) \\ &+ L_g \left(L_f h(X_t) + \tfrac{1}{2} H_b h(X_t)\right) dt \\ &+ L_b \mathcal{L}_{f,b} h(X_t) dW_t \\ &= \left(\mathcal{L}^2_{f,b} h(X_t) + L_g \mathcal{L}_{f,b}\right) dt \\ &+ L_b \mathcal{L}_{f,b} h(X_t) dW_t \\ &= \mathcal{L}^2_{f,b} h(X_t) dt \end{aligned} \tag{15}$$

since $L_b \mathcal{L}_{f,b} h(X_t) = 0$ and $L_g \mathcal{L}_{f,b} h(X_t) = 0$ because $i < r$.

Assume that the following equation is true for k such that $r > k+1$,

$$d^k h(X_t) = \mathcal{L}^k_{f,b} h(X_t) dt \tag{16}$$

then, the differential $d^{k+1} h(X_t)$ is calculated by

$$\begin{aligned} d^{k+1} h(X_t) &= \left(L_f \mathcal{L}^k_{f,b} h(X_t) + \tfrac{1}{2} H_b \mathcal{L}^k_{f,b} h(X_t)\right. \\ &+ L_g \mathcal{L}^k_{f,b} h(X_t) dt + L_b \mathcal{L}^k_{f,b} h(X_t) dW_t \\ &= \left(\mathcal{L}^{k+1}_{f,b} h(X_t) + L_g \mathcal{L}^k_{f,b} h(X_t)\right) dt \\ &+ L_b \mathcal{L}^k_{f,b} h(X_t) dW_t \\ &= \mathcal{L}^{k+1}_{f,b} h(X_t) dt \end{aligned} \tag{17}$$

since $L_b \mathcal{L}^k_{f,b} h(X_t) = 0$ and $L_g \mathcal{L}^k_{f,b} h(X_t) = 0$

In contrast to the case of $r > k+1$, for $k = r-1$, the differential $d^r h(X_t)$ is as follows :

$$\begin{aligned} d^r h(X_t) &= \left(L_f \mathcal{L}^{r-1}_{f,b} h(X_t) + \tfrac{1}{2} H_b \mathcal{L}^{r-1}_{f,b} h(X_t)\right. \\ &+ L_g \mathcal{L}^{r-1}_{f,b} h(X_t) \cdot u dt + L_b \mathcal{L}^{r-1}_{f,b} h(X_t) dW_t \\ &= \left(\mathcal{L}^r_{f,b} h(X_t) + L_g \mathcal{L}^{r-1}_{f,b} h(X_t) \cdot u\right) dt \\ &+ L_b \mathcal{L}^{r-1}_{f,b} h(X_t) dW_t \end{aligned} \tag{18}$$

Consequently, since the filter condition $E_{\mathcal{F}_{D_{d^r h(X_t)}}} L_b \mathcal{L}^{r-1}_{f,b} h(x^0) = 0$ is hold, if the control u is given by $u = \alpha(x) + \beta(x) v$, then u is derived by the equation (18) as follows :

$$\begin{aligned} E_{\mathcal{F}_{D_{d^r h(X_t)}}} & d^r h(X_t) \\ &= E_{\mathcal{F}_{D_{d^r h(X_t)}}} \left(\mathcal{L}^r_{f,b} h(X_t) + L_g \mathcal{L}^{r-1}_{f,b} h(X_t) \cdot u\right) dt \\ &+ E_{\mathcal{F}_{D_{d^r h(X_t)}}} L_b \mathcal{L}^{r-1}_{f,b} h(X_t) dW_t \\ &= \left(\mathcal{L}^r_{f,b} h(X_t) + L_g \mathcal{L}^{r-1}_{f,b} h(X_t) \cdot u\right) dt \Rightarrow \\ v &= E_{\mathcal{F}_{D_{d^r h(X_t)}}} \frac{d^r h(X_t)}{dt} \\ &= \mathcal{L}^r_{f,b} h(X_t) + L_g \mathcal{L}^{r-1}_{f,b} h(X_t) \cdot u \end{aligned} \tag{19}$$

(Q.E.D)

However, in the Proposition 1, since $L_b \mathcal{L}^{k-1}_{f,b} h(x) dW_t$ is not a linear term, the noise term in stochastic feedback linearization system does not represent a linear form, generally. Hence, it is necessary that $b(x)$ have to be constructed in order that $L_b \mathcal{L}^{k-1}_{f,b} h(x) dW_t$ be linear or finds optimal linear, nonlinear filter strategy such that Beneš filter [3]

3. ADAPTIVE CONTROL USING NEURAL NETWORK FOR MINIMUM PHASE STATE FEEDBACK LINEARIZATION SYSTEM

In the paper, neural network is not used as estimated system but used as pure adaptive controller. Therefore, learning equation based on the gradient rule for the conventional error function have to be modified as the gradient rule based on a Lyapunov function for the system. Subsequently, since neural network is not a estimated model, there have to exist a reference model such that

$$\begin{aligned} \dot{z}_m &= A_m z_m + B_m u_c \\ y &= C_m z_m + D_m u_c \end{aligned} \tag{20}$$

where $A_m \in R^{n\times n}, B_m \in R^n, C_m \in R^{1\times n}, D_m \in R$ and $u_c \in R$ is control input. In order to induce the neural network control law, the following assumptions are predefined.

Assumption 1 *There exist a solution P of the Lyapunov equation such that*

$$AP + PA = -Q \tag{21}$$

where P and Q are a positive definite n by n matrix. In addition, the control input of linearization system is defined as follows

$$u = -L(\theta(t), X_t) + u_c \tag{22}$$

where L is a pole placement function and $\theta(t)$ is parameter

Since the neural network is used as the pole placement function, the output of neural network have to be linear function of weight and the output of previous layer. Since the perceptron with pure linear output is a linear combination of weight and state vector, the neural network as pole placement controller is designed with 1 perceptron tuned by the error value defined as follows:

$$e_t = Z_t - z_m \tag{23}$$

where Z_t is the coordinate transformed state vector such that the k-th component of the state is $Z_{tk} = \mathcal{L}^k_{f,b} h(X_t)$. Let the Lyapunov function as follows:

$$V(t) = \frac{\eta}{2}\{e_t^T P e_t + (\theta - \theta^o)^T (\theta - \theta^o)\} \tag{24}$$

where $\eta \in R$ is adaptation gain such that $0 < \eta \ll 1$. So as to analysis the equation (24), set the following assumption.

Assumption 2 *There exist the coordinate transformed linear system and reference model such that*

$$\begin{aligned} dZ_t &= (AZ_t + Bu)dt + \Gamma dW_t \\ Y_t &= CZ_t \\ z_m &= A_m z_m + Bu_c \\ y_m &= Cz_m \end{aligned} \tag{25}$$

and the pole placement function is a linear combination such that

$$L(\theta(t), X_t) = L(\theta(t)) \cdot X_t \;\; L(\theta(t)) \in R^{1\times n} \tag{26}$$

In addition, there exist a relation between A and A_m such that

$$A_m = A - BW(0) \tag{27}$$

From the equation (23), (25), the error dynamics is

$$\begin{aligned} de_t &= (AZ_t - BL(\theta(t))Z_t + Bu_c - A_m z_m - Bu_c)dt + \Gamma dW_t \\ &= (AZ_t - BL(\theta(t))Z_t + A_m Z_t - A_m Z_t - A_m z_m)dt + \Gamma dW \\ &= A_m e_t + (A - BL(\theta(t)) - A_m) Z_t dt + \Gamma dW_t \end{aligned} \tag{28}$$

Since the perceptron is pole placement controller, substitute (??) to the equation (26), then the equation (30) is modified as follows :

$$de_t = A_m e_t + (A - BW(t) - A_m) Z_t dt + \Gamma dW_t \tag{29}$$

Since the equation (27) and $\theta_t = W(t)$, the stochastic differential of Lyapunov function is that

$$\begin{aligned} dV_t &= \tfrac{\eta}{2} de_t^T P e_t + (W(t) - W(0)) \tfrac{\partial W_t}{\partial t}^T dt \\ &= \tfrac{\eta}{2} (A_m e_t + (A - BW(t) - A + BW(0)) Z_t dt + \Gamma dW_t)^T Pe \\ &+ e_t P (A_m e_t + (A - BW(t) - A + BW(0)) Z_t dt + \Gamma dW_t) \\ &+ (W(t) - W(0)) \tfrac{\partial W(t)}{\partial t}^T dt \\ &= \left(\tfrac{\eta}{2} \left(e_t^T (A_m P + P A_m) e_t^T right \right) \right) \\ &- \eta Z_t^T (W(t) - W(0))^T B^T Pe + (W(t) - W(0)) \tfrac{\partial W(t)}{\partial t}^T dt \\ &+ \eta dW_t \Gamma^T P e_t \end{aligned} \quad (30)$$

Since $Z_t^T (W(t) - W(0))^T = (W(t) - W(0)) Z_t$ and $\eta dW_t \Gamma^T P e_t = \eta \|\Gamma\|_P$, set $\frac{\partial W(t)}{\partial t} = \eta e^T P B Z_t^T$, then the equation (referr01) is converted as follows:

$$dV_t = -\frac{\eta}{2} e_t^T Q e_t^T + \eta \|\Gamma\|_P \quad (31)$$

Therefore, the following proposition can be obtained.

Proposition 2 *If there exist positive definite symmetry matrix Q such that $\sup_{\|x\|=1} \|Qx\| > 2 \|Gamma\|_P$, then the learning equation of neural controller which makes system to be stabilized such that*

$$\frac{\partial W(t)}{\partial t} = \eta e^T P B Z_t^T \quad (32)$$

The equation (32) is hold, then the system input is as follows:

$$u = -W(t) Z_t + u_c \quad (33)$$

4. SIMULATION RESULTS

The proposed control law and filtering condition is verified with single input single output (SISO)nonlinear system. In the computer simulation, a considered nonlinear system is the elastic coupling one-link system of which dynamics is as follows

$$J_1 \ddot{q}_1 + F_1 \dot{q}_1 + \frac{K}{N}(q_2 - \frac{q_1}{N}) = T \quad (34)$$

$$J_2 \ddot{q}_2 + F_2 \dot{q}_2 + K(q_2 - \frac{q_1}{N}) + mgd\cos(q_2) = 0 \quad (35)$$

in which J_1 and F_1 represent inertia and viscous friction constants, K the elacity constant of the spring which represents the elastic coupling with the joint, N the transmission gear ratio, T is torque produced at the actuator axis, m and d represent the mass and the position of the center of gravity of the link, respectively. The equation (34) is called as the actuator equation and (35) is called as the link equation. In order to satisfy the filtering condition, a noise effect vector b is set as $(0, 0, \sigma^2, 0)$ such that $\sigma = 0.1$. The characteristic equation of the basic model is $C(s) = s^4 + 5s^3 + 8.75s^2 + 6.25s + 1.5$. In this simulation, the reference model is the basic linear system described as $C(s)$ including LQ controller. In the comparison to the conventional LQ controller, the proposed control law represents a better performance than LQ controller. The mean square error of the proposed algorithm is 10.086 and that of LQ control is 22.11.

5. CONCLUSIONS

In this paper, the pole placement type adaptive neural network controller for a nonlinear system with state feedback linearization is provided. In addition, the filtering condition for state feedback in stochastic nonlinear system. However, the work of pure nonlinear filtering using estimation algebra and the development of a robust filtering algorithm is still remains.

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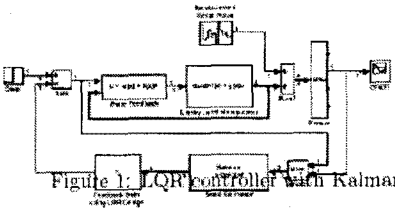

Figure 1: LQR controller with Kalman filtering

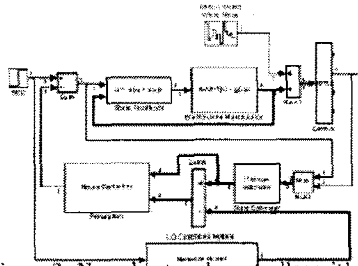

Figure 2: Neural network controller with Kalman filtering

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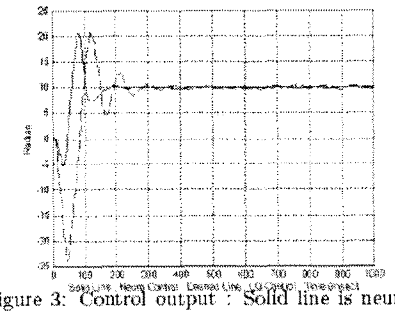

Figure 3: Control output : Solid line is neuro contoller's and dashed line is LQ controller's