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Duffing 시스템의 스위칭 모드 퍼지 모델 기반 제어기의 설계

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Design of Switching-Type Fuzzy-Model-Based Controller for the Duffing System

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Abstract - This paper deals with a design problem of a switching-type fuzzy-model-based controller for a nonlinear system. Takagi-Sugeno(TS) fuzzy model and duffing forced-oscillation system are employed in designing the switching-type fuzzy controller. Finally, we analyze the stability of the global system controlled by the proposed controller.

1. Introduction

Because most of industrial plants are nonlinear, it is difficult to design a controller for a nonlinear system. Recently, the fuzzy-model-based controller has been extensively considered to design the controller in the industrial plant, because it can provide an effective solution to the control of plants that are complex but has the avaliable qualitative knowledge from domain experts for their controllers design. Furthermore, the design problem of a fuzzy-model-based controller has difficulties in the acquisition of expert's knowledge and relies to a great extent on empirical and heuristic knowledge.

Takagi and Sugeno suggested a new kind of fuzz inference system, called Takaki-Sugeno(TS) fuzzy model in 1985[1]. It combines the flexibility of fuzzy logic theory and the rigorous mathematical analysis tools into an unified framework[1-3]. Since TS fuzzy model employs linear state space equations in the consequent parts of the plant rules, it is convenient to apply the conventional linear system theory for the system analysis[9-11].

Cao et al. suggested an alternative method to design a fuzzy-model-based controller, where the stability of the closed loop controlled system is investigated with a switching-type fuzzy-model-based controller and piece-wise Lyapunov function[4-5]. In these approach, a common positive definite matrix must be in the Piecewise Quadratic(PQ) Lyapunov function for satisfying the global stability.

In this paper, we consider a chaotic duffing system which system is approximated by TS fuzzy model. And, the stability analysis of each local system is achieved by PQ Lyapunov function analysis. Then, the global stability of the overall system is determined by the concept "divide and conquer". This concept means that the global system which has several rules is divided into several subsystems and then, a solution is found at each subsystem. The global solution is determined by a conjunction of the solutions of each subsystem.

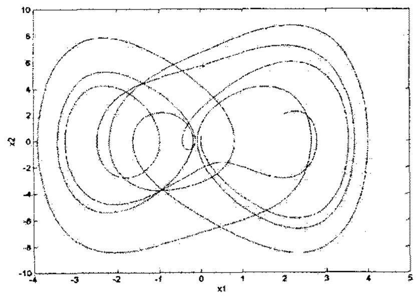

Figure 1. Trajectory of the chaotic Duffing system in the phase plane, with $u(t)=0$ and $x(0)=[2\ 2]^T$.

This paper is organized as follows: In Section 2, first, we linearize the nonlinear duffing system, then, we obtain the fuzzy rules. In Section 3, we detail the switching-type controller and its stability condition. Next we will design the switching-type fuzzy-model-based controller for the given system in Section 4. Finally, conclusions are drawn in Section 5.

2. Duffing Forced-Oscillation System

In this section, we consider a nonlinear system, Duffing forced-oscillation system. And we obtain the fuzzy model of the system.

Consider Duffing forced-oscillation system [6]:

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= -0.1x_2 - x_1^3 + 12\cos(t) + u(t).\end{aligned} \tag{1}$$

As Figure 1, this system is chaotic without control. The trajectory of the system with $u(t)=0$ is shown in the phase plain of (x_1, x_2) in Figure 1 for the initial conditions $\mathbf{x}(0)=[2\ \ 2]^T$ and the time period $t_0=0$ to $t_f=20$.

We will use a controller to control the system state x_1 to track the reference trajectory $y_m(t)=\sin(t)$. In this phase plane, this reference trajectory is a unit circle.

The fuzzy model for this chaotic system is obtained

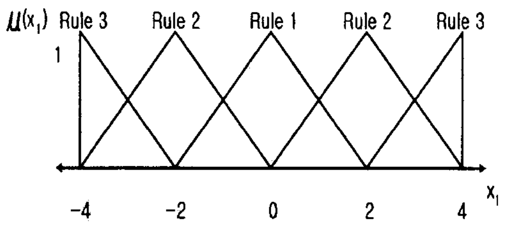

Figure 2. Membership functions for the plant rule

by linearizing the nonlinear equations over a number of operating points in the phase plane. As shown in Figure 1, x_1 is between -4 and 4 for all $t\ (t_0 \le t \le t_f)$. In the manual modeling procedure, x_2 is not included in the fuzzy rule-base since x_2 does not appear in the linearized equations. Therefore, we linearize the nonlinear system (1) on $x_1 = \{-4, -2, 0, 2, 4\}$ which results in five fuzzy rules. However, since the fuzzy rules for $x_1 = \{-2, 2\}$, and $x_1 = \{-4, 4\}$ have the same consequent parts, we finally obtain the following fuzzy rules.[7]

Plant rules :

Rule 1 : IF x_1 is about 0,
THEN $\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1 u$,

Rule 2 : IF x_1 is about ± 2,
THEN $\dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2 u$,

Rule 3 : IF x_1 is ± 4,
THEN $\dot{\mathbf{x}} = \mathbf{A}_3\mathbf{x} + \mathbf{B}_3 u$,

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -12 & -0.1 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ -48 & -0.1 \end{bmatrix}, \quad \mathbf{B}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The membership functions for x_1 are chosen as shown in Figure 2. The rules of the fuzzy-model-based controller are :

Controller rules :

Rule 1 : IF x_1 is about 0,
THEN $u = -\mathbf{K}_1\mathbf{x}$

Rule 2 : IF x_1 is about ± 2,
THEN $u = -\mathbf{K}_2\mathbf{x}$

Rule 3 : IF x_1 is about ± 4,
THEN $u = -\mathbf{K}_3\mathbf{x}$

3. Switching-Type Controller

This section deals with a new design method which is derived from a switching-type controller theory. TS fuzzy model and PQ Lyapunov functions are used to analyze the stability of the switching-type fuzzy-model-based controller.

The continuous-time TS fuzzy system is as follows:

Plant Rule i :
If $x_1(t)$ is F_1^i and $\cdots$ and $x_n(t)$ is F_n^i
Then $\dot{x} = A_i x(t) + B_i u(t)$ (2)

where $F_j^i\ (j = 1, \cdots, n, \quad i = 1, \cdots, r)$ are fuzzy sets.

Before illustrating, we need to define the following assumption.

Assumption 1. *The number of fuzzy rules, which are fired simultaneously for all* $t > 0$, *is* $s > r$.

In this case, we define the subspace $S_l (l = 1, 2, \cdots, m)$ in the entire input spaces as the space where s rules fired concurrently at an instant. The characteristic function of S_l is defined by

$$\eta_l(x(t)) = \begin{cases} 1 & x \in S_l \\ 0 & x \notin S_l \end{cases} \qquad \sum_{l=1}^{m} \eta_l(x(t)) = 1. \tag{3}$$

Define I_l as the set of indices of fuzzy rules in S_l. Then, on every subspace, the fuzzy system can be denoted by

$$\begin{aligned} \dot{x}(t) &= \overline{A}_l(x(t))x(t) + \overline{B}_l(x(t))u(t) \\ &= \sum_{i \in I_l} \mu(x(t))A_i(x(t)) + \sum_{i \in I_l} \mu(x(t))B_i(x(t))u(t), \end{aligned} \tag{4}$$

where $x(t) \in S_l$

Therefore, the global system can be represented using (3) and (4) follows:

$$\dot{x}(t) = \sum_{l=1}^{m} \eta_l(x(t)) \left(\overline{A}_l(x(t))x(t) + \overline{B}_l(x(t))u(t) \right) \tag{5}$$

The above result is summarized as the following theorem.

Theorem 1. *The fuzzy system (2) can be transformed to the piecewise linear time-varying system (5), where each subsystem is the smaller fuzzy system if the number of fuzzy rules fired simultaneously for all* $t > 0$, *is* $s < r$.

In this paper, the stability analysis of the fuzzy-model-based controller is studied with PQ Lyapunov function as a tool for analyzing Lyapunov stability of the fuzzy-model-based controller. In order to use the PQ Lyapunov functions, the Lyapunov function need to be carefully handled, since the employed PQ Lyapunov function candidate is a class of discontinuous functions on the boundary of any two adjacent subspaces. Therefore, we first make the following assumption.

Assumption 2.[5] *If* l*th subsystem is in the* l*th state space, it will stay in the* l*th state space for period of time t where*

$t > \tau$, $\tau > 0$ *is a fixed constant*

and the number of traversing time instants among region S_l *is finite.*

First, to represent a switching-type fuzzy-model-based controller, we consider the autonomous fuzzy system ($u(t) = 0$) and use PQ Lyapunov functions as a tool for analyzing Lyapunov stability of (5) with $u(t) = 0$.

Let $V_l(t) = x(t)^T P_l\, x(t)$ be a Lyapunov function for subspace S_l . Then the global Lyapunov function can be constructed as

$$V(t) = \sum_{l=1}^{m} \eta_l(x(t))x(t)^T P_l x(t) \tag{6}$$

Lemma 1. *The fuzzy system (with* $u(t)=0$*) is quadratically stable if there exists symmetric matrix* P_l *such that*

$$P_l > 0 \quad (7)$$

$$\overline{A}_l(x(t))^T P_l + P_l \overline{A}_l(x(t)) < 0, \ (l=1,2,\cdots,m) \quad (8)$$

proof : Let Lyapunov function be (6). Then the time derivative of the Lyapunov function is

$$\dot{V}(t) = \sum_{l=1}^{m} \eta_l(x(t)) \Big\{ x(t)^T \Big(\sum_{l=1}^{m} \eta_l(x(t)) \ \overline{A}_l(x(t))^T \Big) P_l + P_l \sum_{l=1}^{m} \eta_l(x(t)) \Big(\sum_{l=1}^{m} \eta_l(x(t)) \ \overline{A}_l(x(t)) \Big) x(t) \Big\} \quad (9)$$

$$= \sum_{l=1}^{m} \eta_l(x(t)) x(t)^T (\ \overline{A}_l(x(t))^T P_l + P_l \overline{A}_l(x(t))) x(t)$$

Therefore, if the inequalities (7) and (8) are satisfied, (9) is the fuzzy system (9) negative definite and the proof is completed. □

Theorem 2. *The fuzzy system (2) is quadratically stabilizable via a controller if there exists symmetric matrices* P_l *such that*

$$P_l > 0, \quad (10)$$

$$(\overline{A}_l(x(t)) - \overline{B}_l(x(t)) \ \overline{K}_l)^T P_l + P_l(\overline{A}_l(x(t)) - \overline{B}_l(x(t)) \ \overline{K}_l) < 0 \quad (l=1,2,\cdots,m) \quad (11)$$

where $\overline{K}_l = \sum_{i \in I_l} \mu_i(x(t)) K_i$

proof : Let Lyapunov function be (6). The closed-loop fuzzy system is

$$\dot{x}(t) = \sum_{l=1}^{m} \eta_l(x(t)) \ \overline{A}_l(x(t)) x(t) - \sum_{l=1}^{m} \eta_l(x(t)) \ \overline{B}_l(x(t)) \times \sum_{l=1}^{m} \eta_l(x(t)) \overline{K}_l x(t) \quad (12)$$

$$= \sum_{l=1}^{m} \eta_l(x(t)) (\ \overline{A}_l(x(t)) - \overline{B}_l(x(t)) \overline{K}_l) x(t)$$

Using Lemma 1. the proof is completed.

From theorem 2., the resulting LMI form can be derived as follows[8]:

$$\overline{A}_l(x(t)) X_l + X_l \overline{A}_l(x(t))^T - \overline{B}_l(x(t)) M_l - M_l^T \overline{B}_l(x(t))^T < 0, \quad (13)$$

where $X_l = P_l^{-1}, M_l = \overline{K}_l X_l$

4. Design of Switching-Type Fuzzy-Model-Based Controller

In this section, we design a switching-type fuzzy-model-based controller for the given Duffing forced-oscillation system in Section 2. But, in this case, the global state space can be divided into two subspaces. Therefore, according to Theorem 2, we don't have to design a controller which have three controller gain matrices as in Section 2. The gain matrices which satisfy the inequalities in Theorem 2 are

$$\overline{K}_1 = [\ 38.0869 \ \ 4.8023], \quad \overline{K}_2 = [\ 1251.7 \ \ 23.7]$$

And, there are two symmetric matrices as follows:

$$P_1 = \begin{bmatrix} 47.2709 & 4.3545 \\ 4.3545 & 1.4092 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1305 & 23.6 \\ 23.6 & 1.4 \end{bmatrix}$$

Naturally, the above results satisfy Theorem 2. And, the gain matrices of the switching-type fuzzy-model-based controllers for the TS fuzzy systems guarantee the stability of the total systems. Moreover, the eigenvalues of each subsystem are located in the left half plane. This represents the closed loop system can be stabilizable by the designed controller.

5. Conclusion

In this paper, we have dealt a switching-type fuzzy-model-based controller for a nonlinear Duffing-forced-oscillation system. The proposed method in this paper gives the following advantages:

(1) The total state space can be divided into several subspaces and the stability of the controlled subsystem can be analyzed in each subspaces.
(2) The global Lyapunov stability can be analyzed via each PQ Lyapunov function.
(3) When a fuzzy model is complex and has lots of fuzzy rule like Duffing-forced-oscillation system, the proposed method is very useful to design a controller. Because the controller have only to satisfy the stability conditions of each subsystem.

Though there are many advantages in the proposed method, it requires another application to various type nonlinear systems. The real plant has uncertainties like noises and disturbances. It have to be analyzed in the future work.

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[Reference]

[1] T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control", *IEEE Trans. Syst., Man, and Cybemetics,* Vol. SMC-15, No.1, pp. 116-132, Jan./Feb. 1985.

[2] K. Tanaka, T. Ikeda and H. O. Wang, "Fuzzy Regulators and Fuzzy Observers: Relaxed Stability Conditions and LMI-Based Designs", *IEEE Transactions on Fuzzy Systems,* Vol. 6, No. 2, pp. 250-265, 1998.

[3] K. Tanaka and M. Sugeno, "On Design of Fuzzy Regulators and Fuzzy Observers", *in Proc. 10th Fuzzy Syst. Symp.,* pp. 411-414, 1994.

[4] H. O. Wang, K. Tanaka and M. Griffin, "Parallel Distributed Compensation of Nonlinear Systems by Tagaki and Sugeno's Fuzzy model", *in Proc. FUZZY-IEEE, Yokohama, Japan,* 1995, pp. 531-538.

[5] S. G. Cao, N. W. Rees and G. Feng, "Stability analysis and Design for a class of continuous-time fuzzy control systems", *Int. J. Control,* Vol. 64, pp. 1069-1087, 1996.

[6] G. Chen, X. Dong, "On Feedback Control of Chaotic Continuous-Time Systems", *IEEE Transactions on Circuit Systems - I* : Fund. Theory Appl. Vol. 40, No.9, pp. 591-601, 1993

[7] W. Chang, Y. H. Joo, J. B. Park, and G. Chen, ""Design of Robust Fuzzy-Model-Based Controller with Sliding Mode Control for SISO Nonlinear Systems", *Fuzzy Sets and System, Vol 125, No. 1, pp 1-22,* 2001.

[8] H. J. Lee, J. B. Park and G. Chen, "Robust Fuzzy Control of Nonlinear Systems with parameter Uncertainties", *IEEE Transactions on Fuzzy Systems,* Vol.9, No.2, pp.369-379,2001.

[9] Y. H. Joo, G. Chen and L. S. Shieh, "Hybrid State-Space Fuzzy Model-Based Controller with Dual-Rate Sampling for Digital Control of Chaotic Systems", *IEEE Transactions on Fuzzy Systems,* Vol. 7, pp. 394-408, 1999.

[10] H. O. Wang, K. Tanaka and M. F. Griffin, "An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues", *IEEE Transactions on Fuzzy Systems,* Vol. 4, No 1, pp. 14-23, 1996.

[11] G. Feng, ""Approaches to Quadratic Stabilization of Uncertain Fuzzy Dynamic Systems", *IEEE Transactions on Circuits and Systems,* Vol. 48, No. 6, pp. 760-779, 2001.