한국소음진동공학회 2001년도 추계학술대회논문집, pp. 1044~1049.

고차통계를 이용한 충격/불량신호 탐지

[0] 서종수*, J.K. Hammond**

BLIND IDENTIFICATION OF IMPACTING SIGNAL USING HIGHER ORDER STATISTICS

[0]Jong-Soo Seo*, J.K. Hammond**

ABSTRACT

Classical deconvolution methods for source identification following linear filtering can only be used if the transfer function of the system is known. For many practical situations, however, this information is not accessible and/or is time varying. The problem addressed here is that of reconstruction of the original input from only the measured signal. This is known as 'blind deconvolution'. By using Higher Order Statistics (HOS), the restoration of the input signal is established through the maximisation of higher order moments (cumulants) with respect to the characteristics of the signals concerned.

This restoration is achieved by constructing an inverse filter considering the choice of the initial inverse filter type. As a practical application, an experimental verification is carried out for the restoration of our impacting signal arising in the response of a cantilever beam with an end stop when randomly excited.

1. INTRODUCTION

For many mechanical systems undergoing normal operation, indications of malfunctions and advance warning of system failure may be contained in measurements of physical characteristics.

The problems addressed here relate to obtaining more reliable and consistent detection of the so-called "hidden" signals which are the causes of system malfunctioning. These hidden signals are not directly measurable. The determination of these 'causes' from output variables is an inverse problem.

These problems can sometimes be straightforward when the system through which the causes pass is known. For many physical situations, however, where it is impractical to assume the availability of the system characteristics we require restoration of the original input signal solely from the measured (observed) signal. In this case, the restoration is called blind inversion.

The aim here is to find the input signal from the measured signal alone. One practical representative example of the above situation can be found in condition monitoring, which may require the identification of the 'cause' of a mechanical imbalance or impacting phenomenon which can arise in rotating machinery. For the retrieval of this cause, the statistical properties of the measured signal and their relationships with the unknown linear system are considered; Specifically, higher order statistics (greater than second order) are the key to possible solutions and are the focus of this work [1]. If the input (original) signal cannot be observed, we may be able to utilise its statistical characteristics as the basis of its restoration. In this work, the structure of system responding to the input is assumed to be linear and time invariant (LTI) characterised by the time-domain impulse response sequence. Higher-Order (≥ 3) Statistics (HOS) [2], [3] have been considered in various signal processing areas, and used to find optimum inverse filters to restore the original input signal. In particular, cumulants display the degree of higher-order correlation and also provide a measure of the "departure" from Gaussianity. The advantages of HOS are due to their ability to carry the phase information of a signal or a system and to suppress any (white or coloured) Gaussian additive noise [4].

Under certain conditions, such as for non-Gaussian, independent, identically, distributed (*i.i.d.*) signals, Donoho [5] has shown that the probability distribution of a linear combination of these signals tend to become 'closer' to Gaussian (This is sometimes referred to as partial order) than that of the individual components before the linear combination (e.g., input signals). Based on this, the idea of blind deconvolution is approached by selecting an inverse system that can decrease the Gaussianity of the output of the inverse system. Thus, maximising an appropriately selected function (which can represent the degree of the Gaussianity) with respect to the parameters (coefficients of the linear inverse filter) of the inverse system achieves blind deconvolution. Concerning this 'appropriate' function, Wiggins has proposed an objective function which consists of two cumulants (i.e., the fourth-order cumulant divided by the squared second-order cumulant), which is called 'Minimum Entropy Deconvolution (MED)'. This

* 삼성중공업, 거제조선소, 진동소음연구파트

** Professor of signal processing, ISVR, University of Southampton, UK

objective function is the kurtosis and can be related to the partial order described by Donoho [5]. When the kurtosis of any signal is greater than 3 (or greater than zero, according to another widespread definition of kurtosis), this is referred to the 'super-Gaussianity'. For this case, maximisation of the absolute value of the objective function has been used to yield the reconstruction of the input signal. By pushing the objective functions toward their maximum, one attains blind deconvolution. The procedure of obtaining the coefficients of the inverse filter from which we can reconstruct the unknown input signal may be achieved by (i) (nonlinear) iterative methods using a matrix equation or (ii) a stochastic gradient method using an updating parameter to maximise/minimise the objective function.

2. Theoretical considerations

This section explains the properties of higher order statistics of signals undergoing linear filtering, leading to the use of normalised cumulants in blind deconvolution.

2.1 Relationship between input and output cumulants in linear filtering

A starting point for many problems in signal processing and system theory is the single-input single-output (SISO) linear and time-invariant (LTI) model depicted in Figure 2.1.

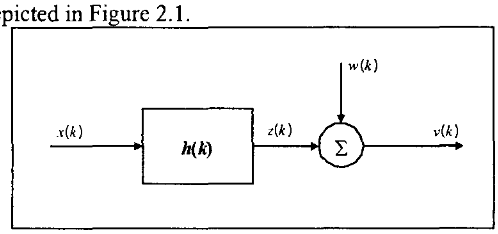


Figure 2.1 Single-channel system

The definitions and calculations of moments and cumulants of signals can be found in relevant literature [2], [3], [6], [8]. We discuss the properties of these statistical parameters from second to fourth order.

The model;

In Figure 2.1, $x(k)$ is a white non-Gaussian sequence input with finite variance σ_x^2. H(z)(=Z{$h(k)$}) is the transfer function of a causal stable system (channel) having impulse response sequence $h(k)$. $w(k)$ is white Gaussian noise with variance σ_w^2. $x(k)$ and $w(k)$ are statistically independent and so therefore are $z(k)$ and $w(k)$. $z(k)$ is the output of the system and $v(k)$ is the output of the system corrupted by Gaussian noise.

The relationship between the signals and system is given by the following equation,

$$v(k) = z(k) + w(k)$$
$$= \sum_{i=-\infty}^{\infty} x(i)h(k-i) + w(k) \qquad (2.1)$$

Higher order properties;

We now construct the relationship between the higher order cumulants (order=3 or 4) of the input x and the output v. From equation (2.1), using the properties of higher order cumulants [6], the kth-order cumulant of $v(k)$ equals the kth-order cumulant of $z(k)$ as $w(k)$ is assumed to be a white (or coloured) Gaussian process.

In the case of a white sequence input,

$$c^{v}_{k}(\tau_1,\ldots,\tau_{k-1}) = c^{z}_{k}(\tau_1,\tau_2,\ldots,\tau_{k-1})$$
$$= \gamma^{x}_{k}\sum_{i_0} h(l-i_0)h(l-i_1+\tau_1)\cdots h(l-i_{k-1}+\tau_{k-1}) \qquad (2.2)$$
$$= \gamma^{x}_{k}\sum_{n=0}^{\infty} h(n)h(n+\tau_1)\cdots h(n+\tau_{k-1})$$

For non-Gaussian $i.i.d.$ input signal $x(n)$, by setting $\tau_1 = \tau_2 = \ldots = \tau_{k-1} = 0$ and $m_1 = m_2 = \ldots = m_{k-1} = 0$, we obtain a result similar to (2.2)

$$c_r^v = c_r^x \sum_k (h_k)^r \qquad (2.3)$$

i.e., the zero lag response cumulant of order r is seen to be the product of the excitation cumulant of order r with the sum of the linear operator's unit-impulse response elements raised to the r-th power. More detailed expansion of the cumulants' relationship can be found in [6].

2.2 Normalisation of cumulants in blind deconvolution

Having established expressions for higher order statistics for stationary inputs and outputs of linear filters, we will now introduce the deconvolution problem and in particular show the concept of normalised higher order statistics are needed.

In the blind deconvolution approach that will be developed, a desirable feature is that it be invariant with respect to signal size. To explain this, Figure 2.2 depicts a single input single output system in which $x(n)$ is the original input sequence (assumed $i.i.d.$), $v(n)$ is measured sequence, and $y(n)$ is output of the cascade system g_n composed of h_n and f_n (i.e. combined convolution-deconvolution operation). From this structure, the relationship of the signals in convolutive terms and cumulants are again written as

$$y = g * x \Leftrightarrow c_k^y = \sum_{i=0}^{\infty} g_i^{\,k} c_k^x \qquad (2.4)$$

where c_k^y and c_k^x are the kth-order cumulant of output and input, respectively.

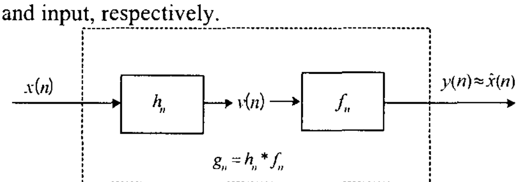


Figure 2.2 Single input-single output system with the convolution-deconvolution form.

In order to make the concept of the cumulant invariant with respect to scalar multiplication, it is necessary to provide a normalisation of the cumulant. Consider the normalised cumulant of order (r, s) associated with random variables $\{x_i\}$, i = 1, 2, ... , n, as defined by

$$K_x(r,s) = \frac{c_r^x}{\left|c_s^x\right|^{r/s}} \qquad (2.5)$$

in which it is assumed that both cumulant c_s^x and c_r^x are nonzero (this may reflect the hypothesis that the input signal is a non-Gaussian process). Typically, the integer parameters are selected so that $r > s$, although this definition is applicable for any choice of those parameters using the definition of a cumulant. It directly follows that this normalised cumulant is scalar multiplication invariant in the sense that $K_{ax}(r,s) = K_x(r,s)$ for any nonzero scalar a. In many applications, the specific selection of s=2 provides a logical choice, since $c_2^x = \sigma_x^2$ (variance) is always nonzero for any non-trivial random variables. This particular selection yields the normalised cumulant relationship

$$K_x(r,2) = \frac{c_r^x}{\sigma_x^r} \qquad (2.6)$$

An important inequality condition relating normalised cumulants between the input and output signal is now described.

Denote the r and sth-order normalised cumulant of the random variables $\{y_i\}$, i = 1,2,...,n, as $K_y(r,s)$, then

$$K_y(r,s) = \frac{\sum_k (g_k)^r}{\left|\sum_k (g_k)^s\right|^{r/s}} K_x(r,s) \qquad (2.7)$$

(see Figure 2.2 to define the g_k)

By taking absolute values on each side, and for a positive even integer s,

$$\left|K_y(r,s)\right| \le \left|K_x(r,s)\right|, \quad \text{for all even and odd } r>s \qquad (2.8)$$

as possible, i.e., adjust **f** to maximise the normalised cumulant of the output of the a certain system (y).

Cumulant based deconvolution problem has been based on the inequality relationship given in the equation (2.8) and the ideal deconvolution operation is performed by maximising the magnitude of the normalised response cumulant $K_y(r,s;\mathbf{g})$, where s is any positive even integer less than r for which cumulant of order s, c_s^x is nonzero (e.g., s=2). This maximisation is to be made with respect to the unit-impulse response $\{g_n\}$ of the combined convolution-deconvolution operation as shown in Figure 2.2. Since the unit-impulse response of the unknown linear convolution operator $\{h_n\}$ is implicitly contained within the observed (measured) data $\{v(n)\}$, this maximisation must be made with respect to the deconvolution operator's unit-impulse response $\{f_n\}$. The required maximisation therefore takes the form

$$\max_{\mathbf{f}} \left|K_y(r,s)\right| = \max_{\mathbf{f}} \left[K_y(r,s)\operatorname{sgn}\left[K_y(r,s)\right]\right] \qquad (2.9)$$

where **f** is an appropriately dimensioned vector whose components are the elements of the unit-impulse response of the deconvolving operator. In words, we desire to find a global maximum of $\left|K_y(r,s)\right|$. Often, the necessary condition which gives a local maximum of this value is found by differentiating and equating to zero with respect to the filter **f**, which will be discussed in next section.

3. Blind Deconvolution (BD process)

The term 'blind deconvolution' addressed here is the problem of restoring an input signal from the measured (observed) signal alone. The measured signal is assumed to be the output of an unknown linear system possibly corrupted by noise. A further key assumption is that (i) the input signal to be restored is non-Gaussian and (ii) the corrupting signal is Gaussian and is independent of the input signal. This blind deconvolution (BD) process is illustrated below:

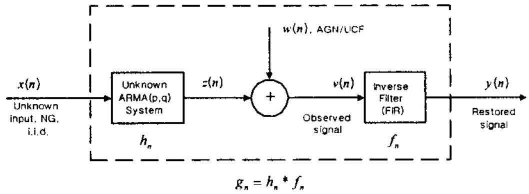


Figure 3.1 The process of convolution-deconvolution in blind source reconstruction problem; NG: Non-Gaussian; AGN: Additive Gaussian Noise; UCF: Unknown Covariance Function.

The inverse filter coefficients fm (denoted **f** in vector notation) is estimated from the maximisation of the higher order cumulant of the output $y(n)$. This results in a constrained non-linear optimisation problem which is solved numerically.

With reference to Figure 3.1, we construct the inverse filters using a normalised cumulant (objective function). This objective function takes the form of the r-th order cumulant of the output signal $y(n)$ divided by the s-th order (s-th moment) of $y(n)$

$$O_y(\mathbf{f}) = \frac{E[(y-m_y)^r]}{\left\{E[(y-m_y)^s]\right\}^{r/s}} = \frac{\frac{1}{N}\sum_{i=0}^{N-1}(y_i - m_y)^r}{\left[\frac{1}{N}\sum_{i=0}^{N-1}(y_i - m_y)^s\right]^{r/s}} \qquad (3.1)$$

where m_y is the mean of y. Similar to the constrained case, the normalising order s is given by 2 which corresponds to the variance of y (σ_y^2). Commonly, the integer r is either 3 or 4 [7].

The maximisation of the objective function is achieved through the differentiation of this objective function with respect to the inverse filter coefficients and equating to zero.

$$\partial O_y(\mathbf{f}) / \partial f_m = 0 \qquad (3.2)$$

The above equation yields inverse filter coefficients as shown in the following equation (3.3) which generally yields a local optimum.

$$\begin{bmatrix} \sum v(n)v(n) & \sum v(n)v(n-1) & \cdots & \sum v(n)v(n-L+1) \\ \sum v(n-1)v(n) & \sum v(n-1)v(n-1) & \cdots & \sum v(n-1)v(n-L+1) \\ \vdots & \vdots & & \vdots \\ \sum v(n-m)v(n) & \sum v(n-m)v(n-1) & \cdots & \sum v(n-m)v(n-L+1) \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_L \end{bmatrix} = \begin{bmatrix} A \cdot \sum y'(n)v(n) \\ A \cdot \sum y'(n)v(n-1) \\ \vdots \\ A \cdot \sum y'(k)v(k-m) \end{bmatrix} \qquad (3.3)$$

where $m=L$ and the term $A = \sum_{n=0}^{N-1} y^2(n) / \sum_{n=0}^{N-1} y'(n)$.

The equation (3.3) can be written equivalently as

$$\mathbf{R}_{vv} \cdot \mathbf{f} = \mathbf{g} \qquad (3.4)$$

where $\mathbf{R}_{vv}$ denotes the symmetry $L \times L$ autocorrelation matrix of the observed signal, f is the $L \times 1$ inverse filter coefficient vector, and **g** is the $L \times 1$ cross-correlation vector between the observed signal and the output of the inverse filter. Using either the constrained or normalised objective function maximisation process, the output signal y(n) can yield the input signal restoration through the convolution of the measured signal $v(n)$ and the inverse filter **f** with length L

$$y(n) = \sum_{m=0}^{L-1} f_m v(n-m) \qquad (3.5)$$

4. Practical consideration on blind deconvolution

The inversion formula defined in the section 3 yields the inverse filter coefficient vector in a non-linear iterative manner. To solve this, an initial inverse filter coefficient vector with a chosen length is selected.

4.1 Effect of initial inverse filter on deconvolution

The aim of this study is to see the effect of this initial inverse filter selection.

Following figure suggests two possible initial inverse filter types (FIR) used in the iterative calculation.

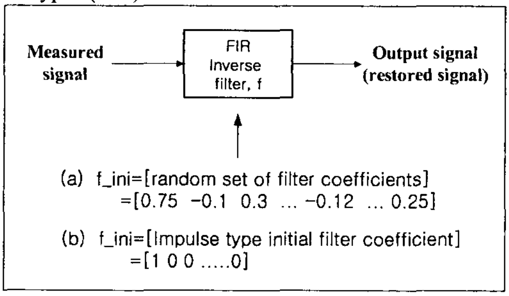


Figure 4.1 Different types of initial inverse filters

According to the central limit theorem and assuming the measured signal is not highly non-Gaussian, any arbitrarily selected inverse filter will make the output signal closer to Gaussian. Thus, from Figure 4.1, if an initial inverse filter is chosen randomly, the statistical property of the output of the inverse filter may be *closer to Gaussian* whereas the other initial inverse filters (b) will not change the statistical properties of the output signal at the first stage of iteration. Accordingly this choice is a better starting point for the iteration.

4.2 Simulation with two different initial inverse filters

To study the effect of filter initialisation, we selected an unknown input signal and output signal.

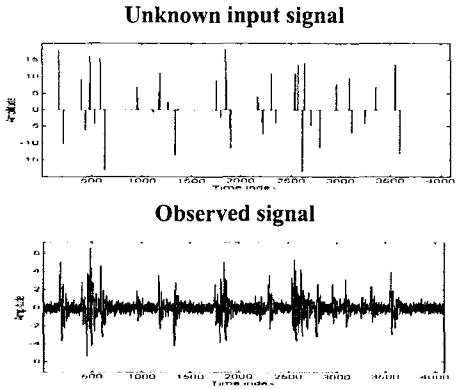


Figure 4.2 Unknown input and observed signal

From above observed signal, the blind deconvolution was carried out using two different initial inverse filter types (the filter length was kept the same). During the iterative calculation of the inverse filters the values of objective function were monitored and compared to each other.

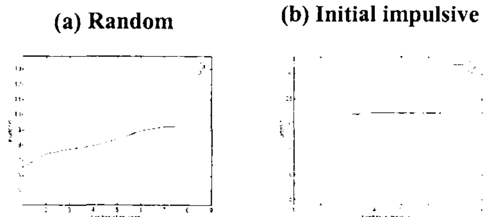
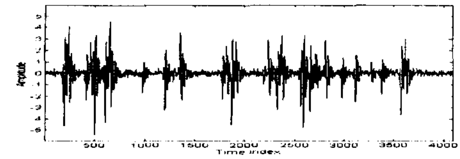


Figure 4.3 The comparison of the objective function values from two different initial inverse type. O_{yi} : objective function of output of the inverse filter at each iteration, O_v : objective function of the measured signal.

As shown in Figure 4.3 (a), the objective function value of the first iteration (first point of the solid line) is smaller than that of the measured signal (dotted line) for the randomly selected initial inverse filter case. On the contrary, if the initial inverse filter is chosen as an impulse type (b), the output of the initial inverse filter can be at least not closer to the Gaussianity than the measured signal. Hence, choosing the initial inverse filter randomly may result in incorrect restoration of signal or need more computational time to achieve the same result as the other types of the initial inverse filter case.

(a) Restored signal from random initial inverse filter



(b) Restored signal from initial impulse inverse filter

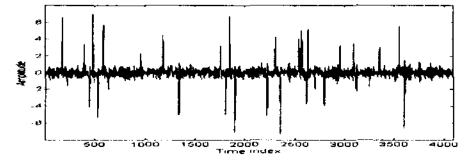


Figure 4.4 Shapes of the restored signals from two different initial inverse filter types.

As can be seen in Figure 4.4, the restored signal from the initial impulse type inverse filter (b) gives significantly better result compared to the result from the random initial inverse filter (a). The resulting restored signals reflect the important effect of selecting the initial inverse filter type. The results are summarised in the table.

Table 4.1 Comparison of restored signals from two different inverse filters (numerical results of Figure 4.4)

	Restored signal by inverse filter	
	Random initial	**Initial impulsive**
Skewness	-0.43702	0.2462
Kurtosis	9.2378	22.3232

5. Application to a mechanical system

In this section, the behaviour of a randomly excited vertical cantilever beam with an endstop is investigated experimentally. The aim of this section is the practical verification of blind signal separation and recovery of the impacting signal. In the context of 'condition monitoring' of a mechanical system, this might be a key element related to fault detection. The aim would be to obtain a diagnostic to avoid false alarms. In a practical system, there may be (unexpected) nonlinearities and other hidden effects among the signals and structures. Accordingly, in our study, the experimental conditions are controlled closely to assure that the mathematical models used in previous chapters are relevant.

5.1 Experimental layout and signals

The experimental layout is shown in Figure 5.1 in which a cantilever beam is driven from a random source through an exciter. A broad band Gaussian excitation is the excitation. Impacting is induced by placing an end stop restricting the motion of the beam (at the beam tip). The impacting signal is generated by the end stop incorporating a force transducer which is placed 1.5 mm

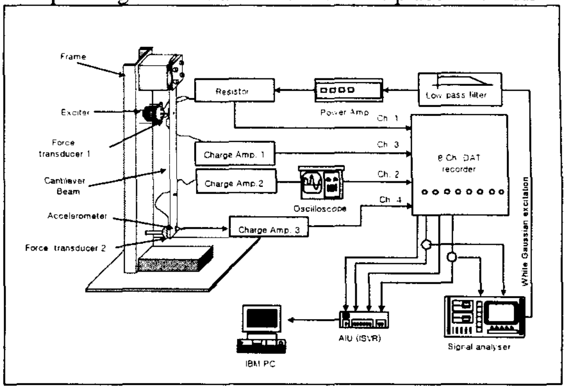


Figure 5.1 Experimental set-up

behind from the beam steady state position. The accelerometer is attached at the free end of the cantilever beam to collect the signal (Ch. 4) mixed with the vibration signal of the beam and impacting signal caused by the end stop. Note the impacting signal (Ch. 2) is captured so as to assess the performance of the inversion process – normally of course this would be unavailable. The whole structure of Figure 5.1 can be thought as a system shown below;

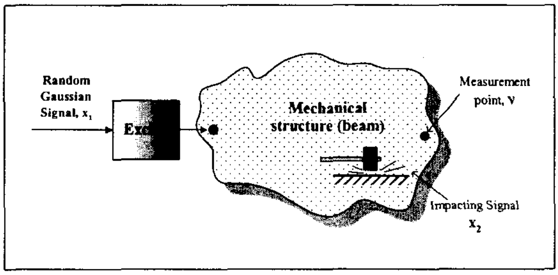
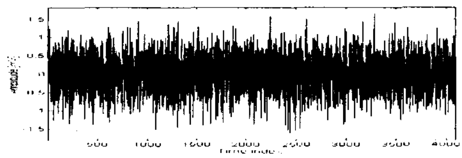


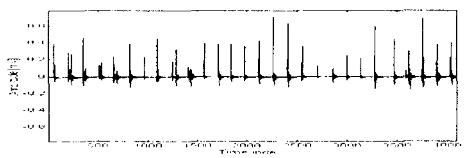
Figure 5.2 Structural components of beam excitation

This structure is a complicated system as the impacting signal $\mathbf{x}_2$ is **nonlinearly** related to the excitation signal $\mathbf{x}_1$, and so the analysis of this situation thus becomes a very complex problem.

$\mathbf{x}_1$



$\mathbf{x}_2$



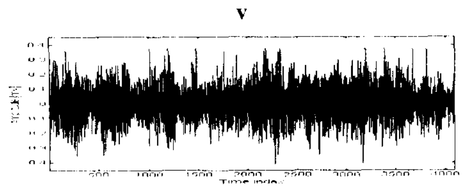


Figure 5.3 Signals used in experiment; x_1: unknown Gaussian excitation signal, x_2: impacting signal, v: accelerometer signal.

Higher order statistical properties of each signal are listed in Table 5.1.

Table 5.1 Statistical properties of each signal

Signals Statistical parameters	Input signals (unknown)		Observed signal
	x_1	x_2	v
Skewness	-0.0186	9.454	0.066
Kurtosis	2.9001	128.533	2.983

5.2 Reconstruction of impacting signal from accelerometer signal (v)

The initial FIR inverse filter is selected as an impulse (initial) type with 39 coefficients [8] for the third order case as;

$$\mathbf{f}_{b\_ini} = \underbrace{[1\quad 0\quad 0\quad \dots\quad 0]}_{39}, \quad \mathbf{f}_a = 1$$

The shapes of restored signals from the third order Wiener optimisation are shown below.

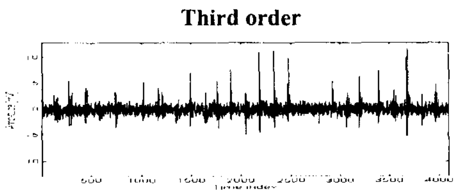


Figure 5.4 Restored impacting signal

As the measurement position of the accelerometer (channel 2, signal **v**) is quite close to the impacting point of the beam, the impulsive nature of signal in Figure 5.4 shows a clear pulses as the input impacting signal.

6. Summary and conclusion

Based on the study of this impacting signal reconstruction using higher order normalised cumulant maximisation, summaries and conclusions are presented relating to four stages of the work.

Stage 1 (Section 2) We begin with the explorations of the characteristics of higher order cumulants of signals and their properties through convolution. Based upon theory and simulation, this stage provides evidence of the validity of the application of higher order statistics to the source signal reconstruction problem.

Selecting the value of the normalised cumulant which incorporates the second order (s=2) and higher (third or fourth) order (r=3 or 4) cumulant, the inequality condition of this value is established. This condition now provides the key motivation in the blind deconvolution problems and hence is *employed for the blind reconstruction* of source signal.

Stage 2 (Section 3) This stage has been devoted to the fundamental consideration of the utilisation of the Higher Order Statistics in order to reconstruct an unknown impacting signal from only a measured signal. Starting from the basic Wiener optimisation approach for the FIR system, the blind deconvolution procedure has been justified utilising the objective function.

Stage 3 (Section 4) This stage provides guides to solutions to open questions in blind deconvolution namely, initial filter coefficient vector selection. From this, it is demonstrated that it becomes natural to take the *initial inverse filter* as *impulsive* in form.

Stage 4 (Section 5) In this stage, we used an experimental impacting cantilever beam to validate the blind deconvolution process from computer simulations. Thus, we succeeded to reconstruct the impacting signal through single channel BD process. The experimental results for restoring the faulty impacting signal gave acceptable information to identify the cause of mechanical system.

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