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Optimum Shape Design of a Rotating-Shaft Using ESO Method
ESO 법을 이용한 회전축의 형상최적화

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Key Words : ESO(Evolutionary Structural Optimization,진화적 구조최적화), Rotor shaft system(회전축계), Optimum shape design(형상 최적설계), Sensitivity analysis(감도해석)

ABSTRACT

본 논문에서는 최근의 진화적 구조최적화(ESO) 전략을 회전축의 형상최적화에 적용하였으며, 각 계산 스텝마다 단위 유한요소의 크기를 변경함으로써 기존의 방법보다 빠르고 정확한 최적형상에 수렴하는 새로운 방법을 제시하였다. 축요소의 직경을 시스템 설계변수로 하였으며, 축중량의 감소, 공진배율(Q-factor)의 감소 및 충분한 위험속도의 분리여유를 갖도록 목적함수를 설정하였다. 불평형응답 및 굽힘응력의 구속조건을 부가하였으며, 목적함수에 대한 설계변수의 감도해석을 수행하였다. 전동기축계에 대한 적용 결과로부터 주파수와 동적 구속조건하의 로터베어링 시스템에 대한 축 형상 최적화에 ESO 법이 효과적으로 이용될 수 있음을 확인하였다.

1. INTRODUCTION

In the design of modern rotating machinery, it is often necessary to increase the performance of rotor-bearing systems. This aim requires designing a system compact and lightweight which greatly saves fuel usage during its operation. Since the critical speed range influences the performance and safety of the whole system, it may be necessary and better to constrain the critical speeds and the resonance response in the design process to avoid large vibrations. And the minimization of response amplitudes within the operating range of the rotor system may be the most primary design object. The problem of weight minimization usually arises from the revision of an existing rotor-bearing system to increase the system performance. Many papers have shown that the system parameters, including the distribution of the mass and stiffness of the shaft and the coefficients of the bearings, have an influence on the dynamic characteristics of a rotor-bearing system.[1~3] The optimum shape design of rotor system with restrictions on critical speeds using genetic algorithm has been studied by Choi and Yang. [4] In this paper, the present study will focus on the design of a rotor-bearing system with minimum shaft weight, minimum Q factor and enough separation margin of the critical speed under the requirements of dynamic behaviors such as dynamic stress and steady-state unbalance response, to increase the performance of a rotor-bearing system.

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ESO in its original form optimizes a structure by slowly removing elements with low stress, approaching towards a fully stressed design. [5] The primary goal of the research and development of ESO is to provide the engineering industry with a practical and ``user-friendly" optimization method to assist in the design process. Hence ESO has been extended to accommodate various optimization criteria and is becoming a more practical method. Some of these researches include the implementation of stiffness and displacements as optimization criteria and the applications in multiple load, non-linear, dynamic and buckling problems.[6~9] Querin et al. [10] extended the ESO method to add as well as remove elements, namely bidirectional ESO (BESO). This means that the initial design no longer had to be the maximum design domain. Thus the solution time may be reduced especially if the user specifies a near optimal topology to be the initial design. However, this knowledge is not always available and the typical long solution time of ESO has been an obstacle to its practical applicability as a design tool.

In traditional ESO method, the size of FE element in each step is usually fixed. To get fine shape of optimum, the design model should be divided more detail, and it takes very long time because FE analysis is executed in each step. To overcome these demerits of ESO shown in previous works, new method which is faster and more accurate is investigated in this paper by means of applying variable FE element in each step. As the design variable is diameter of each element, the FE element that should be removed or added is type of a shell which has an unit width of direction to diameter. Prior to applying this new approach, eigenvalue and sensitivity analysis of initial model is executed, and then calculate the sensitivity numbers of object function for the diameter of each element. And the sensitivity numbers of each element are compared and some elements are added or removed(in this model, increase or decrease the diameter of element) in proportion to sensitivity numbers. As the iteration number increase, the size of element becomes more precisely and finally convergent to optimum shape.

The proposed ESO method is used to find the optimum shape of a rotor shaft and bearing so that the optimized rotor-bearing system can yield the minimum shaft weight, Q factor and enough separation margin of critical speed with the dynamic constraints. The results show that the new ESO algorithm can reduce the weight of the shaft and Q factor and yield the critical speeds as far from operating speed as possible with dynamic behavior constraints.

2. EIGENVALUE AND SENSITIVITY ANALYSIS

The system equations that describe the behavior of the entire rotor-bearing system are formulated by following equation

$$M\ddot{p} + C\dot{p} + Kp = Q^u \tag{1}$$

where, $M(=M_T + M_R)$ is the mass matrix, M_T, M_R are the translational and rotational mass matrices, $C(=-\Omega G + C_b)$, $K(=K_b + K_s)$ are the damping and stiffness matrices, G is a gyroscopic matrix, K_b, C_b are the stiffness and damping matrices of bearing, and Q^u is a force vector, respectively.

2.1 Eigenvalue Analysis

In setting up the complex eigenvalue problem for the whirl frequencies of the system governed by Eq. (1), it is convenient to write the system equation in the first order state vector form

$$A\dot{q} + Bq = 0 \tag{2}$$

where

$$A=\begin{bmatrix} C & M \\ M & 0 \end{bmatrix},\ B=\begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix},\ q=\begin{bmatrix} p \\ \dot{p} \end{bmatrix}$$

For assumed harmonic solution $q=\bar{q}e^{\lambda t}$ of Eq. (2), the associated eigenvalue problem is

$$(A\lambda + B)\bar{q} = 0 \tag{3}$$

where λ is the eigenvalue. The eigenvalues are usually complex eigenvalues and conjugate roots

$$\lambda_i = \alpha_i \pm i\omega_i \tag{4}$$

where α_i , ω_i are the growth factor and the damped natural frequency of *i*th mode, respectively. The Q factor Q_i in the critical speed is expressed in term of the real and imaginary parts of complex eigenvalue.

$$Q_i = \frac{1}{2\zeta_i} = -\frac{\sqrt{\alpha_i^2 + \omega_i^2}}{2\alpha_i} \tag{5}$$

where ζ_i is the damping ratio of *i*th mode.

2.2 Sensitivity Analysis

In the ESO method, a key issue is to evaluate the efficiency of material used in the design domain. For a static optimization problem, the efficiency of material can be evaluated by considering the stress level of each particular element. If the stress level of an element is very low, it means that the material of this element is not used efficiently and therefore can be removed. For the natural frequency optimization problem, there is not any external dynamic load in the system because the system is in a free vibration state. Thus, it is impossible to use the stress level to determine the efficiency of material for an evolutionary natural frequency optimization problem. In order to solve this problem, it is essential to evaluate the individual contribution of an element to the natural frequency concerned since finite elements are basic cells of the design domain. Although the sensitivity of a natural frequency, which is usually expressed in the differentiation sense, can be used to evaluate the efficiency of an element, the contribution factor of an element to the natural frequency is used in this study to evaluate the efficiency of the element because it is expressed in the difference sense and therefore may be most suitable to the finite element analysis.

2.2.1. Sensitivity analysis of eigenvalue

It can be expressed to Eq. (6) considering *i*th mode complex eigenvalue and eigenvector from Eq. (2)

$$(\lambda_i A + B)\varphi_i = 0 \tag{6}$$

where,

$$A=\begin{bmatrix} C & M \\ M & 0 \end{bmatrix},\ B=\begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix},\ \varphi_i=\begin{bmatrix} \varphi_{1i} \\ \lambda_i\varphi_{1i} \end{bmatrix}$$

Taking the derivative of Eq. (6) with respect to design parameter d_j gives

$$\left(\frac{\partial\lambda_i}{\partial d_j}A + \lambda_i\frac{\partial A}{\partial d_j} + \frac{\partial B}{\partial d_j}\right)\varphi_i + (\lambda_i A + B)\frac{\partial\varphi_i}{\partial d_j} = 0 \tag{7}$$

Then premultiplying Eq. (7) by φ_i^T

$$\frac{\partial\lambda_i}{\partial d_j}\varphi_i^T A\varphi_i + \lambda_i\varphi_i^T\frac{\partial A}{\partial d_j}\varphi_i + \varphi_i^T\frac{\partial B}{\partial d_j}\varphi_i + \varphi_i^T(\lambda_i A + B)\frac{\partial\varphi_i}{\partial d_j} = 0 \tag{8}$$

Solving Eq. (8) to obtain the eigenvalue sensitivity gives

$$\frac{\partial\lambda_i}{\partial d_j} = -\frac{\lambda_i\bar{p}_i' + \bar{q}_i'}{\bar{p}_i} \tag{9}$$

where,

$$\bar{p}_i = \varphi_i^T A\varphi_i = \varphi_{1i}^T C\varphi_{1i} + 2\lambda_i\varphi_{1i}^T M\varphi_{1i}$$

$$\overline{p}_i' = \varphi_i^T \frac{\partial A}{\partial d_j}\varphi_i = \varphi_{1i}^T \frac{\partial C}{\partial d_j}\varphi_{1i} + 2\lambda_i \varphi_{1i}^T \frac{\partial M}{\partial d_j}\varphi_{1i}$$
$$\overline{q}_i' = \varphi_i^T \frac{\partial B}{\partial d_j}\varphi_i = \varphi_{1i}^T \frac{\partial K}{\partial d_j}\varphi_{1i} + \lambda_i^2 \varphi_{1i}^T \frac{\partial M}{\partial d_j}\varphi_{1i} \quad (10)$$

2.2.2. Sensitivity analysis of Q factor

Similarly by differentiating Eq. (5) with respect to design parameter d_j, the derivatives of Q factor can be obtained as

$$\frac{\partial}{\partial d_j}Q_i = \frac{\alpha_{ij}}{2\alpha_i^2}\sqrt{\alpha_i^2+\omega_i^2} - \frac{\alpha_i\alpha_{ij}+\omega_i\omega_{ij}}{2\alpha_i\sqrt{\alpha_i^2+\omega_i^2}} \quad (11)$$

where

$$\alpha_{ij} = \frac{\partial \alpha_i}{\partial d_j} = \mathrm{Re}\left(\frac{\partial \lambda_i}{\partial d_j}\right), \quad \omega_{ij} = \frac{\partial \omega_i}{\partial d_j} = \mathrm{Im}\left(\frac{\partial \lambda_i}{\partial d_j}\right)$$

2.2.3. Sensitivity of weight

The total weight of shaft can be expressed as follows

$$W = \sum_{i=1}^{N_e} \rho l_i \pi \frac{(d_{Oi}-d_{Ii})^2}{4} \quad (12)$$

where, l_i, d_{Oi}, d_{Ii} is the length, outer diameter and inner diameter of ith element, ρ is density of element and N_e is the total number of element, respectively.

As the outer diameter of each element is taken to design variable in this paper, the derivatives of other element are zeros and finally the derivatives of total shaft weight for jth element diameter are of the form

$$\frac{\partial W}{\partial d_j} = \frac{\rho l_j \pi}{2}(d_{Oj} - d_{Ij}) \quad (13)$$

3. OPTIMIZATION APPROACH

3.1 Formulation of Optimum Design

In this study, the objective function $F(x)$ is composed with the shaft weight $W(x)$, natural frequency $\omega_i(x)$ and Q factor $Q_i(x)$. For this exmple, operating speed(3000 rpm) is between the 2nd natural frequency (2259 cpm) and the 3rd forward natural frequency (9071cpm). The operating speed is near the 2nd forward natural frequency and the 3rd natural frequency is far from the operating speed sufficiently. So the objective function is as follows;

$$F(x) = \alpha\frac{W(x)}{W_0} + \beta\frac{\omega_2(x)}{\omega_0} + \gamma\frac{Q_2(x)}{Q_0} \rightarrow \mathit{Minimize} \quad (14)$$

where α, β, γ are weighting factors. Each value of the objective function is divided by the reference value to make the objective function dimensionless and all value of three items having equal value range because three items in the objective function have a different unit and scale. The constraints on the bending stress and unbalance response are taken as follows:

$$g_1(x) = |\sigma_{\max}| - \sigma^* \le 0$$
$$g_2(x) = |\delta_{\max}| - \delta^* \le 0 \quad (15)$$

where $\sigma_{\max}$ and $\delta_{\max}$ denote the maximum bending stress and response in the steady-state. σ^* and δ^* represent the allowable stress and allowable steady-state response, respectively.

From Eqs. (10), (11), (13) and (14), the sensitivity number SN of objective function is defined as follows

$$SN = \frac{\partial F}{\partial d_j} = \frac{\alpha}{W_0}\frac{\partial W}{\partial d_j} + \frac{\beta}{\omega_{20}}\frac{\partial \omega_2}{\partial d_j} + \frac{\gamma}{Q_0}\frac{\partial Q_2}{\partial d_j} \quad (16)$$

Hence, the solution procedures using sensitivity number of multi-objective optimization problem are outlined as follows.

3.2 ESO Procedure Using Sensitivity Number

The solution procedures using sensitivity number (SN) of multi-objective optimization problem are outlined as follows.

***Step* 1:** FE analysis of new model which produces total weight W, natural frequency ω_2 and Q factor Q_2 and calculates the value of objective function $F(x)$ using Eq. (14).

***Step* 2:** Sensitivity analysis which produces SN of objective function for each diameter of element using Eqs. (9), (11), (13), and (14).

***Step* 3:** Change the diameter of each element in proportion to SN numbers. If the sensitivity number of ith element is positive, decrease the diameter of this element, because sensitivity number means the change of objective function per positive unit change of design variable and the goal is to minimize the objective function. And then it needs to be determined how much change will be accomplished. In this paper, following changing criterion was used.

$$\delta d_i = \begin{cases} -\dfrac{SN_i}{SN_{\max}}(d_{i\max} - d_i)\ C_P & : SN_i < 0 \\ \dfrac{SN_i}{SN_{\max}}(d_i - d_{i\min})\ C_P & : \mathit{Otherwise} \end{cases} \quad (17)$$

where, δd_i is the amount of change of ith element, SN is the sensitivity number of ith element, SN_{max} is the maximum absolute value of sensitivity number among all element, d_i is the diameter of ith element, C_p is the changing rate of the shaft at each iteration, $d_{i\max}, d_{i\min}$ are the maximum and minimum allowable diameter of ith element at present iteration(jth).

If the sign of SN of this element in present iteration is changed from previous iteration, there must be the optimum between present diameter and previous diameter. So it needs to change the limit of diameter as follows:

$$\begin{cases} d_{i,j\max} = d_{i,j-1} & : SN_{i,j}SN_{i,j-1} < 0,\ SN_{i,j} < 0 \\ d_{i,j\min} = d_{i,j-1} & : SN_{i,j}SN_{i,j-1} < 0,\ SN_{i,j} \geq 0 \end{cases} \quad (18)$$

***Step* 4:** Repeat Step 1 to Step 3 until following convergence criterion C_C is satisfied.

$$\frac{|F_{j-1} - F_j|}{F_{j-1}} \leq C_C \quad (19)$$

4. NUMERICAL RESULT AND DISCUSSION

In order to illustrate how the ESO can be used to find the optimum value of a shaft diameter, a numerical example is shown and discussed below. In example, ESO is used to minimize the shaft weight, Q factor and yield the critical speed so far from the operating speed as possible. The optimum model is compared with an original model in the unbalance response, Campbell diagram. Figure 1 shows the three-phase induction motor of a numerical calculation model. The principle data for this motor are listed in Table 1.

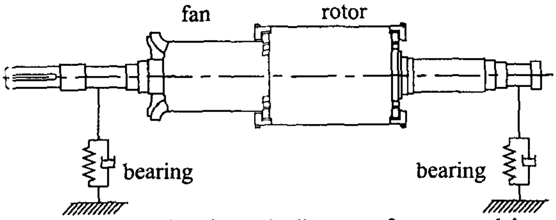

Fig. 1. Schematic diagram of motor model

Table 1. The configuration data of induction motor.

Motor	2 Pole, 50 Hz, 2200 kW
Shaf t	$L = 2.847$ m, $W = 5.207$ kN $E = 206$ GN/m^2, $G = 83.06$ N/m^2 $\gamma = 77$ kN/m^3
Bear ing	2 lobe bearing (preload factor = 0.5) (C = 0.1 mm, L = 150 mm, D = 125 mm)

The side constraints of the design variables are given by

$$12.5\text{mm} \leq d_i \leq 20.5\text{mm}, \quad i = 1 \text{ to } 19$$

$$d_1 \sim d_4, d_{10}, d_{11}, d_{17} \sim d_{19} : \text{will not change}$$

In Eq. (14), W_0, ω_0 and Q_0 are set to initial value and $\alpha = \beta = 1$, $\gamma = 0.004$. The weighting factor of weight and natural frequency is set to be same and weighting factor of Q factor is smaller than others because Q factor is much influenced by bearing than shape of shaft. In ESO algorithm, the changing rate of shaft diameters at each iteration is set to 55% of changeable region (C_P =0.55) and the convergence criterion factor (C_C) is set to 0.00005.

Figure 2 shows the shape of an original, intermediate and optimum shaft. The gray area shows fixed elements. It can be seen that which element is more effective to decrease the objective function. Figure 3 shows the change of shaft weight, 2nd natural frequency and objective function in each iteration. We know that these parameters are convergent within several times of iteration from Fig. 3.

Figures 4~5 are shown a comparison of critical speeds and unbalance responses before and after optimization. Comparing with an original model, the 2nd critical speed decreased from 2259cpm to 1985cpm and so the operating speed sufficiently away from resonance region. To analysis the unbalance response, the allowable residual unbalance is calculated on the basis of ISO G6.3. Obtained the allowance residual unbalance is divided into two and applied to both ends of a core. Figure 5 is shown the unbalance response in the center of core. The unbalance response of an optimum model (43.4μm) in the operating speed is reduced about 11.5μm compared to an original model (54.9μm) in Fig. 5. Also the maximum unbalance response at the bearing is satisfied the vibration limit value (50.8μm) of API standards in bearing, sufficiently.

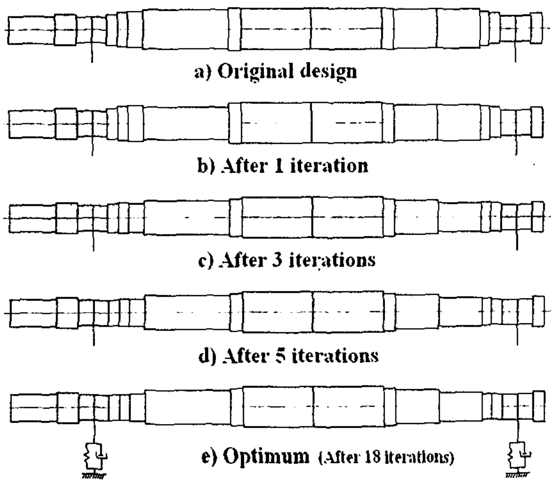

Fig. 2. Comparison of shaft shapes

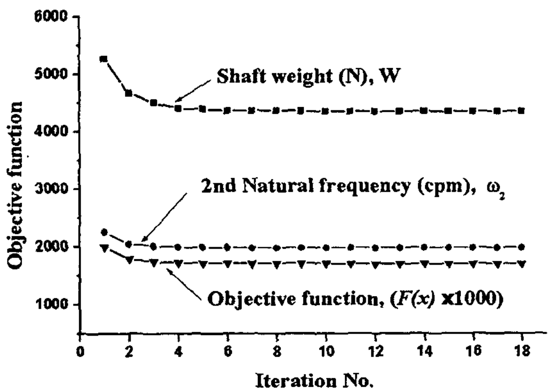

Fig. 3. Convergence characteristics of objective function

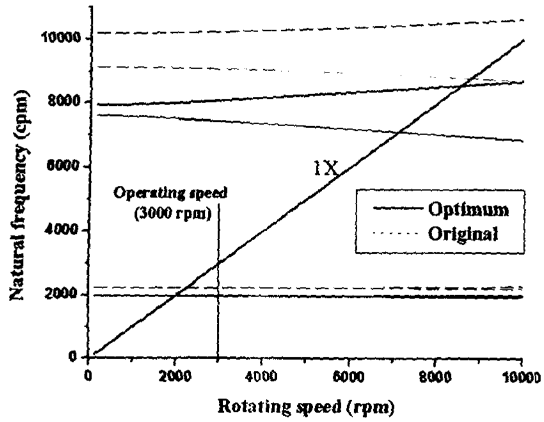

Fig. 4. Campbell diagram

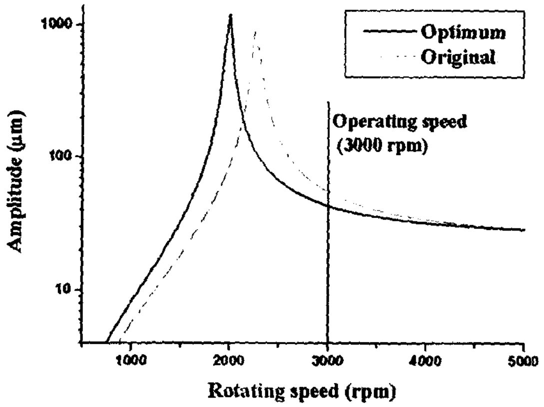

Fig. 5. Unbalance response

5. CONCLUSIONS

In this paper, the dynamic optimal design of a motor shaft was studied by using advanced ESO method, considering the diameter of the motor shaft. The sensitivity number of rotating system is investigated to apply the ESO method which was used to optimize the structural system. Through the optimization to minimize shaft weight and Q factor and to avoid the resonance region sufficiently in some stress and dynamic response constraints, the results show the capability of ESO method to rotating shaft. The total shaft weight, critical speeds and the unbalance response of a motor can be significantly improved by the modification of shaft diameters, even without changing each element length of the rotor-bearing system.

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