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# Semi-active Damping Control for Vibration Attenuation: Maximum Dissipation Direction Control


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**Key Words :** Semi-active Control (반능동 제어), Variable Damping (가변감쇠), Least Damped Mode (최소감쇠 모드), Magneto-rheological Fluid (자기유변유체)


## ABSTRACT

A practical and effective semi-active on-off control law is developed for vibration attenuation of a natural, multi-degree-of-freedom suspension system, when its operational response mode is available. It does not need the accurate system parameters and dynamics of semi-active actuator. It reduces the total vibratory energy of the system including the work done by external disturbances and the maximum energy dissipation direction of the semi-active control device is tuned to the operational response mode of the structure. The effectiveness of the control law is illustrated with a three degree-of-freedom excavator cabin model.


## INTRODUCTION

As the need for reduced noise and vibration increases, the suspension systems of machines and structures are becoming even more complex and more important than ever. Among others, the semi-active suspension system is known to be a good candidate for practical applications because it combines the advantages of passive and active suspension systems. It provides far better performance than the passive suspension system, not requiring high power actuators or supplies. It costs less and its performance is often not better than the active suspension system, although the controller implementation remains almost identical.

Semi-active control laws, which are often developed by modifying active control laws, require an accurate and yet robust mathematical model of the structure and the control devices. Clipped-optimal control is perhaps one of the most commonly used semi-active control algorithms, due to its robustness to change of the system parameters. On the other hand, semi-active control devices possess inherent nonlinearity so that development of optimal control laws becomes challenging. As the suspension technology advances, many passive suspension systems have been replaced by semi-active suspension systems. The stringent performance requirement for semi-active suspension systems is the simple implementation, not the best isolation. Thus, numerous semi-active on-off control algorithms have been developed and adopted for semi-active control systems, which are robust to modeling uncertainties. The 'sky-hook' damper control algorithm has been commonly adopted for vehicle suspension systems and demonstrated its improved performance over passive systems when applied to a single degree-of-freedom system[1]. Recently, a control algorithm based on Lyapunov direct stability theory has been proposed for electrorheological fluid dampers[2, 3]. It reduces the responses by minimizing the cost function, the rate of change of a Lyapunov function, where the state weighting matrix is to be properly selected.

## SEMI-ACTIVE CONTROL SYSTEM

The semi-active control system originates from a passive control system which has been subsequently modified to allow for adjustment of mechanical properties. The mechanical properties of the system may be adjusted based on feedback of the excitation and/or the measured response. The control force in a semi-active control system normally acts to oppose the motion of the system, promoting the global stability of the structure. Semi-active control systems maintain the reliability of passive control systems and, yet, take the advantage of the adjustable parameter characteristics of active control systems. Among others, energy dissipation devices which dissipate energy through various mechanisms such as shearing of viscous fluid, orificing of fluid, and sliding friction, have commonly been modified to behave in a semi-active manner[4]. Without loss of generality, a semi-active control force, $F_i$, may be modelled as a linear damper with controllable damping, i.e.

$$F_i = v_i(t)\dot{r}_i(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{y}}), \qquad (1)$$

where $\dot{r}_i(\mathbf{q}, \mathbf{y})$ is the relative velocity vector between two ends of the $i$ th semi-active device; $\mathbf{q}$ and $\dot{\mathbf{q}}$ denote the $n$ dimensional vectors with components of generalized coordinates $q_k$ and generalized velocities


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$\dot{q}_k$, $k = 1$ to $n$, for an $n$ degree-of-freedom system; $\mathbf{y}(t)$ and $\dot{\mathbf{y}}(t)$ are the $m$ dimensional external disturbance vectors with components of dispacements $y_j$ and velocities $\dot{y}_j$, $j = 1$ to $m$, where $m$ is the number of external disturbances; $v_i(t)$ is the variable damping coefficient with $0 \le v_{i\min} \le v_i(t) \le v_{i\max}$, $i = 1$ to $p$, where $p$ is the number of semi-active devices; $v_{i\min}$ and $v_{i\max}$ are the smallest and largest allowable values for $v_i$. For a semi-active vibration control system, Rayleigh's dissipation function $R$ can be expressed as

$$R = R_o + R_v, \qquad (2)$$

where $R_o = \frac{1}{2}\sum_{i=1}^{s}\left(c_i \dot{r}_i^2\right)$, $c_i$ is the damping coefficient of passive mounts or dampers, $i = 1$ to $s$, where $s$ is the number of the passive devices; and $R_v = \frac{1}{2}\sum_{i=1}^{p}\left(v_i \dot{r}_i^2\right)$. Here $R_o$ and $R_v$ are the Rayleigh's dissipation functions derived from the original dissipative forces and the semi-active control forces, respectively.

## CONTROL STRATEGY

For a natural system, the Hamiltonian $H$ reduces to the total energy of the system and its rate of change can be expressed as

$$\dot{H} = \dot{H}_d + \dot{H}_i \qquad (3)$$

where $\dot{H}_d = \sum_{k=1}^{n}\left(\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)\dot{q}_k - \frac{\partial L}{\partial q_k}\dot{q}_k\right)$, $\dot{H}_i = -\sum_{j=1}^{m}\frac{\partial L}{\partial y_j}\dot{y}_j$ and $L$ is Lagrangian. Here, $\dot{H}_i$ is the energy inflow rate due to the external disturbances applied to the system and $\dot{H}_d$ is the change rate of the system energy. When there are dissipative forces derived from the Rayleigh's dissipation function $R$ in Eq. (2), we can obtain

$$\dot{H}_d = \dot{H}_{do} + \dot{H}_{dv} \qquad (4)$$

where $\dot{H}_{do} = -\sum_{i=1}^{s} c_i \dot{r}_i\left(\sum_{k=1}^{n}\frac{\dot{r}_i}{\partial \dot{q}_k}\dot{q}_k\right)$ and $\dot{H}_{dv} = -\sum_{i=1}^{p} v_i \dot{r}_i\left(\sum_{k=1}^{n}\frac{\dot{r}_i}{\partial \dot{q}_k}\dot{q}_k\right)$. Here, $\dot{H}_{do}$ is the energy dissipation rate due to the original passive dampings in the system. The variable damping coefficients of the semi-active control devices should be set to be dissipative for interested generalized coordinates of the system and thus to make $\dot{H}_{dv}$ negative. But, because of the semi-active nature of control forces restrictions and the presence of external disturbances, it is impractical to keep $\dot{H}_{dv}$ always negative. However, it becomes feasible to keep $\dot{H}_{dv}$ negative while the semi-active control is activated. For that purpose, the variable damping coefficient $v_i(t)$ may be determined as

$$if\ \dot{r}_i\left(\sum_{k=1}^{n}\rho_{ik}\frac{\partial \dot{r}_i}{\partial \dot{q}_k}\dot{q}_k\right) \ge 0,\ then\ v_i = v_{i\,max}, \qquad (5a)$$

$$if\ \dot{r}_i\left(\sum_{k=1}^{n}\rho_{ik}\frac{\partial \dot{r}_i}{\partial \dot{q}_k}\dot{q}_k\right) < 0,\ then\ v_i = v_{i\,min}. \qquad (5b)$$

where $\rho_{ik}$ is the weighting factor imposed on the $k$ th generalized coordinate associated with the $i$ th semi-active device. For most of practical cases where we can assume that $v_{i\min}$ is zero, $\dot{H}_{dv}$ can be expressed as

$$\begin{aligned}
\dot{H}_{dv} &\approx -\sum_{i=1}^{p}\alpha_i\left(\sum_{k=1}^{n}\rho_{ik}\frac{\partial \dot{r}_i}{\partial \dot{q}_k}\dot{q}_k\right)\left(\sum_{k=1}^{n}\frac{\partial \dot{r}_i}{\partial \dot{q}_k}\dot{q}_k\right) \\
&= -\sum_{i=1}^{p}\alpha_i\left(\sum_{k=1}^{n}\sum_{l=1}^{n}\rho_{ik}\frac{\partial \dot{r}_i}{\partial \dot{q}_k}\frac{\partial \dot{r}_i}{\partial \dot{q}_l}\dot{q}_k\dot{q}_l\right) \\
&= -\sum_{i=1}^{p}\alpha_i\dot{\mathbf{q}}^T\mathbf{D}_i\dot{\mathbf{q}}\left(\equiv \sum_{i=1}^{p}\dot{H}_{dvi}\right) = -\dot{\mathbf{q}}^T\left[\sum_{i=1}^{p}\alpha_i\mathbf{D}_i\right]\dot{\mathbf{q}} \\
&= -\dot{\mathbf{q}}^T[\alpha_1\mathbf{D}_1 + \alpha_2\mathbf{D}_2 + \cdots + \alpha_p\mathbf{D}_p]\dot{\mathbf{q}}
\end{aligned} \qquad (6)$$

where $\alpha_i = \frac{v_i\|\dot{r}_i\|}{\left\|\sum_{k=1}^{n}\rho_{ik}\frac{\partial \dot{r}_i}{\partial \dot{q}_k}\dot{q}_k\right\|}$ ; $\mathbf{D}_i$ is an $n \times n$ real symmetric matrix with elements of $\rho_{ik}\frac{\partial \dot{r}_i}{\partial \dot{q}_k}\frac{\partial \dot{r}_i}{\partial \dot{q}_l}$. Note that, by the control action (5), $\dot{H}_{dv}$ becomes a quadratic form of generalized coordinates only. Contrary to $\dot{H}_{dv}$, $\dot{H}_{do}$ explicitly contains both the generalized coordinates and the external disturbances. So, it can be divided into two terms with the coordinates and the external disturbances, respectively, that is,

$$\dot{H}_{do} = \dot{H}_{doc} + \dot{H}_{doe} \qquad (7)$$

and $\dot{H}_{doc}$, the term with the coordinate, in general, can be represented in the quadratic form of

$$\dot{H}_{doc} = -\dot{\mathbf{q}}^T\mathbf{C}\dot{\mathbf{q}} \qquad (8)$$

where $\mathbf{C}$ is the passive system damping matrix with elements of $\frac{\partial \dot{r}_i}{\partial \dot{q}_k}$ and $c_i$.

## WEIGHTING FACTOR DETERMINATION

### The Maximum Principle

For a real symmetric matrix $\mathbf{A}$, the quadratic form $Q(\dot{\mathbf{q}}) = \dot{\mathbf{q}}^T\mathbf{A}\dot{\mathbf{q}}$ produces a real number for every vector $\dot{\mathbf{q}}$ in $\Re^n$. Since $Q(\dot{\mathbf{q}})$ is a continuous function in $\Re^n$, it attains a maximum on the closed, bounded set of vectors $\|\dot{\mathbf{q}}\| = 1$. The quadratic form attains the maximum $\lambda_1$ at $\dot{\mathbf{q}} = \boldsymbol{\eta}_1$ where $\lambda_1$ is the largest eigenvalue of the matrix $\mathbf{A}$ and $\boldsymbol{\eta}_1$ is the eigenvector corresponding to $\lambda_1$. Then for every unit vector $\dot{\mathbf{q}}$ orthorgonal to $\boldsymbol{\eta}_1$, $Q(\dot{\mathbf{q}}) \le Q(\boldsymbol{\eta}_1)$. But, on the subset $\dot{\mathbf{q}} \cdot \boldsymbol{\eta}_1 = 0$ and $\|\dot{\mathbf{q}}\| = 1$, $Q(\dot{\mathbf{q}})$ again attains a maximum $\lambda_2$ at $\dot{\mathbf{q}} = \boldsymbol{\eta}_2$ where $\lambda_2$ is the second eigenvalue of the matrix $\mathbf{A}$ and $\boldsymbol{\eta}_2$ is the eigenvector corresponding to $\lambda_2$. Continuing in this way we can get a set of mutually orthorgonal unit vectors $\boldsymbol{\eta}_k$ at which the local extremal value $\lambda_k$ is attained on the subset $\|\dot{\mathbf{q}}\| = 1$. We can also easily find that $\boldsymbol{\eta}_k$ is in fact the $k$th eigenvector of $\mathbf{A}$ and that the value $\lambda_k$ is the corresponding eigenvalue, $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_n$ [5].

From simple calculation, it is easily derived that the matrix $\mathbf{D}_i$ has only two non-zero eigenvalues irrespective of the matrix dimension, $n$, except when all weighting factors are identical. The eigenvalues are obtained to be

$$\lambda_{pos}^{\mathbf{D}_i} = \frac{1}{2}\left( \sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \right)^2 \rho_{ik} + \sqrt{\sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \right)^2} \right) \tag{9}$$

$$\lambda_{neg}^{\mathbf{D}_i} = \frac{1}{2}\left( \sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \right)^2 \rho_{ik} - \sqrt{\sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \right)^2} \right) \tag{10}$$

and the eigenvector $\boldsymbol{\eta}_{pos}^{\mathbf{D}_i}$ corresponding to $\lambda_{pos}^{\mathbf{D}_i}$ can also be easily obtained as

$$(\boldsymbol{\eta}_{pos}^{\mathbf{D}_i})_j = \frac{\dfrac{\partial \dot{r}_i}{\partial \dot{q}_j}\left( 1 + \rho_{ij}\sqrt{\sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \right)^2} \right)}{\sqrt{\sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \right)^2 \left( 1 + \rho_{ik}\sqrt{\sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \right)^2} \right)^2}} \tag{11}$$

where $\sum_{k=1}^{n} \left( \frac{\partial \dot{r}_i}{\partial \dot{q}_k} \rho_{ik} \right)^2 = 1$ and the superscript $\mathbf{D}_i$ denotes that the eigensolutions are calculated from the matrix $\mathbf{D}_i$. When all weighting factors are identical, leading to the 'Sky-hook control', there exists only one non-zero, positive eigenvalue. The weighting factors then should be properly assigned such that the eigenvector $\boldsymbol{\eta}_{pos}^{\mathbf{D}_i}$ corresponding to $\lambda_{pos}^{\mathbf{D}_i}$ is aligned with the excessive vibrational motion of interest. In this case, because the eigenvalue $\lambda_{neg}^{\mathbf{D}_i}$ is negative, the vibration motion or mode corresponding to $\boldsymbol{\eta}_{neg}^{\mathbf{D}_i}$ is little damped.

### Target Mode Selection

For proper assignment of the weighting factors, the excessive dominant vibrational motion of interest, namely the target mode should be specified to be aligned with the eigenvector $\boldsymbol{\eta}_{pos}^{\mathbf{D}_i}$ corresponding to $\lambda_{pos}^{\mathbf{D}_i}$. The specified vibrational motion of interest may vary depending on the vibrational chracteristics of the system related to the energy storage and dissipation elements, the excitation and operational conditions, and the user's subjective requirement on the performance. In practice, the forced vibrations of many systems are often dominated by the so-called operational deflection shape, which can be a single natural mode or a combination of many natural modes. When the target mode is chosen as the dominant operational deflection shape of the interested system, its general form becomes a combination of natural modes. On the other hands, the least damped mode, which often causes excessive vibration, may be selected as the target mode. For example, we can select the target mode such that

$$\boldsymbol{\eta}_{target}^{i} \equiv \boldsymbol{\eta}_{n}^{\mathbf{C}} \tag{12}$$

where $\boldsymbol{\eta}_n^{\mathbf{C}}$ is the eigenvector corresponding to the least eigenvalue $\lambda_n^{\mathbf{C}}$ and $\boldsymbol{\eta}_{target}^{i}$ is the target mode imposed on the $i$th semi-active device. Note that, in this case, the amount of energy dissipation is significantly increased along with the target mode $\boldsymbol{\eta}_n^{\mathbf{C}}$.

### Optimization

Equations (9) and (10) suggest that, in order to make $\lambda_{pos}^{\mathbf{D}_i}$ large and $\lambda_{neg}^{\mathbf{D}_i}$ small, all the weighting factors should be positive. With such constraint, the actuator eigenvector $\boldsymbol{\eta}_{pos}^{\mathbf{D}_i}$ may not become identical to the specified target mode. Thus we find the $n$ dimensional weighting vector $\mathbf{x}_i = \{\rho_{i1} \ \ \rho_{i2} \ \ \cdots \ \ \rho_{in}\}$ for the $i$th semi-active control device which minimizes the cost function

$$f(\mathbf{x}_i) = 1 - Corr\left(\boldsymbol{\eta}_{pos}^{\mathbf{D}_i}(\mathbf{x}_i), \boldsymbol{\eta}_{target}^{i}\right) \tag{13}$$

subject to the $n$ inequality constraints

$$\rho_{ik} \ge 0; \ k = 1 \text{ to } n, \tag{14}$$

where $Corr(\mathbf{a},\mathbf{b}) = \dfrac{|\mathbf{a}^T\mathbf{b}|}{\|\mathbf{a}\|\|\mathbf{b}\|}$ which is a measure of alikeness between two vectors $\mathbf{a}$ and $\mathbf{b}$. Note that, when two vectors $\mathbf{a}$ and $\mathbf{b}$ are in line, $Corr(\mathbf{a},\mathbf{b}) = 1$, whereas $Corr(\mathbf{a},\mathbf{b}) = 0$ for $\mathbf{a} \perp \mathbf{b}$.

## ILLUSTRATIVE EXAMPLE

As an example, vibration control of a three degree-of-freedom excavator cabin suspension system is considered. Figure 1 shows the dynamic model of the excavator cabin with one semi-active linear damper in the suspension, accounting for the bounce, pitch and roll motions of the cabin. For simplicity, it is assumed that the principal moment of inertia axes of the cabin coincide with the Cartesian coordinates. The semi-active control device is assumed to be placed at point 1 as shown in Figure 1. The equations of motion are obtained as

$$
\begin{aligned}
& m\ddot{z} + c_1\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_1\right) + c_2\left(\dot{z} - l_1\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_2\right) \\
& + c_3\left(\dot{z} - l_1\cos\theta_y\dot{\theta}_y + w_1\cos\theta_x\dot{\theta}_x - \dot{y}_3\right) + c_4\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y + w_1\cos\theta_x\dot{\theta}_x - \dot{y}_4\right) \\
& + k_1\left(z + l_2\sin\theta_y - w_2\sin\theta_x - y_1\right) + k_2\left(z - l_1\sin\theta_y - w_2\sin\theta_x - y_2\right) \\
& + k_3\left(z - l_1\sin\theta_y + w_1\sin\theta_x - y_3\right) + k_4\left(z + l_2\sin\theta_y + w_1\sin\theta_x - y_4\right) \\
& + v\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_1\right) = 0
\end{aligned}
\tag{15a}
$$

$$
\begin{aligned}
& I_{yy}\ddot{\theta}_y + c_1 l_2\cos\theta_y\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_1\right) - c_2 l_1\cos\theta_y\left(\dot{z} - l_1\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_2\right) \\
& - c_3 l_1\cos\theta_y\left(\dot{z} - l_1\cos\theta_y\dot{\theta}_y + w_1\cos\theta_x\dot{\theta}_x - \dot{y}_3\right) + c_4 l_2\cos\theta_y\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y + w_1\cos\theta_x\dot{\theta}_x - \dot{y}_4\right) \\
& + k_1 l_2\cos\theta_y\left(z + l_2\sin\theta_y - w_2\sin\theta_x - y_1\right) - k_2 l_1\cos\theta_y\left(z - l_1\sin\theta_y - w_2\sin\theta_x - y_2\right) \\
& - k_3 l_1\cos\theta_y\left(z - l_1\sin\theta_y + w_1\sin\theta_x - y_3\right) + k_4 l_2\cos\theta_y\left(z + l_2\sin\theta_y + w_1\sin\theta_x - y_4\right) \\
& + v l_2\cos\theta_y\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_1\right) = 0
\end{aligned}
\tag{15b}
$$

$$
\begin{aligned}
& I_{xx}\ddot{\theta}_x - c_1 w_2\cos\theta_x\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_1\right) - c_2 w_2\cos\theta_x\left(\dot{z} - l_1\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_2\right) \\
& + c_3 w_1\cos\theta_x\left(\dot{z} - l_1\cos\theta_y\dot{\theta}_y + w_1\cos\theta_x\dot{\theta}_x - \dot{y}_3\right) + c_4 w_1\cos\theta_x\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y + w_1\cos\theta_x\dot{\theta}_x - \dot{y}_4\right) \\
& - k_1 w_2\cos\theta_x\left(z + l_2\sin\theta_y - w_2\sin\theta_x - y_1\right) - k_2 w_2\cos\theta_x\left(z - l_1\sin\theta_y - w_2\sin\theta_x - y_2\right) \\
& + k_3 w_1\cos\theta_x\left(z - l_1\sin\theta_y + w_1\sin\theta_x - y_3\right) + k_4 w_1\cos\theta_x\left(z + l_2\sin\theta_y + w_1\sin\theta_x - y_4\right) \\
& - v w_2\cos\theta_x\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_1\right) = 0
\end{aligned}
\tag{15c}
$$

where $z$, $\theta_y$ and $\theta_x$ are the linear displacement and rotation angles of the cabin mass center along the z, y and x axes; $\dot{y}_j$ and $y_j$, $j=1, 2, 3, 4$, are the linear velocities and displacements at point $j$ along the z axis, acting as external distubances to the cabin. The system parameters and damping coefficient of the semi-active control device are listed in Table 1. Assuming the small motion, $\dot{H}_{doc}$ can be approximated as a quadratic form of

$$\dot{H}_{doc} = -\dot{\mathbf{q}}^T\mathbf{C}\dot{\mathbf{q}} \tag{16}$$

where

$$
\mathbf{C} = \begin{bmatrix} c_1 + c_2 + c_3 + c_4 & l_2(c_1 + c_4) - l_1(c_2 + c_3) & w_1(c_3 + c_4) - w_2(c_1 + c_2) \\ l_2(c_1 + c_4) - l_1(c_2 + c_3) & l_2^2(c_1 + c_4) + l_1^2(c_2 + c_3) & w_2(l_1c_2 - l_2c_1) - w_1(l_1c_3 - l_2c_4) \\ w_1(c_3 + c_4) - w_2(c_1 + c_2) & w_2(l_1c_2 - l_2c_1) - w_1(l_1c_3 - l_2c_4) & w_2^2(c_1 + c_2) + w_1^2(c_3 + c_4) \end{bmatrix}
$$

and $\dot{\mathbf{q}}^T = \{\dot{z}\ \ \dot{\theta}_y\ \ \dot{\theta}_x\}$. The three eigenvalues and the corresponding eigenvectors of the matrix $\mathbf{C}$ are obtained as $\lambda_1^{\mathbf{C}} = 120.3$, $\boldsymbol{\eta}_1^{\mathbf{C}} = \{-0.99\ \ 0.14\ \ 0\}^T$; $\lambda_2^{\mathbf{C}} = 107.2$, $\boldsymbol{\eta}_2^{\mathbf{C}} = \{-0.14\ \ -0.99\ \ 0\}^T$; and $\lambda_3^{\mathbf{C}} = 27.7$, $\boldsymbol{\eta}_3^{\mathbf{C}} = \{0\ \ 0\ \ 1\}^T$, respectively. Superscript $\mathbf{C}$ means that the values are calculated from the matrix $\mathbf{C}$. Note that the rolling motion is decoupled from the other two eigenvectors and least damped, when the four supporting passive dampers are identical. The target mode is determined as

$$\boldsymbol{\eta}_{target} \equiv \boldsymbol{\eta}_3^{\mathbf{C}} = \{0\ \ 0\ \ 1\}^T \tag{17}$$

so that the semi-active damper can extract energy mostly from the least damped rolling motion. The proposed control law gives the variable damping of the semi-active control device as

$$
\begin{aligned}
& if\left(\dot{z} + l_2\cos\theta_y\dot{\theta}_y - w_2\cos\theta_x\dot{\theta}_x - \dot{y}_1\right)\left(\rho_1\dot{z} + \rho_2 l_2\cos\theta_y\dot{\theta}_y - \rho_3 w_2\cos\theta_x\dot{\theta}_x\right) \ge 0, v = v_{\max} \\
& else \qquad , v = v_{\min}
\end{aligned}
\tag{18}
$$

By the control action, $\dot{H}_{dv}$ can be approximated as a quadratic form given by

$$\dot{H}_{dv} = -\alpha\dot{\mathbf{q}}^T\mathbf{D}\dot{\mathbf{q}} \tag{19}$$

where $\alpha = \dfrac{v\left\|\dot{z} + l_2\dot{\theta}_y - w_2\dot{\theta}_x - \dot{y}_1\right\|}{\left\|\rho_1\dot{z} + \rho_2 l_2\dot{\theta}_y - \rho_3 w_2\dot{\theta}_x\right\|}$;

$$
\mathbf{D} = \begin{bmatrix} \rho_1 & \dfrac{\rho_1 + \rho_2}{2}l_2 & -\dfrac{\rho_1 + \rho_3}{2}w_2 \\ \dfrac{\rho_1 + \rho_2}{2}l_2 & \rho_2 l_2^2 & -\dfrac{\rho_2 + \rho_3}{2}l_2 w_2 \\ -\dfrac{\rho_1 + \rho_3}{2}w_2 & -\dfrac{\rho_2 + \rho_3}{2}l_2 w_2 & \rho_3 w_2^2 \end{bmatrix}.
$$

**Table 1. System Parameters**

| Symbol | Content | Value |
|---|---|---|
| $m$ | Mass | 602 kg |
| $I_{xx}$ | Moment of inertia w.r.t. X-axis | 244.6 kgm$^2$ |
| $I_{yy}$ | Moment of inertia w.r.t. Y-axis | 258.9 kgm$^2$ |
| $l_1$ | Length from mass center to mount 2 or 3 along X-axis | 961 mm |
| $l_2$ | Length from mass center to mount 1 or 4 along X-axis | 931 mm |
| $w_1$ | Length from mass center to mount 3 or 4 along Y-axis | 480 mm |
| $w_2$ | Length from mass center to mount 1 or 2 along Y-axis | 480 mm |
| $c_i$ | $i$ th mount damping coefficient | 30 Ns/m |
| $k_i$ | $i$ th mount stiffness coefficient | 20 kN/m |
| $v_{max}$ | Max. damping coefficient of semi-active mount | 300 Ns/m |
| $v_{min}$ | Min. damping coefficient of semi-active mount | 0 Ns/m |

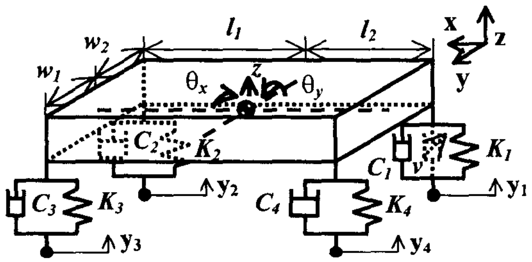


**Figure 1. Dynamic Model of Three Degree-of-Freedom Cabin Suspension System**

For the 'Sky-hook Damper' control, since $\{\rho_1 \;\; \rho_2 \;\; \rho_3\} = \{0.69 \;\; 0.69 \;\; 0.69\}$, we obtain $\lambda^{D}_{pos} = 1.45$ and $\boldsymbol{\eta}^{D}_{pos} = \{0.69 \;\; 0.64 \;\; -0.33\}^T$. This action may not be desirable because the least damped rolling motion is least attenuated.

For the target mode $\{0 \;\; 0 \;\; 1\}^T$, the optimization (13) gives: $\{\rho_1 \;\; \rho_2 \;\; \rho_3\} = \{0 \;\; 0 \;\; 2.08\}$, $\lambda^{D}_{pos} = 0.96$ and $\boldsymbol{\eta}^{D}_{pos} = \{-0.42 \;\; -0.39 \;\; 0.82\}^T$. Note that $\boldsymbol{\eta}^{D}_{pos}$ is not identical to $\boldsymbol{\eta}_{target} \equiv \boldsymbol{\eta}^{C}_{3}$, giving the measure of alikeness of 0.6657. The inevitable discrepancy is due to the inadequate location of the semi-active device and the inequality constraints imposed on the weighting factors, Eq. (14). Note that the least damped rolling mode is most attenuated. When the system is subject to disturbances, the response vector can be decomposed as

$$\mathbf{q} = u_1 \boldsymbol{\eta}^{C}_{1} + u_2 \boldsymbol{\eta}^{C}_{2} + u_3 \boldsymbol{\eta}^{C}_{3}. \qquad (20)$$

where $u_i = \mathbf{q}^T \cdot \boldsymbol{\eta}^{C}_{i}$ is the component of $\mathbf{q}$ projected to $\boldsymbol{\eta}^{C}_{i}$.

In the simulations, we compared the performances of the following 4 cases:

1. Original system,
2. Passive damping control system which merely adds a passive damper of $v_{max}$ at point 1,
3. Sky-hook damper control system which is equivalent to $\rho_1 = \rho_2 = \rho_3 = 0.69$,
4. Proposed on-off damping control system with $\rho_1 = 0$, $\rho_2 = 0$ and $\rho_3 = 2.08$,

subject to an impulse disturbance input at pt. 1, given by

$$y_1 = (0.1)\delta(t) \qquad (21)$$

where $\delta(t)$ is a Dirac delta function. Figures 2, 3 and 4 show the impulse responses associated with the three components $u_i$, $i = 1, 2, 3$, of $\mathbf{q}$, respectively, for the above four cases. Note that the roll motion, $u_3$, is much larger than the bounce and pitch dominant motions, $u_1$ and $u_2$, respectively because of the damping characteristics of the original system. Responses of the passive and the sky-hook damper control systems are not distinguable because, except $t = 0$, there are no external disturbances. The proposed on-off control system gives smaller (larger) values of $u_3$ ($u_1$ and $u_2$) than the passive and the sky-hook damper control systems. Figures 5, 6 and 7 are the Fourier transforms of the impulse responses of and $u_1$, $u_2$ and $u_3$, respectively, normalized by the intensity of the impulse input. The results shown in the frequency domain also confirm that the proposed on-off control system shows better attenuation for the roll motion than the passive and the sky-hook damper control systems.

## CONCLUSION

Using Lagrange's equations and Lyapunov direct method, an efficient semi-active on-off damping control law for vibration attenuation of a multi-degree-of-freedom vibratory system has been developed. It minimizes the total vibratory energy of the structure, including the work done by external disturbances, whereas the dissipative energy of the semi-active control device is transformed into the weighted quadratic form and is maximized for the specified vibrational response of the system by a proper assignment of the weighting factors. The vibrational response vector of interest, namely the target mode, is determined at will, considering the vibrational chracteristics of the system and the user's subjective requirement on the performance. Features of the proposed scheme are: it is robust to the system parameters as well as the dynamics of semi-active control devices; it needs theoretically a single semi-active control device; and it needs only the velocity feedback when the system is linearized.

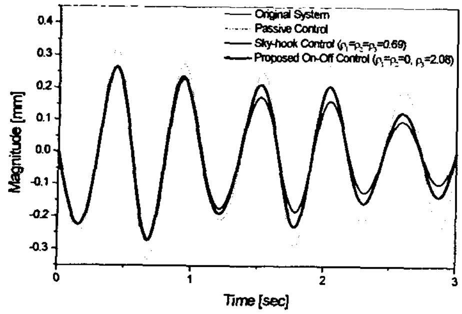


**Figure 2. Bounce Dominant Motion,** $u_1(t)$

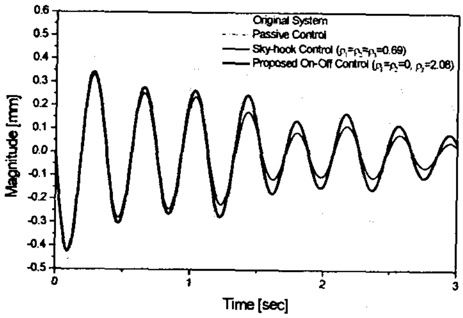


**Figure 3. Pitch Dominant Motion,** $u_2(t)$

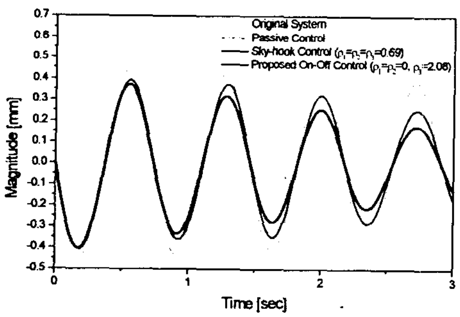


**Figure 4. Roll Motion,** $u_3(t)$

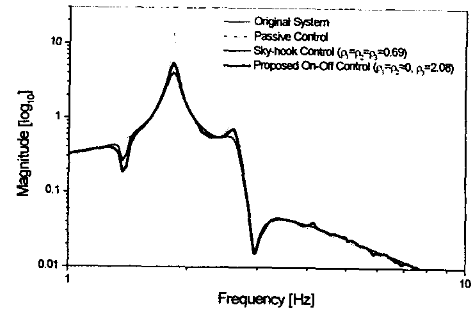


**Figure 5. Frequency Response Function : Bounce Dominant Motion**

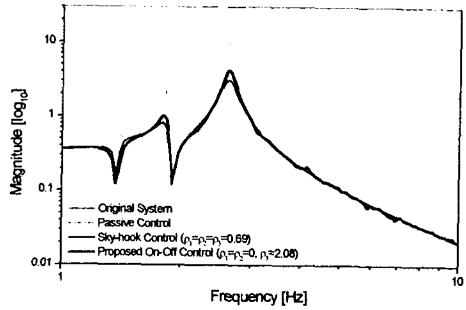


**Figure 6. Frequency Response Function : Pitch Dominant Motion**

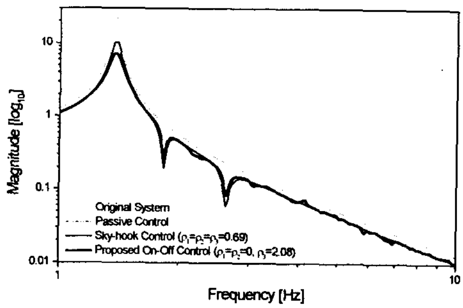


**Figure 7. Frequency Response Function : Roll Motion**

## REFERENCES


[1] D. C. Karnopp, M. J. Crosby and R. A. Harwood, "Vibration control using semi-active force generators", Journal of Engineering for Industry, Vol. 96, No. 2, pp. 619 ~ 626, 1974

[2] G. Leitmann, "Semi-active control for vibration attenuation", Journal of Intelligent Material Systems and Structures, Vol. 5, pp. 841 ~ 846, 1994

[3] G. Leitmann and E. Reithmeier, "A control scheme based on ER-materials for vibration attenuation of dynamical systems", Applied Mathematics and Computation, Vol. 70, pp. 247 ~ 259, 1995

[4] M. D. Symans, M. C. Constantinou, "Semi-active control systems for seismic protection of structures: a state-of-the-art review", Engineering Structures, Vol. 21, pp. 469 ~ 487, 1999

[5] J. P. Keener, Principles of Applied Mathematics: Transformation and Approximation, Addison-Wesley Publishing Company, 1988