

# Dynamics of an atomic wave packet in a standing wave quantized field

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The purpose of this work is to investigate the dynamics of an atomic wave packet whose center-of-mass motion is quantized in a resonant standing wave cavity field. The mechanical aspect of the matter-field interaction has been extensively studied in the theme of atomic beam deflection, diffraction, or reflection<sup>1)2)</sup> by a standing-wave field. The effect caused in the behavior of spontaneous emission by the atomic center-of-mass motion, classical and quantized, in a standing wave cavity mode has been studied<sup>3)</sup>, and recently the one-atom laser with quantized atomic center-of-mass motion has been investigated<sup>4)</sup>.

In most of these studies, the atoms are treated as plane waves in the coordinate space. The plane wave approximation is valid only when the spatial extent of the atomic wave packet is substantially larger than the wavelength of the field so that the initial atomic momentum is well-defined whereas the position uncertainty is completely indefinite. The opposite limit of approximation is such that atoms are regarded as spatially well-localized point masses (or plane waves in the momentum space). Obviously, the atom-field mechanical interactions then cannot be correctly incorporated due to the pre-assumed momentum translational symmetry. Note that most fundamental models such as the Jaynes-Cummings model<sup>5)</sup> are tacitly based on this approximation.

There are, however, situations in which the size of atomic wave packet is comparable to the wavelength of the field and consequently both the momentum and the position uncertainty are finite. Investigation of such a situation is the aim of this article: We wish to incorporate the Heisenberg uncertainty issue into the atom-field interaction as well as the atom-field momentum exchange process, and see if this will give any discernible differences in the predictions made by cavity quantum electrodynamics (cavity-QED).

For this purpose, we will look into a particular physical situation, i.e., the single atom detection scheme recently adopted by a few work groups<sup>6)7)</sup>. The idea is based on the so called vacuum Rabi splitting (VRS)<sup>8)9)</sup>. An empty cavity with loss rate  $2\chi$  will contain  $|\epsilon/\chi|^2$  quanta of light in it at steady state when it is driven by a resonant coherent field of amplitude  $\epsilon$ . Thus the steady state cavity transmission power will be at the constant value of  $\chi|\epsilon/\chi|^2$  quanta. When a two-state atom is placed in the cavity, however, things are different. The eigen-energies of the atom-field composite system are no longer the atomic or the cavity resonance frequency<sup>10)</sup>. Assuming for simplicity that the cavity is tuned to the atomic transition so that  $\omega_a = \omega_c \equiv \omega_0$ , we have, in the weak driving field limit, the maximum transmission occurring at  $\omega = \omega_0 \pm g$  (VRS) where  $g$  is the coupling strength of the atom and the field. Consequently, for the probe field tuned to the cavity resonance, the transmission power will drop. Thus, whenever the power drop is monitored, one can tell that an atom is traversing the cavity at that moment.

The question, however, arises when the spatial structure of the cavity field is not uniform as in the case of a standing-wave field in a Fabry-Perot-type cavity. Note that in most cavity-QED

experiments, the Fabry-Perot-type cavities are used in order to achieve the strong coupling regime where various damping rates are small compared to the atom-field coupling strength. The non-uniform field brings about a position-dependent atom-field coupling. I.e., when the atom traverses the cavity along a passage through or near a node of the field mode, the atom-field interaction will be negligible—as long as the atomic wave packet remains sufficiently well localized during the interaction. Then in the aforementioned single atom detection system, the transmission signal may not show a notable change although an atom has just passed through the cavity, and one may fail to detect the passing atom. If, however, the atomic wave packet gets significantly broadened through the interaction with the field, the atom-field coupling can be strong enough to provide a visible change in the transmission signal even when an atom passes through a node. So, the question is how significantly an atomic wave packet would get broadened through the atom-field interaction in realistic experimental situations. We aim to provide a quantitative answer to this question in the present work through a quantum trajectory<sup>11)</sup> calculation.

Let us note that the dynamics of an atomic wave packet is indeed previously studied by a number of authors<sup>12)13)14)</sup>, particularly seeking the possibility of position measurement of a two- or three-level atom in a standing wave field beyond the resolution of the so-called Heisenberg microscope. The difference of these studies from our present work lies in different measurement schemes, as well as in the large atom-field detuning. Different measurement processes bring about different back-actions to the system, and consequently different outcomes of the measurements such as atom localization, etc. Also in case of a large atom-field detuning, the atom-field momentum exchanges do not occur essentially, and the effect of the wave packet spreading via the momentum exchanges does not play a significant role. We consider in this work the system in a direct detection scheme where the atom and field are on resonance, beyond the Raman-Nath regime, including the atomic and cavity decay processes.

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