An Improved Method for Evaluation of Network Reliability with Variable Link-Capacities

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ABSTRACT

We propose a new method to evaluate the network reliability which greatly reduces the intermediate steps toward calculations of maximum capacity flow by excluding unnecessary simple paths contained in the set of failure simple paths. By using signed simple paths and signed flow, we show that our method is more efficient than that of Lee and Park (2001a) in the number of generated composite paths and in the procedure for obtaining minimal success composite paths. Numerical examples are given to illustrate the use and the efficiency of the method.

Keywords: Maximum capacity flow, Signed simple path, Signed flow, Flow augmenting simple path, Unilateral link, Saturated link, Flowless link

1. INTRODUCTION

Recently, network reliability and maximum capacity flow (MCF) transmitted through the network are popularly considered as important measures for the performance of network. A network is generally modeled as a graph $G(V, E)$ which V and E represent a node set and a link set, respectively. The network with variable link capacities is referred to as a flow network, and a number of methods for evaluating the performance of the network have been proposed under the assumption that all simple path (sp) sets are known. The network is operating if a required amount of flow from the source to the terminal is successfully transmitted.

Misra and Prasad (1982), Aggarwal et al. (1982), and Aggarwal (1988) propose the methods which generate composite paths, but these methods give some incorrect results in computing maximum capacity flow. Varshney et al (1994) modify the method of Aggarwal (1988) and incorporate the multistate links with corresponding probabilities. Schanzer (1995) shows that the methods of Aggarwal (1988) and Varshney et al (1994) are incorrect in some cases. Rai and Soh (1991) propose a method correcting the drawbacks of the preceding results, but it generates a large number of redundant composite paths (*cp*) and needs extra efforts of converting the given simple path into minimal cuts. Lee and Park (2001a) define the concept of eligibility and additivity and suggest a new method which reduces both the possible occurrence of redundancy and the number of composite paths considered for the capacity computation significantly. A *sp*, P, having additivity guarantees that P is a minimal *sp* which is added to an existing *cp*, C and a P having eligibility on C with respect to a set of *sp*, S, assures that the subnetwork induced by $C \cup P$ is composed of *sp*'s only in S. Lee and Park (2001b) also introduce an efficient method that selects the flow augmenting simple paths to evaluate MCF of the given network.

This paper suggests a method which deletes the unnecessary simple paths from the set of failure simple paths (FSP) prior to the computation of MCF. Then, the numbers of computing MCF and generating composite paths are fewer than those required in the method by Lee and Park (2001a). For developing our method, we use the concepts introduced in Lee and Park (2001a, 2001b). Section 2 gives notations and assumptions, and an improved algorithm is described in Section 3. Section 4 gives some numerical examples to illustrate the use and the efficiency of the method.

2. NOTATIONS AND ASSUMPTIONS

Notations

k	number of composition needed to generate the current *cp*, $k = 0, 1, 2, \cdots$.
$C(k)$	current *cp* at level k
FSP(k)	set of available failure signed *sp*'s at level k
ELGSP(k)	set of signed *sp*'s reserved for checking eligibility at level k
$P_{(k:n_k)}$	current additive signed *sp* on $C(k)$, where $n_k = 1, \cdots, N_k$
$W(C)$	MCF of the subnetwork induced by C

$W_{(k:n_k)}$	$= W(C(k) \cup_{P \in \mathrm{FSP}(k)} P)$
$W_{(k:ALL)}$	$= W(C(k) \cup P_{(k:n_k)} \cup_{P \in \mathrm{FSP}(k)} P)$
$w(k)$	augmented amount of flow by P with respect to $f(k-1)$
$sign(x)$	integer valued function; $+1$ if $x > 0$, 0 if $x = 0$, -1 if $x < 0$
$f_i(k)$, $f(k)$	flow on link i, flow pattern of the network at level k

Assumptions

1. The nodes are perfect and each has no capacity limit.
2. The links are independent and either function or fail with known probabilities.
3. All the links are undirected and each link flow is bounded by the link capacity.
4. No flow can be transmitted through a failed link.
5. The *sp* of the network, considering connectivity only, is known.

3. ALGORITHM

We present an algorithm to eliminate the unnecessary computations of MCF, which such computations are needed in the method by Lee and Park (2001a). Also, the links which do not increase MCF in success *cp* are deleted. The deletion helps to obtain minimal success *cp* (*mscp*), efficiently.

The link i which is in a *sp*, P, has the direction of $a \rightarrow b$ if it appears as an edge (a, b), and the direction of $b \rightarrow a$, otherwise. The moving direction of the flow of positive amount through link i is called *the flow direction in link i*. Also, the link i in P is said to be *saturated in P* if the flow direction of $f_i(\cdot)$ is the same with the link direction of i and the absolute value of $f_i(\cdot)$ is equal to the capacity of link i. Let a link i connect two nodes, a and b, and we define an order on the nodes as $a < b$. The sign of i has '+' as $a \rightarrow b$, and '−', otherwise, but we omit the '+' sign. Additional concepts which are needed for our algorithm are defined in the following.

Definition 1 (Lee and Park (2001b)).

1) A simple path P in which each edge is represented as a link signed by its link directon in P is said to be a *signed simple path P*. Similarly, the flow on link i which is signed by its flow direction is said to be the *signed flow on i*.

2) A simple path P is said to be a *flow augmenting simple path* (*fasp*) with respect to flow pattern if there is no saturated link in P.

3) A link is said to be *unilateral* if the link direction of it is the same in all simple paths containing it and *flowless* if the amount of flow on link i is zero.

A simple path and flow mean a signed *sp* and signed flow in the following. The methods for checking equivalence and additivity are modified as well.

3.1 Basic Procedure

Sort all *sp* in descending order of the magnitude of MCF. If there are *sp* having the same magnitude of MCF, the one with the smaller number of links is chosen first. If the number of links is tied again, the *sp* having the smaller link index is chosen first. At level k, compute an augmented amount of flow for each *sp* in FSP and selecet $P_{(k:n_k)}$ which gives the maximal increase of MCF among the *sp*'s. As we generate a new *cp*, delete the '−' signs of links in $P_{(k:n_k)}$. In addition, if there are saturated unilateral links in $P_{(k:n_k)}$, delete all *sp* including the links in FSP except $P_{(k:n_k)}$. If the flowless links remain in success *cp*, remove those links from the *cp*. Then, the number of comparison for obtaining *mscp* can be reduced. In checking eligibility and additivity, we ignore the '−' signs because it can be done by the comparison of unions of the existing *cp* and each of *sp* in FSP or a set of *sp* with eligibility. The whole process is stopped when the retreat is required at $k = 0$.

3.2 Algorithm

In this subsection, we present only those steps which need to be modified from Lee and Park (2001a). Let c_i be the capacity of link i, N_k be the number of additive simple paths on $C(k)$, and W_{min} be a required amount of capacity flow for the network. Some notations have the superscript, *, when P is substituted by $P_{(k:n_k)}$.

M1. (Initialization)

Set $w(0) = 0$, $W_{(-1;n_{-1})} = 0$, $f_i(-1) = 0$, and $f(0) = (0, 0, \cdots, 0)$.

M2. (Check Equivalence and Additivity)

For each $P \in \text{FSP}(k)$, remove all paths, P', if $P' \cup C(k)$ is the same with $P \cup C(k)$, but not P itself. Then, P is an additive simple path if there is no P'' in $\text{FSP}(k)$ satisfying $P'' \cup C(k) \subset P \cup C(k)$.

M3. (Compute MCF with Additive Choices)

For each $i \in P$, $w_i(k) = |f_i(k-1)|$ if $sign(i) + sign(f_i(k-1)) = 0$, otherwise $w_i(k) = c_i - |f_i(k-1)|$. Set $w(k) = min_{i \in P} w_i(k)$ and $W_{(k:n_k)} = W_{(k-1:n_{k-1})} + w(k)$.

M4. (Compute Flow Pattern)

$f_i(k) = f_i(k-1) + sign(i) \times w^*(k)$ for each $i \in P_{(k:n_k)}$, and $f_i(k) = f_i(k-1)$ for each $i \notin P_{(k:n_k)}$.

(Record Success *cp*)

When a success *cp*, $C(k) \cup P_{(k:n_k)}$, is obtained, we remove flowless links still remaining in the success *cp*. Record the *cp* and go to M6.

(Advance)

As $W^*_{(k:n_k)} < W_{min}$, FSP$(k+1)$ = FSP$(k) - \{P | i \in P\}$ if for $i \in P_{(k:n_k)}$, i is unilateral and $|f_i(k)| = c_i$.

M5. (Retreat)

Set $k = k-1$. If $k > 0$, retreat $f(k-1)$ for $f(k)$. Otherwise, set $f(0) = (0, 0, \cdots, 0)$.

M6. (Update ELGSP)

Remove non-additive paths P from FSP(k) if $P_{(k:n_k)} \cup C(k) \subset P \cup C(k)$.

4. EXAMPLES

EXAMPLE 1. Consider the network shown in Figure 4.1, and suppose that the capacity vector is given as $c = (1, 1, 2, 2, 2, 1, 1)$ and $W_{min} = 4$. In this network, the order of nodes is $s < a < b < t$, and all links except 4 are unilateral. Then, we have 9 signed simple paths : (1,5), (1,4,6), (1,4,7), (2,5), (2,4,6), (2,4,7), (3,−4,5), (3,6), (3,7). The saturated unilateral links

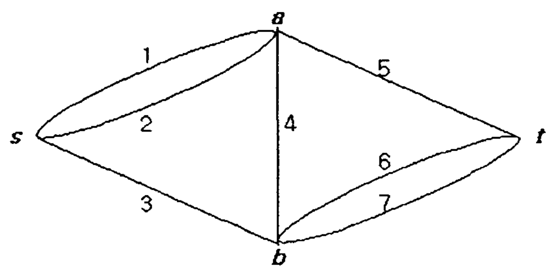

Figure 4.1: A 7-Branch Network

are marked by '*s*' in flow patterns. The 'success' in the last columns of Table 4.1 and 4.2 has

'Y' when a success *cp* is obtained, and has 'N', otherwise.

Table 4.1. Process for Figure 4.1

no.	k	$C(k)$	N_k	n_k	$P_{(k:n_k)}^{W(k:n_k)}$	FSP(k)	$f(k)$	ELGSP(k)	success
1	0	∅	-	-	-	(1,5), (1,4,6), (1,4,7), (2,5), (2,4,6), (2,4,7), (3,−4,5), (3,6), (3,7)	(0,0,0,0,0,0,0)	∅	-
3			9			$(3,-4,5)^2,(1,5)^1,(2,5)^1,(3,6)^1,(3,7)^1,(1,4,6)^1,(1,4,7)^1,(2,4,6)^1,(2,4,7)^1$			
4				1	$(3,-4,5)^2$	$(1,5)^1,(2,5)^1,(3,6)^1,(3,7)^1,(1,4,6)^1,(1,4,7)^1,(2,4,6)^1,(2,4,7)^1$	$(0,0,s,-2,s,0,0)$		N
	1	(3,4,5)	-	-		(1,4,6),(1,4,7) (2,4,6),(2,4,7)			
2									
3			4			$(1,4,6)^3,(1,4,7)^3$ $(2,4,6)^3,(2,4,7)^3$			
4				1	$(1,4,6)^3$	$(1,4,7)^3,(2,4,6)^3,(2,4,7)^3$	$(s,0,s,-1,s,s,0)$		N
	2	(1,3,4,5,6)	-	-	-	(2,4,7)			
2									
3			1			$(2,4,7)^4$			
4				1	$(2,4,7)^4$	∅	$(s,s,s,0,s,s,s)$		Y

For $k = 2$, the success *cp*, $(1,3,4,5,6) \cup (2,4,7)$, is obtained, but link 4 is flowless. This implies that link 4 is not needed to acheive MCF of 4. Therefore, the success *cp*, $(1,2,3,5,6,7)$,

Table 4.2. Process for Figure 4.1 (Continued)

no.	k	$C(k)$	N_k	n_k	$P_{(k:n_k)}^{W(k:n_k)}$	FSP(k)	$f(k)$	ELGSP(k)	success
5	1	(3,4,5)	4	1	$(1,4,6)^3$	$(1,4,7)^3,(2,4,6)^3,(2,4,7)^3$	$(0,0,s,-2,s,0,0)$	∅	-
6								(1,4,6)	
4				2	$(1,4,7)^3$	$(2,4,6)^3,(2,4,7)^3$	$(s,0,s,-1,s,0,s)$		N
	2	(1,3,4,5,7)	-	-	-	(2,4,6)		(6)	
2						∅			

is recorded in the set of success *cp*. At this stage, go to M6. Since $n_2 = N_2 = 1$, go to M5 and the subscquent process is explained in Table 4.2.

We omit the remaining detailed process and give only the brief explanation. In the subsequent process, the other success *cp*, $(1,5) \cup (2,5) \cup (3,6) \cup (3,7)$, is obtained, but is equal to the success *cp* which was obtained in the previous step. If the algorithm given in Lee and Park (2001a) is applied, two success *cp*'s, (1,2,3,4,5,6,7) and (1,2,3,5,6,7), are obtained. Both this new algorithm and the algorithm of Lee and Park (2001a) give (1,2,3,5,6,7) as a *mscp*. But, the number of comparisons for checking *mscp* is fewer for the new algorithm than that of Lee and Park (2001a). Moreover, the numbers of *cp*'s for computing MCF including the numbers of $W_{(k:ALL)}$ are 18 and 24 for our new method and Lee and Park (2001a)'s, respectively. Hence, the new method is more effcient in checking *mscp* and in reducing the unnecessary computations of MCF. As a result, the network reliability, R, is computed as $p_1p_2p_3p_5p_6p_7$, and the reliability is 0.53144 when $p_i = 0.9$ for all i.

REFERENCES

Aggarwal, K.K. (1988), "A fast algorithm for the performance index of a telecommunication network", IEEE Transactions on Reliability, **37**, 65-69.

Aggarwal, K.K., Chopra, Y.C. and Bajwa, J.S. (1982). "Capacity consideration in reliability analysis of communication systems", IEEE Transactions on Reliability, **31**, 177-181.

Lee, S.M. and Park, D.H. (2001a). "An efficient method for evaluation of network reliability with variable link-capacities", IEEE Transactions on Reliability, *to be appeared.*

Lee, S.M. and Park, D.H. (2001b). "An algorithm for finding maximum flow in networks", IEEE Transactions on Reliability, *submitted for publication.*

Misra, K.B. and Prasad, P. (1982). "Comments on: Reliability evaluation of a flow network", IEEE Transactions on Reliability, **31**, 174-176.

Rai, S. and Soh, S. (1991). "A computer approach for reliability evaluation of telecommunication networks with heterogeneous link-capacities", IEEE Transactions on Reliability, **40**, 441-451.

Schanzer, R. (1995). "Comment on : Reliability modeling and performance of variable link-capacity networks", IEEE Transactions on Reliability, **44**, 620-621.

Varshney, P.K., Joshi, A.R. and Chang, P.L. (1994) "Reliability modeling and performance evaluation of variable link capacity networks", IEEE Transactions on Reliability, **43**, 378-382.