Study on the Strain-Rate Dependent Constitutive Equation using Elastoplastic-Viscoplastic Constitutive Model.
Bounding Surface 모델을 이용한 변형율속도 의존적인 구성 관계식에 관한 연구

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개 요 : 응력-변형률 관계의 모델링에 있어서 creep, stress relaxation, strain rate effect 등의 묘사는 중요한 지반거동중의 하나인 시간 의존적 거동에 대한 simulation은 있어서 대단히 중요한 요소라 할 수 있다. 특히 지반은 변형률 속도에 대하여 때로는 매우 다른 거동 특성을 보이기 때문에 지반의 모델링에 있어서 변형율 속도를 고려한 구성방정식의 제시는 큰 비중을 차지한다 하겠다. 본 연구에서는 변형율에 따라 변화하는 지반의 거동특성을 보다 현실에 가깝게 묘사하기 위한 시간 의존적 구성모델을 제시하는데 있다. Bounding Surface Model의 Stress Invariant 부분을 Perzyna(1966)와 Adachi and Oka(1982)의 변형율 속도 의존적인 구성관계 이론을 이용하여 발전시켰다. 제안된 구성모델은 다양한 변형율 속도에 적용에 있어서 기존의 방식보다 간단히 모델 정수들을 결정 할 수 있다. 지반거동의 수치적인 해석을 위하여 기존의 Bounding Surface Model에 사용되었던 Program Code를 발전 시켜 사용하였으며, 엄격히 시행된 실내시험의 결과와 비교/검증하였다.

Key Words : elastoplasticity, viscoplasticity, constitutive model, strain-rate effect, bounding surface model

1. Introduction

Constitutive laws or models represents a mathematical model that describes our ideas of the behavior of a material and let our knowledge of material behavior enter into engineering design. For instance, the stress-strain behavior of geologic materials, such as clays, are in generally highly nonlinear, elastoplastic, viscoplastic, anisotropic and both time and load path dependent. They have been modeled by several constitutive models. The coupling of plastic behavior with time dependency comes under the general heading of elasto-viscoplasticity. An elasto-viscoplastic formulation which combined the Critical State theory (Roscoe et al., 1963; Roscoe and Burland, 1968; Schofield and Wroth, 1968)was developed by Perzyna's elastic-viscoplastic continuum theory(1963,1966). Many modified versions of Perzyna's theory have been developed (Adachi and Oka, 1982; Zienkiewicz, et al., 1975, 1977; Sekiguchi, 1984; Katona and Mulert, 1984; Katona, 1984). However, these models were applicable to normally consolidated clays only. Kaliakin (1985), Kaliakin and Dafalias (1990a, 1990b, 1991), Al-Shamrani (1991), and Al-Shamrani and Sture (1994) combined the elastoplastic bounding surface model and the viscoplastic model based on Perzyna's viscoplastic theory for isotropic and anisotropic cohesive soils. The bounding surface originally introduced by Dafalias and Popov(1976) and Dafalias(1975). In this study, based upon the concept of the bounding surface in stress space, a proposed constitutive equations (Adachi and Okano, 1974), which describes the relations between stress and strain rate, is implanted in bounding surface model. Since this implantation of constitutive relation is related with the viscoplastic part in bounding surface. For the experimental verification of the developed rate-dependent constitutive model and determination of model parameter values, a series of laboratory tests were performed. In addition, as the previous study on soil modelling, a very homogeneous and undisturbed soil sample was required. Thus, remolded YangSan clay sample was used in this laboratory experiments. A series of triaxial tests and oedometer tests is performed to determine model parameters. Comparing between numerical prediction and proposed relationship of stress-strain rate was accomplished to verify the developed rate-dependent constitutive model. For the numerical verification and use of the developed rate-dependent constitutive model, the acquired constitutive equation is implanted into a computer program code ANCALBR8. ANCALBR8 is derived from CALBR8 developed by Kaliakin(1992) for anisotropic bounding surface model. The program CALBR8 used the isotropic elastoplastc-viscoplastc bounding surface model with a main subroutine CLAYVP.

2. Development of Rate-Dependent Constitutive Model

Taking into account the stress-strain relations, we should obtain the rate dependent constitutive equation. Let us denote by $\dot{\varepsilon}_{ij}$ the strain tensor by $\dot{\sigma}_{ij}$ the stress tensor in cartesian coordinates. For an elastic-viscoplastic body it will be assumed that the strain tensor can be represented in the form of the sum(Perzyna, 1966);

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{vp} \tag{1}$$

By using Drucker's postulate(1959),

$$\dot{\varepsilon}_{ij}^{vp} = \langle \Phi(F) \rangle \frac{\partial f}{\partial \sigma_{ij}} \tag{2}$$

$$\langle \Phi(F) \rangle = \begin{cases} 0 & \text{for } F \leq 0 \\ \Phi(F) & \text{for } F > 0 \end{cases} \tag{3}$$

Using the stress and strain deviator tensor($\dot{\varepsilon}_{ij}^{e} = \dot{e}_{ij}^{e} + \frac{1}{3}\dot{\varepsilon}_{ij}\delta_{ij}$), the generalized Hook's law ($e_{ij}^{e} = \frac{1}{2G} s_{ij}$), and the result of consolidation and swell tests($e_{ij}^{e} = \frac{1}{2G} s_{ij}$) , we obtain the following rate dependent constitutive equation.

$$\dot{\varepsilon}_{ij} = \frac{1}{2G}\dot{s}_{ij} + \frac{\chi}{3(1+e)} \frac{\dot{\sigma}_m'}{\sigma_m'} \delta_{ij} + \Phi(F) \frac{\partial f_d}{\partial \sigma_{ij}'} \tag{4}$$

where G is the elastic shear modulus, χ is swelling index, e is void ratio and δ_{ij} is Kronecker's delta.

Adachi and Oka's stress-strain rate constitutive relation

Adachi and Okano(1974) introduced two parameters $\eta^* = \sqrt{2J_2}/\sigma_m'$ and $\psi^* = \dot{\varepsilon}_{kk}^{p}/\sqrt{2I_2^{p}}$. Then suggested the state boundary surface at the equilibrium state are given from Eq. (19) as follows.

$$\sigma_m' exp\left(\frac{\sqrt{2J_2}}{M^* \sigma_m'} \right) = \sigma_{my}' \tag{5}$$

We define the static equilibrium state as a state at which deviatoric strain rate components $\dot{e}_{ij}$ as well as volumetric strain rate $\dot{\upsilon}$ becomes zero. Therefore, any deformation processes with definite strain rate are regarded as in non-eqilibrium state, namely, in dynamic state. Perzyna(1963) pointed out that the difference of the dynamic and static behaviors of materials occurred due to the strain rate sensitivity of the materials and defined this rate sensitive behaviors as viscoplastic. Then, he assumed the existence of the static yield function as follows,

$$F(\sigma_{ij}, \varepsilon_{ij}^{p}) = f(\sigma_{ij}, \varepsilon_{ij}^{p})/k_s = 1 \tag{6}$$

where k_s is the work hardening parameter. In order to construct constitute equations, we have to assign the yield function. According to the original critical state energy theory(Eq. (26); Roscoe et al., 1963), the following static yield function is assumed to be valid.

$$\ln \sigma_{my}'^{\,s} = f_s = \sqrt{2J_2}^{(s)}/M^* \ \sigma_m'^{(s)} + \ln \sigma_m'^{\,s} = k_s \tag{7}$$

where $\sqrt{2J_2} = \sqrt{S_{ij}S_{ij}}$ is the second invaliants of deviatoric stress S_{ij}, M^* is defined as the value of stress ratio $\sqrt{2J_2}/\sigma_m'$ at the critical state and the superscript (s) denotes the values at the static equilibrium states. The strain-hardening parameter ks is assumed to be given by $\ln \sigma_{my}'$, namely, σ_{my}' represents strain-hardening effect in the change of stress state from $\sigma m' = 0$ to $\sigma m' = \sigma_{my}'$. Thus we define

$$k_s = \ln \sigma_{my}'^{(s)} \tag{8}$$

The dynamic yield function f_d should be the same functional form of f_s because F=0 expresses the statical yield condition. f_d is express as

$$f_d = \sqrt{2J_2}/M^* \ \sigma_m' + \ln \sigma_m' = k_d \tag{9}$$

where k_d , in the same way as the static strain-hardening parameter k_s, is

$$k_d = \ln \sigma_{my}'^{(d)} \tag{10}$$

According to the outcome of the previous works (Adachi and Okano 1974; Oka, 1979), the function form of $\Phi(F)$ in Eq. (2) is assumed to be follows

$$\Phi(F) = c_0 \exp[m' \ln(\sigma_{my}'^{(d)}/\sigma_{my}'^{(s)})] \tag{11}$$

by substituting both dynamic and static yield function of Eq. (10) and (12) into Eqs (14), $\Phi(F)$ is rewritten as follows

$$\Phi(F) = c_0 \exp\left\{ m' \left[\frac{\sqrt{2J_2}}{M^* \sigma_m'} + \ln \sigma_m' - \frac{\sqrt{2J_2}^{(s)}}{M^* \sigma_m'^{(s)}} - \ln \sigma_m'^{(s)} \right] \right\} \tag{12}$$

Under undrained conditions, the total strain rate component $\dot{\varepsilon}_{11}$ is equivalent $\dot{e}_{ij}$ because Eq.(5) is always satisfied.

$$\dot{\varepsilon}_{11} = \dot{e}_{ij} = \frac{\dot{S}}{2G} + \frac{1}{M^* \sigma_m'} \Phi(F) \frac{S_{ij}}{\sqrt{2J_2}} \tag{13}$$

We continue to discuss the problem in a simple case of conventional axisymmetric triaxial compression, i.e., $\sigma'_1 > \sigma'_2 = \sigma'_3$. Under this specific condition, the following relations are reduced

$S_{11} = 2/3(\sigma_1 - \sigma_3)$, $\sqrt{2J_2} = \sqrt{2/3}(\sigma_1 - \sigma_3)$, $\varepsilon_{11} = e_{11} = 2/3(\varepsilon_1 - \varepsilon_3)$

Using these relations Eq. (4) results in as follows

$$\dot{\varepsilon}_{11} = \frac{\dot{S}_{ij}}{2G} + \frac{\sqrt{2/3}}{M^* \sigma_m'} \Phi(F) \tag{14}$$

$$\Phi(F) = c_0 \exp\left\{ m' \left[\frac{q}{M \sigma_m'} + \ln \sigma_m - \frac{q^{(s)}}{M \sigma_m'^{(s)}} - \ln \sigma_m'^{(s)} \right] \right\} \tag{15}$$

where $q = (\sigma_1 - \sigma_3)$ and $M = \sqrt{3/2} M^*$.

Assuming $\dot{\varepsilon}_{11} = \dot{\varepsilon}_{11}^{p}$, namely elastic shear strain rate $\dot{e}_{11}^{e} = \dot{S}_{11}/2G$ to be negligible, the next relation is obtained from Eq. (17) and (18) by comparing the state P1 and P2 shown in Fig. 3.

$$\ln \dot{\varepsilon}_{11}^{(1)} = \ln \frac{\sqrt{2/3}}{M^* \sigma_m'} + \ln c_0 + \left\{ m' \left[\frac{q^{(1)}}{M \sigma_m'} + \ln \sigma_m - \frac{q^{(s)}}{M \sigma_m'^{(s)}} - \ln \sigma_m'^{(s)} \right] \right\} \tag{16}$$

$$\ln \dot{\varepsilon}_{11}^{(2)} = \ln \frac{\sqrt{2/3}}{M^* \sigma_m'} + \ln c_0 + \left\{ m' \left[\frac{q^{(1)}}{M \sigma_m'} + \ln \sigma_m - \frac{q^{(s)}}{M \sigma_m'^{(s)}} - \ln \sigma_m'^{(s)} \right] \right\} \tag{17}$$

so,

$$\ln \left(\dot{\varepsilon}_{11}^{(1)} / \dot{\varepsilon}_{11}^{(2)} \right) = \frac{m'}{M^*} \times \left(\sqrt{2J^{(1)}}/\sigma_m' - \sqrt{2J^{(2)}}/\sigma_m' \right) \tag{18}$$

where the superscripts (1) and (2) correspond to the states P1 and P2. The stress ratio and logarithm of the strain rate have the linear relationship. If the material properties such as m′ and M* can be find properly, we obtain the different deviatoric stress invariant value at the different strain rate condition, through Eq. (18).

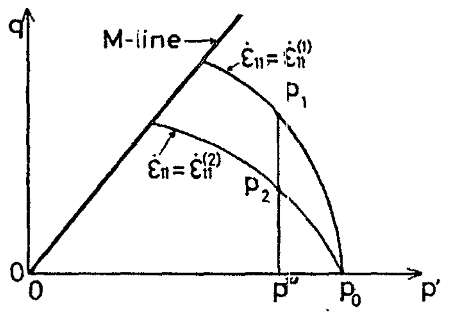

Fig. 3. Typical ESP for NC clay under undrained condition.

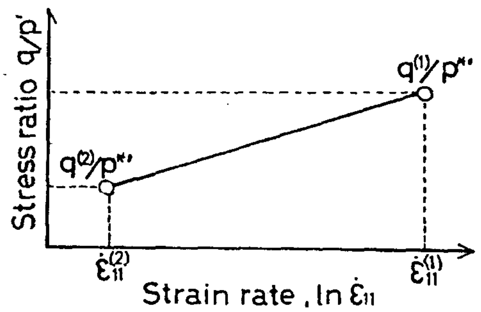

Fig. 4. Determination of the viscoplastic parameter, m′.

Bounding Surface Model for Anisotropic Cohesive soil

The bounding surface for anisotropic cohesive soils are expresses as

$$F(\sigma_{ij},\ \delta^{a}_{ij},\ I^{a}_{0},\ R)=0 \tag{19}$$

To implant the relations between strain rate and stress into the bounding surface model, the viscosity function is used. To better account for the influence of the level of shear stress, as well as of mean normal effective stress on the time dependent response of cohesive soils, the following expression for viscoplastic function was developed (Kaliakin, 1985)

$$\hat{V}= V\{\exp-[J/(NI)]\} \tag{20}$$

In Eq. (23), V is a constant viscoplastic parameter, and I, J, and N are as previously defined. For the prediction of behaviour in different strain rate Eq. (38) is modified and implanted as follows

$$J^{(2)}=\frac{1}{2}\left[\frac{M^{*}\times\sigma'_{m}}{m'}\ln(\ \dot{\varepsilon}_{11}{}^{(1)}/\ \dot{\varepsilon}_{11}{}^{(2)})-\sqrt{2J^{(1)}}\right]^{2}$$

$$=\frac{1}{2}\left[\frac{M^{*}\times\frac{1}{3}I'}{m'}\ln(\ \dot{\varepsilon}_{11}{}^{(1)}/\ \dot{\varepsilon}_{11}{}^{(2)})-\sqrt{2J^{(1)}}\right]^{2} \tag{21}$$

$$\hat{V}=V\left\{\exp-\left[\frac{\left[\frac{1}{2}\left\{\frac{M^{*}\times\frac{1}{3}I'}{m'}\ln(\ \dot{\varepsilon}_{11}{}^{(1)}/\ \dot{\varepsilon}_{11}{}^{(2)})-\sqrt{2J^{(1)}}\right\}^{2}\right]}{(NI')}\right]\right\} \tag{22}$$

where superscript (1) and (2) correspond to the states P1 and P2. If it need not to observe other strain rate behaviors, $\dot{\varepsilon}_{11}{}^{(1)}$ and $\dot{\varepsilon}_{11}{}^{(2)}$ are same value in Eq. (25), so that Eq. (26) returend to Eq. (23) format. Utilizing the concept of bounding surface elastoplasticity -viscoplasticity, as presented in the preceding section, it is focused to recast inviscid bounding surface model into a rate-dependent formation.

3. Laboratory Experiments

Specimen

A cohesive soil, which was from YangSan, Kyung-Nam, Korea, was used in this paper. Index properties of the sample is summarized in Table 1. Fig. 5. presents the soil classification of this sample.

Class	Properties
ω_n (%)	55.7 ~ 64.6
ω_L (%)	42.8 ~ 52.6
ω_p (%)	24.7 ~ 30.6
Ip	18.1 ~ 28.0
Gs	2.70 ~ 2.71
e_0	1.722 ~ 1.925
Soil class	CL

Table 1. Index properties of the samples

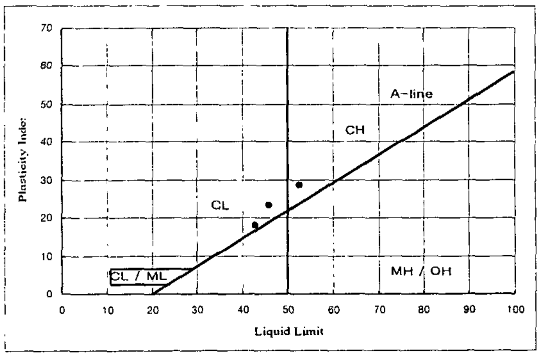

Fig. 5. Plasticity Chart (Unified System).

Slurry Consolidometer Technique

In order to prepare soil specimens that have same stress history, a slurry consolidometer technique is used. A slurry consolidometer was made of stainless steel, and it was used to prepare 10cm diameter specimens by sedimentation and consolidation under K_0 condition from the powdered soil-water mix.

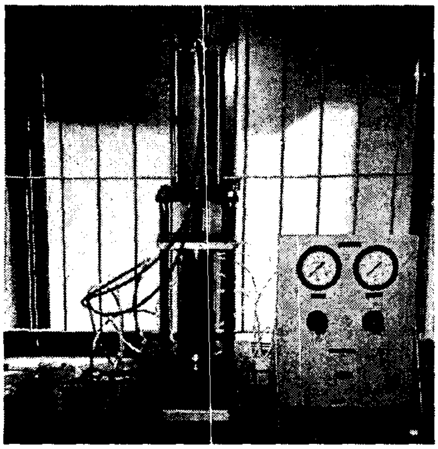

Fig. 6. Slurry consolidometer.

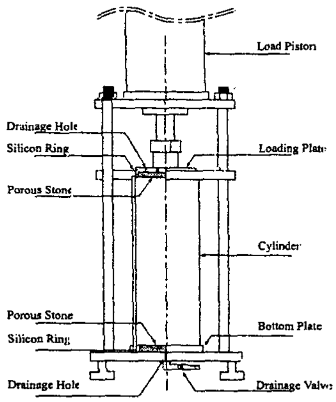

Fig. 7. Schematic shown for slurry consolidometer.

Three anisotropic consolidated-undrained compression triaxial test are conducted. To observe the strain rate effect, the strain-control type undrained shearing was conducted at the rate of 0.1%/min, 0.01%/min, and 0.001%/min. Table 2. shows triaxial test program. Fig. 11. shows the Effective stress path Characteristics of Yang-San remolded clay. The effective stress path progresses approximately vertically, i.e. elastically, from the point of K_0-consolidation for all the samples, irrespective of a difference in strain rate. After the effective stress path has reached the extended yield surface, plastic strain is produced and it progresses with the further yielding up to the critical state. It can be considered that the magnitude of this elastic portion is dependent on the viscoplastic component significantly (Sekiguchi and Ohta, 1977). Fig. 13. shows the Stress and Strength Characteristics of relationship between strength and strain rate. This figure is considered equivalent to in Fig. 1,2. For the compression tests, the S_u/σ'_{vc} value seems to be approximately in linear relationship with a logarithm of the strain rate.

Test No.	Slurry consolidation σ'_{vc} (kPa)	K_0-consolidation in Triaxial apparatus			Strain control Undrained Shearing (rate)
		σ'_v (kPa)	σ'_h (kPa)	σ'_h/σ'_v (kPa)	
1	176	98	200	0.49	0.1 %/min
2	176	98	200	0.49	0.01 %/min
3	176	98	200	0.49	0.001 %/min

Table 2. Program of Triaxial Tests.

4. Verification of Developed Model

The results of triaxial tests should be observed in the numerical prediction, as well. In the numerical prediction, just change the value of strain rate parameter, we predicted the soil behaviors at the different strain rate condition. For these numerical prediction based on the developed rate-dependent constitutive model, the acquired constitutive equation was implanted into a computer program code ANCALBR8. ANCALBR8 is derived from CALBR8 developed by Kaliakin(1992) for anisotropic bounding surface model. The program CALBR8 used the isotropic elastoplastc-viscoplastc bounding surface model with a main subroutine CLAYVP.

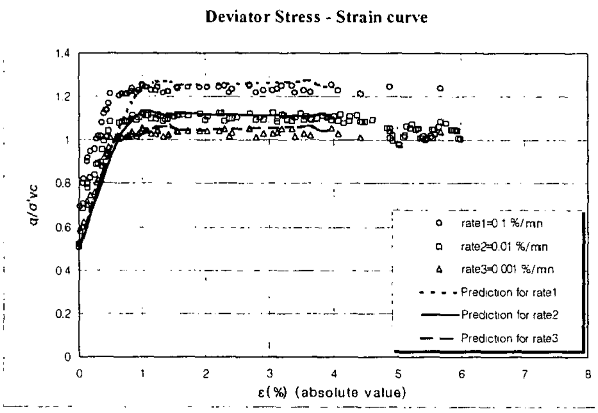

Fig. 10. Comparison of deviator stress.

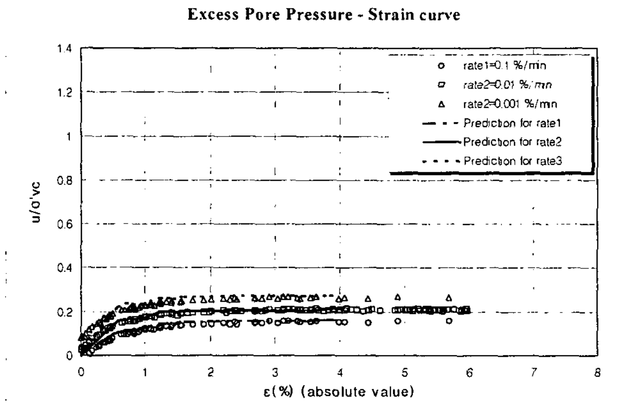

Fig. 11. Comparison of excess pore pressure.

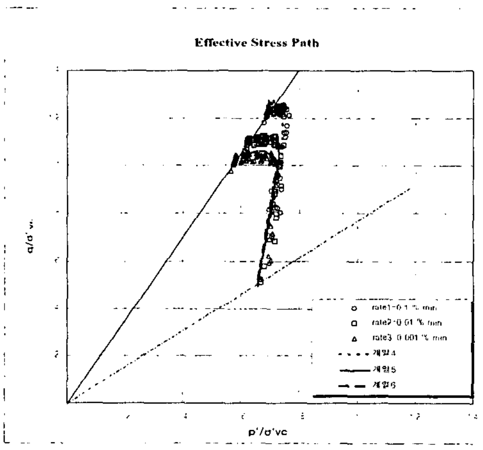

Fig. 12. Comparison of effective stress.

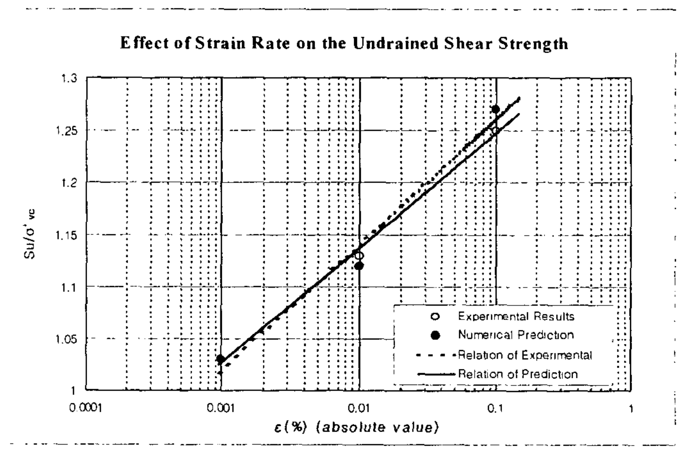

Fig. 13. Comparison of relation of strength-strain rate.

Above comparison of numerical predictions with experimental results were obtained by the modified computer program code ANCARBR8. Concerning the conventional method, all parameter values might be altered with the different strain rate. However, in the developed rate dependent model, just the ratio of strain rate parameter value is changed. Below Fig. 10, 11, 12, and 13 demonstrate the comparison.

5. Summary and Conclusion

The major scope of this study is to develope a rate-dependent constitutive soil model. At first, a rate-dependent constitutive equation within the framework of Roscoe's energy theory(e.g., Roscoe and Burland (1986)) was acquired. The anisotropic bounding surface model proposed by Kaliakin and Dafalias(1990a) successfully used to aided to develope a rate-dependent constitutive equation, implanting above acquired rate-dependent equation into it. In addition, the developed rate-dependent constitutive model well predicted the results of cohesive soil behavior. Briefly, a rate-dependent constitutive model was deveoped by using both a rate-dependent constitutive equation and a anisotropic soil model. The following summarized conclusions can be obtained from this study;

1. Based on the Roscoe's energy theory and Adachi and Oka's function, the relation between stress and strain rate of cohesive soil is obtained. This relation means a rate-dependent equation.

2. The acquired rate-dependent constitutive equation can be used to predict the soil behavior, as being aided by a soil model. In other words, implanting the stress-strain rate relation into the anisotropic bounding surface model, we obtain the rate-dependent soil model.

3. The developed rate-dependent constitutive model was verified by a series of experiments and numerical analysis in this paper. In conclusion, this constitutive model well predicted the rate-dependent soil behavior.

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