

웨이브릿 응용 개관

Overview of Wavelet Applications

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Abstract

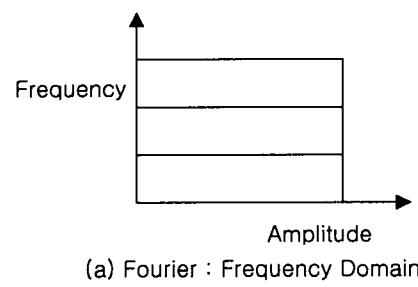
본 연구는 Fourier, Shannon, Gabor, Wavelet Filters 간의 상호비교를 통해 Wavelet Analysis의 장점을 도출하고 학제간 모든 과학기술에 적용될 수 있는 분야를 유형화하고 기존 연구를 고찰한다.

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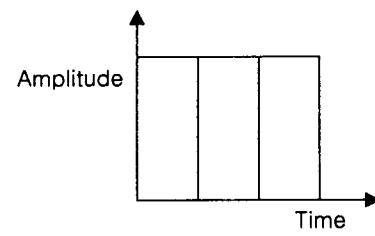
1. Introduction

1.1 Definition

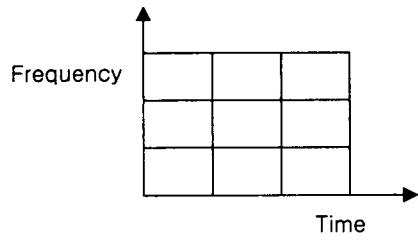
- Small Wave
- Ondelette
- Figure 1. Fourier, Shannon, Gabor, Wavelet, Heisenberg Rectangles (Mallat, 1999)
- Father Heavy Side Function
- Mother Haar Function
- Daughter Wavelet Function



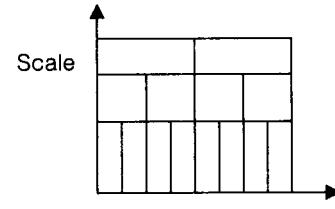
(a) Fourier : Frequency Domain



(b) Shannon: Time Domain



(c) Gabor : STFT



(d) Wavelet Analysis

Figure 1.

1.2 Why Wavelets ? (Ogden, 1997)

- Good Time - Frequency Localization
- Fast Algorithms
- Simplicity of Form
- Interdisciplines

1.3 Applications

- Analysis : Detection of Crashes, Edges or Other Events
(Nievergelt, 1999)
- Compression : Reduction of Storage
- Smoothing : Attenuation of Noise
- Synthesis : Reconstruction after Compression of Other Modification
- One - dimensional Arrays : Sounds or Other Time - Series
- Two - dimensional Arrays : Pictures or Maps
- Three - dimensional Arrays : Spatial Diffusion
- Lossless (Orthogonal) Transforms : Orthogonal and Unitary Matrices
(Strang et al. 1997)
- Invertible (Biorthogonal) Transforms : Invertible Matrices
- Lossy Transforms : Not Invertible
- Lowpass Filter : Moving Average
- Highpass Filter : Moving Difference
- Analysis Bank : Filtering and Down Sampling
- Synthesis Bank : Upsampling and Filtering
- Table 1. DT and CT

Table 1

Discrete Time	Continuous Time
Filter bank tree	Multiresolution
Downsampling $w \rightarrow w/2$	Rescaling $t \rightarrow 2*t$
Lowpass Filter	Averaging with $\phi(t)$
Highpass Filter	Detailing with $w(t)$
Orthogonal Matrices	Orthogonal bases
Analysis bank output	Wavelet Coefficients
Synthesis bank output	Sum of Wavelet transform
Product of filter matrices	Fast Wavelet transform

2. Interdisciplinary Applications of Wavelets

2.1 Mathematics

- Phase Space (Combes et al., 1990)
- Numerical Analysis (Ruskai et al., 1992)
- Approximation (Chui et al., 1995)
- Partial Differential Equations (Dahmen et al., 1997)
- Linear Algebra (Frazier, 1999)
- Renormalization (Battle, 1999)
- Orthogonal Systems (Walter et al., 2001)

2.2 Electronical and Electrical Engineering

2.2.1 Signal Processing

- Multiscale (Cohen et al., 1995)
- Multirate (Fliege, 1994)
- Multiresolution (Akansu, et al., 1992)

2.2.2 Image Processing

- Multidimensional (Basu et al., 1996)
- Subband (Akansu et al., 1996)
- Pattern Recognition (Tang et al., 2000)

2.2.3 Video Compression

- Multimedia Computing (Li et al., 1997)

2.2.4 Optical Engineering

- Wavelet Applications (SPIE, 1997)

2.3 Statistics

- Statistical Applications and Data Analysis (Ogden, 1997)
- Time Series Analysis (Percival et al., 2000)
- Bayesian Inference (Müller et al., 1999)
- S - Plus (Bruce et al., 1996)
- Multivariate Polysplines (Ognyan 1999)

2.4 Computer Science

- Computer Graphics (Stollnitz et al., 1996)
- Lifting (Silverman et al., 1999)

2.5 Physics

- Fractals (Wornell, 1996)
- Distributions (Saichev, 1997)
- Quantum Mechanics (Steeb, 1998)

2.6 Other Applications

- Medicine & Biology (Akay, 1998)
- Geoscience (Georgiou et al., 1994)
- Chemical Engineering (Motard et al., 1994)
- Vibrations (Newland, 1993)
- Acoustics (Meyer, 1992)
- Finance and Economics (Ramazan, 1999)
- Neural Networks (Starck et al., 1998)

3. Electronic Resources and Wavelet Software (Hubbard, 1998)

3.1 Electronic Resources

- Wavelet Digest
- Fractals
- Fingerprints
- JPEG
- Music
- Reproducible Resources
- Steerable Pyramids
- Wavelets

3.2 Free Wavelet Software

- Denoise
- EPIC
- Fraclab
- Liffpack
- Matlab PyrTools
- Rice Wavelet Toolbox
- Splus WaveTresh
- WAILI, Wavelets with Integer Lifting
- WabBox Software
- WaveLab
- Wavelet Image Compression Construction Kit
- wplib
- XWPL and WPLab

3.3 Commercial Wavelet Software

- Numerical Recipies
- S+WAVELETS
- WavBox Software and FirWau Filter Libraries

- Wavelet Explorer
- Wavelet and Filter Banks Design Toolkit
- Wavelet Packet Laboratory for Windows
- Wavelet Toolbox
- Wavepak

4. Conclusions

- Wavelet Properties : Local Optimization, Multiresolution, Efficiency
- Various Interdisciplinary Applications : Math, EE, Statistics, CS, Physics, etc.

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