Buyer's EOQ problem with inventory-level-dependent demand rate when the supplier allows day-terms credit

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Abstract

In today's business transactions, it is more and more common to see that the buyers are allowed some grace period before they settle the account with the supplier. In this regard, we analyze the problem of determining the buyer's EOQ when the supplier allows day-terms credit. For the analysis, it is assumed that the buyer's demand rate is a function of the on-hand inventory level and the relevant mathematical model is developed.

I. Introduction

Trade credit affects the conduct of business for many reasons and the supplier usually expects that the increased sales volume can compensate the capital loses incurred during the credit period. In this regard, many research papers appeared which deal with the EOQ problem under the condition of day-terms supplier credit. Chu *et al.*[2] and Chung[3] analyzed the effects of trade credit on the buyer's EOQ. Recently, Hwang and Shinn[5] examined the joint price and lot size determination problem when the supplier permits the delay in payments for an order of a product assuming that the consumer's demand rate is represented by a constant price elasticity function which is a decreasing function of retail price. However, for certain commodities, such as consumer goods, food grains, stationery items etc., the buyer's demand rate may depend on the size of the quantity on hand. Such a situation generally arises for a consumer-goods type of inventory and the demand rate may go up or down if the on-hand inventory level increases or decreases. In this regard, some research papers evaluated an inventory system where the demand rate has been assumed to be dependent on the on-hand inventory.[1,3] With this type of product, the probability of making a sale would increases as the amount of the product in inventory increases. It is, therefore, likely to have an effect of increasing the size of each order. Also, the availability of opportunity to delay the payments effectively reduces the buyer's cost of holding inventories, and thus is likely to result in larger order quantity. In this regard, we analyze the problem of determining the buyer's EOQ for an inventory-level-dependent demand rate items when the supplier offers a fixed credit period.

II. Assumptions and Notations

The assumptions and notations of this paper are as follows:

(1) Replenishments are instantaneous with a known and constant lead time.

(2) No shortages are allowed and the demand rate is linearly dependent on the level of inventory.

(3) The supplier proposes a certain credit period and the purchasing cost of the products sold during the credit period is deposited in an interest bearing account with rate I. At the end of the period, the credit is settled and the buyer starts paying the capital opportunity cost for the items in stock with rate R($R \geq I$).

C : unit purchase cost.

P : unit retail price.

S : ordering cost.

tc : credit period set by the supplier.

H : inventory carrying cost, excluding the capital opportunity cost.

R : capital opportunity cost(as a percentage).

I : earned interest rate(as a percentage).

Q : order size.

T : replenishment cycle time.

$q(t)$: inventory level at time t.

D : annual demand rate, as a function of the on-hand inventory,

$D = \alpha + \beta q(t)$ where α and β are non-negative constants.

III. Determination of buyer's EOQ

In this paper, it is assumed that the buyer's demand rate is represented by a linear function of the on-hand inventory and so, the demand rate at time t, will take the form

$$D = \alpha + \beta q(t), \ \alpha \text{ and } \beta \geq 0. \tag{1}$$

As stated by Baker and Urban[1], the buyer's inventory differential equation is

$$dq(t)/dt = -\alpha - \beta q(t) \tag{2}$$

With the boundary condition, $q(T) = 0$, the buyer's inventory level is derived as:

$$q(t) = (e^{\beta(T-t)} - 1)\alpha/\beta, \quad 0 \leq t \leq T. \tag{3}$$

Also, from the fact that $q(0) = Q$, the quantity ordered per cycle is

$$Q = (e^{\beta T} - 1)\alpha/\beta. \tag{4}$$

Now, we formulate the annual net profit $\Pi(T)$. The annual net profit consists of the following five elements.

(1) Annual sales revenue = $PQ/T = \alpha P(e^{\beta T} - 1)/\beta T$.

(2) Annual ordering cost = S/T.

(3) Annual purchasing cost = $CQ/T = \alpha C(e^{\beta T} - 1)/\beta T$.

(4) Annual inventory carrying cost = $\frac{H}{T}\int_0^T q(t)dt = \frac{H\alpha}{\beta^2 T}(e^{\beta T} - \beta T - 1)$.

(5) Annual capital opportunity cost = $\begin{cases} \frac{CR}{T}\left(\int_{tc}^{T} q(t)dt - CI\left(Qtc - \int_0^{tc} q(t)dt\right)\right) & \text{for } tc \le T \\ \frac{\alpha CI}{\beta^2 T}(e^{\beta}T(1-\beta tc) - \beta(T-tc) - 1) & \text{for } tc > T. \end{cases}$

Then, depending on the relative size of tc to T, the annual net profit $\Pi(T)$ has two different expressions as follows:

$$\Pi_1(T) = \alpha(P-C)(e^{\beta T}-1)/\beta T - \alpha H(e^{\beta T} - \beta T - 1)/\beta^2 T - S/T$$
$$- \alpha C\{R(e^{\beta(T-tc)} - \beta(T-tc) - 1)/\beta^2 T - Ie^{\beta T}(e^{-\beta tc} + \beta tc - 1)\}, \text{ for } tc \le T \tag{5}$$

$$\Pi_2(T) = \alpha(P-C)(e^{\beta T}-1)/\beta T - \alpha H(e^{\beta T} - \beta T - 1)/\beta^2 T - S/T$$
$$- \alpha CI\{e^{\beta T}(1-\beta tc) - \beta(T-tc) - 1\}/\beta^2 T, \text{ for } tc > T. \tag{6}$$

By using a truncated Taylor series expansion for the exponential function, the model can be approximated as

$$\Pi_1(T) = \alpha\left(P - C\left(1 - \left(R + \frac{I\beta tc}{2}\right)tc\right)\right) - \frac{1}{T}\left(S + \frac{\alpha C(R-I)tc^2}{2}\right) - \frac{\alpha T}{2}\left(H - P\beta + C\beta + CR - \frac{CI\beta^2 tc^2}{2}\right), \tag{7}$$

$$\Pi_2(T) = \alpha(P - C(1 - Itc)) - \frac{S}{T} - \frac{\alpha T}{2}(H - P\beta + C\beta + CI - CI\beta tc). \tag{8}$$

Note that the annual net profit, $\Pi(T)$, at $T = tc$ is obtained on substituting $T = tc$ in (13) or (14) and we have the following relationship between $\Pi_i(tc)$, $i = 1, 2$,

$$\Pi_1(tc) > \Pi_2(tc). \tag{9}$$

For the normal condition($R \ge I$), $\Pi(T)$ is a concave function of T. And so, there exists a unique value T_i, which maximizes $\Pi_i(T)$ and they are:

$$T_1 = \sqrt{\frac{2S_1}{\alpha H_1}} \text{ where } S_1 = S + \frac{\alpha C(R-I)tc^2}{2} \text{ and } H_1 = H - (P-C)\beta + CR - \frac{CI\beta^2 tc^2}{2}, \tag{10}$$

$$T_2 = \sqrt{\frac{2S}{\alpha H_2}} \text{ where } H_2 = H - (P-C)\beta + CI(1-\beta tc). \tag{11}$$

Based on the above results, we develop the following solution procedure to determine an optimal replenishment cycle time T^* for this approximate model.

Solution algorithm

Step 1. Determine T_1 by equation (10).

Step 2. If $T_1 \ge tc$, then obtain $\Pi_1(T_1)$ by equation (7) and go to step 3.

Otherwise, obtain $\Pi_1(tc)$ by equation (7) and go to step 3.

Step 3. Determine T_2 by equation (11).

Step 4. If $T_2 < tc$, then obtain $\Pi_2(T_2)$ by equation (8) and go to step 5.
Otherwise, go to step 5.

Step 5. Select the replenishment cycle time (T^*) with the maximum annual net profit value evaluated in steps 2 and 4.

IV. Numerical Example

To illustrate the solution algorithms, the following problem is considered.
S = \$50, C = \$3, P = \$3.9, H = \$0.1, R = 0.15(= 15%), I = 0.1(= 10%), tc = 0.3, $\alpha = 3200$ and $\beta = 0.3$.

Step 1. From equation (10), $T_1 = 0.4$.

Step 2. Since $T_1 \geq tc(=0.3)$, compute $\Pi_1(tc)$ by equation (7) and go to Step 3.

Step 3. From equation (11), $T_2 = 0.55$.

Step 4. Since $T_2 \geq tc(=0.3)$, go to Step 5.

Step 5. From the results in steps 2 and 4, T^* becomes 0.4 with its maximum annual net profit \$2698.

V. Conclusions

In this paper, we analyzed the EOQ problem under the condition of day-terms supplier credit. For the analysis, it is assumed that the buyer's demand rate is a function of the on-hand inventory level. After formulating the mathematical model, we proposed the solution procedure which lead to an optimal retailing policy for the model developed. With an example problem, the solution algorithm presented is illustrated.

[References]

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