Algorithms for Reliability Calculation of Multistate System

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Abstract

This paper studies the structure and reliability of homogeneous s-coherent multistate system. We describe efficiency of inclusion-exclusion algorithm and pivotal decomposition algorithm for reliability calculation of 2-states system which developed in (Lee 1999) [10]. We extend our method, applied in [10], to the case when components of the system are given multi-states. As an application, the high pressure injection system of a pressurized water reactor is modeled as a multistate system composed of homogeneous s-coherent multistate subsystems. And Several examples are illustrated.
Key word: inclusion-exclusion algorithm, pivotal decomposition algorithm, multistate system

1. Introduction

The reliability literature of the past 10 years contains many papers with reliability calculation of coherent structure. This paper describes [6] and discussed the structure and reliability of homogeneous s-coherent multistate system. Problems related to the coherent structure of the system are based on [6]. The path set and cut set method for determining system reliability [1] is used. In section 2 and 3, we propose an inclusion-exclusion algorithm and a pivotal decomposition algorithm by the use of inclusion-exclusion formula and the decomposition rule which proposed in [6]. In section 4, we compare the computational speed between two algorithms. In section 5 faintly survey homogeneous coherent multistate systems.

2. Algorithms for Reliability calculation

Inclusion- Exclusion Algorithm

Input : n, p_1, p_2, $\cdots$, p_n, N_0 or ε

Output : $h(p)$

step 1. set $i = 1$

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step 2. while $i \leq N_0$ do step 3~5

step 3. set $S_i = \sum_{1 \leq r_1 < \cdots < r_i \leq n} P_{r_1} \cdots P_{r_i}$ (*compute* S_i)

$h(p) = \sum_{i=1}^{n} (-1)^{i-1} S_i$

step 4. if $|S_i - S_{i-1}| < \varepsilon$

output $h(p)$

stop

step 5. set $i = i + 1$

$S_i = S_{i+1}$

step 6. output (method failed after N_0 iterations, N_0 or $h(p)$)

Pivotal decomposition algorithm

Algorithm for series system

Input : $n, p_1, p_2, \cdots, p_n$

Output : $h(p)$

step 1. $i = 1$

step 2. while $i \leq n$ do step 3~4

step 3. set $h(0_i, p) = 0$

$h(1_i, p) = p_{i+1} \cdot h(1_{i+1}, p) + (1 - p_{i+1}) \cdot h(0_{i+1}, p)$

$h(p) = p_i \cdot h(1_i, p) + (1 - p_i) \cdot h(0_i, p)$ (compute $h(p)$)

step 4. $i = i + 1$

step 5. output $h(p)$

Algorithm for parallel system

Input : $n, p_1, p_2, \cdots, p_n$

Output : $h(p)$

step 1. $i = 1$

step 2. while $i \leq n$ do step 3~4

step 3. set $h(1_i, p) = 1$

$h(0_i, p) = p_{i+1} h(1_{i+1}, p) + (1 - p_{i+1}) h(0_{i+1}, p)$

$h(p) = p_i \cdot h(1_i, p) + (1 - p_i) \cdot h(0_i, p)$ (compute $h(p)$)

step 4. $i = i + 1$

step 5. output $h(p)$

3. Numerical Examples

Now, we consider the following example in order to compare two algorithms in computational complex system.

Example 1. The bridge structure is shown in the following diagram

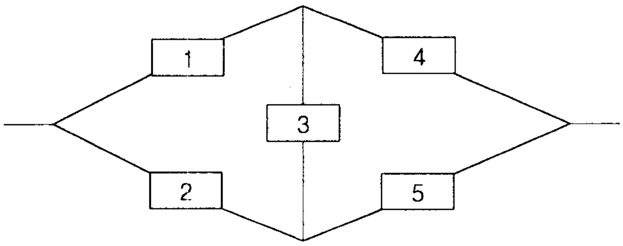

Fig.1 5-component bridge system

There are 4 minimal path sets

$P_1=\{1,4\},\ P_2=\{1,3,5\},\ P_3=\{2,5\},\ P_4=\{2,3,4\}$

By inclusion-exclusion algorithm, reliability function is

$$h(p)=p_1p_4+p_1p_3p_5+p_2p_5+p_2p_3p_4-(p_1p_3p_4p_5+p_1p_2p_4p_5+p_1p_2p_3p_4+p_1p_2p_3p_5+p_1p_2p_3p_4p_5$$
$$+p_2p_3p_4p_5)+(p_1p_2p_3p_4p_5+p_1p_2p_3p_4p_5+p_1p_2p_3p_4p_5+p_1p_2p_3p_4p_5)-p_1p_2p_3p_4p_5$$
$$=p_1p_4+p_1p_3p_5+p_2p_5+p_2p_3p_4-(p_1p_3p_4p_5+p_1p_2p_4p_5+p_1p_2p_3p_4+p_2p_3p_4p_5)+4p_1p_2p_3p_4p_5$$

By pivotal decomposition algorithm, reliability function is

$$h(p)=p_1p_4+(1-p_1p_4)[p_1p_3p_5+(1-p_1p_3p_5)\{p_2p_5+(1-p_2p_5)p_2p_3p_4\}].$$

Thus pivotal decomposition algorithm is more useful for reliability calculation in 5-bridge structure.

Example 2. (8-component complex system) Consider complex system in fig. 2.

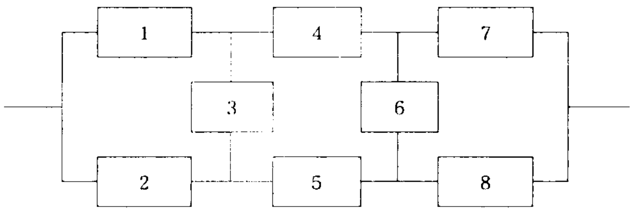

Fig.2 8-component complex system

There are 8 minimal path sets;

$P_1=\{1,4,7\}$, $P_2=\{2,5,8\}$, $P_3=\{1,3,5,8\}$, $P_4=\{1,4,6,8\}$, $P_5=\{2,3,4,7\}$,
$P_6=\{2,5,6,7\}$, $P_7=\{1,3,5,6,7\}$, $P_8=\{2,3,4,6,8\}$

System reliability is

$$h(p) = p_1p_4p_7 + (1-p_1p_4p_7)\ [p_2p_5p_8+(1-p_2p_5p_8)\{p_1p_3p_5p_8+(1-p_1p_3p_5p_8) \cdot \{p_1p_4p_6p_8+(1-p_1p_4p_6p_8)\{p_2p_3p_4p_7)+(1-p_2p_3p_4p_7)\{p_2p_5p_6p_7+(1-p_2p_5p_6p_7) \cdot \{p_1p_3p_5p_6p_7+(1-p_1p_3p_5p_6p_7)p_2p_3p_4p_6p_8\}\}\}\}\}]$$

4. Computational Results

This section discusses the results obtained when algorithms inclusion-exclusion and pivotal decomposition were programed and run on a variety of complex system reliability calculation problems.

Table 3 displays elapsed computation time in CPU seconds for each of the two algorithms.

table 1. CPU Time for the algorithms

system(Algorithm)	# Min Paths	1	2	3	4	5	mean	CPU sec
2-out-of-4:G system (inclusion-exclusion)	6	59.54s	59.49s	59.05s	59.15s	59.19s	59.28s	5.93ms
5-component bridge system (inclusion-exclusion)	4	30.63s	30.67s	30.36s	30.54s	30.41s	30.52s	3.05ms
5-component bridge system (pivotal decomposition)	4	4.29s	3.99s	4.07s	4.12s	3.94s	4.08s	0.41ms
7-component bridge system (inclusion-exclusion)	7	59.25s	59.60s	59.53s	59.69s	59.61s	59.54s	5.95ms
7-component bridge system (pivotal decomposition)	7	4.30s	4.13s	4.23s	4.20s	4.25s	4.22s	0.42ms
8-component bridge system (inclusion-exclusion)	8	67.02s	66.56s	66.99s	66.80s	66.88s	66.85s	6.69ms
8-component bridge system (pivotal decomposition)	8	3.90s	3.78s	3.94s	3.98s	3.86s	3.89s	0.39ms

* Operation Environment

PC - IBM 586 CPU - Pentium II 350MHz

RAM - 128MB (Synchronous DRAM 100MHz)

Compiler - Microsoft Visual C++ 6.0

0.08s/10000(iteration) = 8μs = 0.008ms

By above table, we suggest that pivotal decomposition algorithm provides an efficient method for complex system. Now we faintly survey the multistate coherent systems.

5. Multistate systems

Binary component and systems are inadequate for modeling some real world situation. Components might have more than two discrete states or their states might be continuous variables that can not be modeled satisfactorily by only two states. Most systems can be decomposed into a number of disjoint subsystems consisting of lower order subsystems and so on, down to the elementary component level. Consider for example the High-Pressure-Injection-System of the Pressurized Water Reactor which uses the three Reactor Coolant System pumps $P_i (i=1,2,3)$ to inject water into the primary system cold legs through one of two redundant pipes (p_1 and p_2) or into the primary system hot legs through one of two redundant pipes (p_3 and p_4). The Required flow of the system is equal to the capacity of at least one pump. Any of four pipes can provide this flow under the required pressure. The manually operated valves are closed under usual conditions, where as the check valves allow only in-flow. For analysis purpose, the HIPS can be modeled as composed of two subsystem: that of the pumps and that of the pipes, each sub system being a system of its elementary components. The components of each subsystem are equivalent as far as determining the state of the over all system is concerned. Suppose that these component, each subsystem, and the system itself are binary. Then one can model the HPIS as two subsystem in series, each subsystem being a parallel system of its components. In the sense that all component of each subsystem are equivalent and that their relative locations in the subsystem do not matter, the two subsystems in the example can be called homogeneous. Similarly, the overall system is a homogeneous system of the two subsystems. The purpose of this section is to study the conditions under which the concept of homogeneity can be extended to multistate systems of multistate components and to establish simple algorithms calculating the reliability of the system.

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