Adaptive Eigenvalue Decomposition Approach to Blind Channel Identification

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Abstract—Blind adaptive channel identification of communication channels is a problem of important current theoretical and practical concerns. Recently proposed solutions for this problem exploit the diversity induced by antenna array or time oversampling, leading to the so-called, second order statistics techniques. And adaptive blind channel identification techniques based on a off-line least-squares approach have been proposed. In this paper, a new approach is proposed that is based on eigenvalue decomposition. And the eigenvector corresponding to the minimum eigenvalue of the covariance matrix of the received signals contains the channel impulse response. And we present a adaptive algorithm to solve this problem. The performance of the proposed technique is evaluated over real measured channel and is compared to existing algorithms.

I. INTRODUCTION

In recent years, the interest in blind channel identification problem has received considerable attention. The basic blind channel identification problem involves a channel model where only the observation signal is available for processing in the identification channel. Earlier blind channel identification approaches mostly depend on higher order statistics (HOS), because the second order statistics (SOS) does not contain phase information for stationary signal[1]-[4]. In HOS-based methods, because the performance index as the optimization criterion is nonlinear with respect to estimation parameters and these methods require a large amount of data samples. These methods have the disadvantage that their computational complexity may be large. See, for example, [1] and references therein. Since the seminal work by Tong *et al.* the problem of estimating the channel response of multiple FIR channel driven by an unknown input symbol has interested many researchers in signal processing and communication fields. This is achieved by exploiting assumed cyclostationary properties, induced by over-sampling or antenna array at the receiver[1][2].

Up to date, the implementation of the SOS based methods have been mostly block based algorithm rather than adaptive algorithms. Most communication channels are time-varying in practice. Therefore, the algorithms should be able to track the change of the channel impulse response. Moreover, in a fast fading channel, the multipath channels in wireless communications vary rapidly, and we only have a few data samples corresponding to the same channel characteristics. The adaptive algorithms for blind adaptive channel identification based on SOS have been shown. Heath proposed the adaptive algorithm based on least-squares (LS) method[8]. Blind channel identification technique has been developed in adaptive algorithm based on vector-correlation method[5][8][9]. But most algorithms neglected the effect of channel noise.

In this paper, a novel adaptive blind identification algorithm is proposed by exploiting a constrained adaptive filter in noisy environment. We show that the minimization of the error variance, subject to a specific constant norm constraint, permits the derivation of asymptotically noise-free case. And it can be implemented adaptively at low cost using LMS-like algorithm. Most notations are standard: vectors and matrices are boldface small and capital letters, respectively; the matrix transpose, the complex conjugate, the Hermitian, and convolution are denoted by $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, and $\otimes$, respectively; $\mathbf{I}_P$ is the $P\times P$ identity matrix; $E(\cdot)$ is the statistical expectation.

II. PROBLEM FORMULATION

Let $x(t)$ be the signal at the output of a noisy channel

$$x(t)=\sum_{k=-\infty}^{\infty}s(k)h(t-kT)+v(t) \tag{1}$$

where $s(k)$ denotes the transmitted symbol at time kT, $h(t)$ denotes the continuous-time channel impulse response, and $v(t)$ is additive noise. The fractionally spaced discrete time model can be obtained either by time oversampling or by the sensor array at the receiver. As is shown in [2], the single channel system can be considered as the multichannel system by the sampling the received signal at a rate faster than the input symbol rate. The source signal $s(n)$ then passes through M equivalent symbol rate linear filters. And as shown in Fig. 1, $x_i(\cdot)$ denotes the output from the ith channel with the noisy FIR channel impulse response $\{h_i(\cdot)\}$, which is driven by the same input $s(\cdot)$. Clearly, for linearly modulated communication signals, $x_i(\cdot)$, $a_i(\cdot)$, $s(\cdot)$, $v_i(\cdot)$, and $h_i(\cdot)$ are related as follows

$$x_i(n)=\sum_{k=0}^{L}h_i(k)s(n-k)+v_i(n) \\ =a_i(n)+v_i(n), i=1,\cdots,M \qquad (2)$$

where L is the maximum order of the M channels.

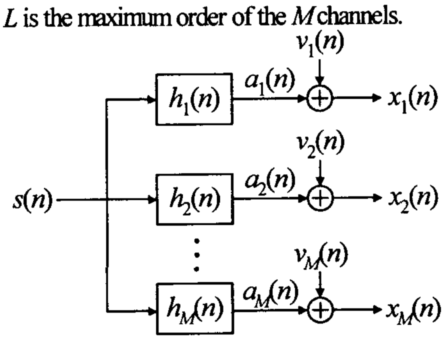

Figure 1. Equivalent SIMO model with M subchannels.

The blind identification problem can be stated as follows: Given the observation of channel output $\{x_i(n), i=1,\ldots M, n=L,\ldots,N\}$, determine the channels and further recover the input signals $\{s(\cdot)\}$. As in classical system identification problems, certain conditions about the channel and the source must be satisfied to ensured identifiability. In the multichannel blind identification case, three conditions are shared by many different approaches. We assume the following throughout in this paper about the channel and source conditions.

A1) Subchannels do not share common zeros, or in other words, they are coprime.

A2) The noise $\mathbf{v}(n)$ is zero mean, white with known covariance, no cochannel correlation, and uncorrelated with source signal.

A3) The channel has known order L.

Assumption 1 provides the necessary and sufficient condition to the unique solution for the blind channel identification problem. This condition has been regarded as the major difficulty of blind algorithms using the SOS[4]. In this paper, consider a special case, when the channel output is two times oversampled or there are two antennas at the receiver, this is equivalent to two channel representation (M=2). In the absence of noise, it is apparent that the output of each subchannel is

$$x_1(n)=h_1(n)\otimes s(n) \\ x_2(n)=h_2(n)\otimes s(n) \qquad (3)$$

Then

$$h_2(n)\otimes x_1(n)=h_2(n)\otimes[h_1(n)\otimes s(n)] \\ =h_1(n)\otimes[h_2(n)\otimes s(n)] \\ =h_1(n)\otimes x_2(n) \qquad (4)$$

Obviously, the above equation is not applicable for a single channel system. We can write (4) as

$$[\mathbf{X}_1(L)\ \ \text{-}\mathbf{X}_2(L)]\begin{bmatrix}\mathbf{h}_2(n)\\ \mathbf{h}_1(n)\end{bmatrix}=0 \qquad (5)$$

where $\mathbf{h}_m=[h_m(L),\ldots, h_m(0)]^T$, m=1, 2, and

$$\mathbf{X}_m(L)=\begin{bmatrix}x_m(0) & \cdots & x_m(L)\\ \vdots & \ddots & \vdots\\ x_m(N-L) & \cdots & x_m(N)\end{bmatrix} \qquad (6)$$

Let us define as follows:

$$\mathbf{h}=[\mathbf{h}_2^T\ \mathbf{h}_1^T]^T,\ \mathbf{X}=[\mathbf{X}_1 \vdots \mathbf{X}_2]^T \qquad (7)$$

In the noise free context, $\mathbf{h}$ is the null space of $\mathbf{X}$, and equivalently (5) can be written as follows[3]

$$\mathbf{X}(L)\mathbf{h}=0 \qquad (8)$$

Equation (8) provides the unique solution for the identification problem if and only if subchannels are coprime, i.e., they do not share any common zeros. When the channel is corrupted by additive noise, we can estimate h by solving the following LS problem

$$\min_{\hat{\mathbf{h}}}\|\mathbf{X}(L)\hat{\mathbf{h}}\|^2 \qquad (9)$$

where $\hat{\mathbf{h}}$ is subject to nontrivial constraints, e.g., $\|\hat{\mathbf{h}}\|=1$ or $\mathbf{c}^*\hat{\mathbf{h}}=1$ for a constant vector $\mathbf{c}$. Although the treatment of the noise is in (9) may be statistically optimal, it is perhaps a natural simple way of formulating this problem.

III. Proposed Method

A. Principle of the Proposed Method

We assume that the channel is linear and time invariant within small time interval; therefore, we have the following relation as described in (4)

$$\mathbf{x}_1^H(n)\mathbf{h}_2=\mathbf{x}_2^H(n)\mathbf{h}_1 \qquad (10)$$

where

$$\mathbf{x}_i(n)=[x_i(n),\cdots,x_i(n-L+1)]^T, i=1,2 \qquad (11)$$

and the channel impulse response vector of length L are defined as

$$\mathbf{h}_i=[h_{i,0}\ h_{i,1}\cdots h_{i,L-1}]^T, i=1,2 \qquad (12)$$

This linear relation follows from (5). The covariance matrix of the two received signals is given by

$$\mathbf{R}_x=\begin{bmatrix}\mathbf{R}_{x1x1} & \mathbf{R}_{x1x2}\\ \mathbf{R}_{x2x1} & \mathbf{R}_{x2x2}\end{bmatrix} \qquad (13)$$

where

$$\mathbf{R}_{xixj}=E[\mathbf{x}_i^H(n)\mathbf{x}_j^H(n)],\ i,j=1,2 \qquad (14)$$

Consider the $2L\times1$ vector as follows:

$$\mathbf{h}=\begin{bmatrix}\mathbf{h}_2\\ -\mathbf{h}_1\end{bmatrix} \qquad (15)$$

From (10) and (13), it can be seen that $\mathbf{R}_x\mathbf{h}=\mathbf{0}$, which means that the vector $\mathbf{h}$ is the eigenvector of the covariance matrix $\mathbf{R}_x$ corresponding to the eigenvalue 0. Moreover, if the two channel impulse response $\mathbf{h}_1$ and $\mathbf{h}_2$ have no common zeros and the autocorrelation matrix of the source signal $s(n)$ is full rank, which is assumed in the rest of this paper, the covariance matrix $\mathbf{R}_x$ has one and only one eigenvalue equal to zero. Consider the noisy channel case as described in (1) and let M=2. It follows from (1) that

$$\begin{aligned} \mathbf{x}^H\mathbf{h} &= \sum_{k=0}^{L} x_2^*(n-k)h_1(k) - \sum_{k=0}^{L} x_1^*(n-k)h_2(k) \\ &= \sum_{k=0}^{L} v_2^*(n-k)h_1(k) - \sum_{k=0}^{L} v_1^*(n-k)h_2(k) = \mathbf{v}^H\mathbf{h} \end{aligned} \quad (16)$$

If the correlation matrix of the vector $\mathbf{x}$ is denoted by $\mathbf{R}_x$, a direct of conclusion of (16) will be

$$\mathbf{R}_x\mathbf{h} = E[\mathbf{x}\mathbf{x}^H]\mathbf{h} = E[\mathbf{x}\mathbf{v}^H]\mathbf{h} = E[\mathbf{v}\mathbf{v}^H]\mathbf{h} = \mathbf{R}_v\mathbf{h} = \sigma_v^2\mathbf{h} \quad (17)$$

We note from (17) that $\mathbf{h}$ is the eigenvector of the correlation matrix $\mathbf{R_X}$ and σ_v^2 is the corresponding eigenvector of $\mathbf{R_X}$. The knowledge of σ_v^2 is not required in the practical case, but it can be obtained as a by product if wanted.

$$\sigma_v^2 = \frac{\mathbf{h}^H\mathbf{R}_x\mathbf{h}}{\mathbf{h}^H\mathbf{h}} \quad (18)$$

B. Adaptive Implementation

In practice, it is simple to estimate iteratively the eigenvector corresponding to the minimum eigenvalue of $\mathbf{R}_x$, by using an algorithm similar to the Frost algorithm that is a simple constrained LMS algorithm[6]. In the following, we show how to apply these techniques to our problem. Minimizing the quantity $\mathbf{h}^H\mathbf{R}_x\mathbf{h}$ with respect to $\mathbf{h}$ and subject to $\|\mathbf{h}\|^2=\mathbf{h}^H\mathbf{h}=1$ will give us the optimum weight $\mathbf{h}_{opt}$.

Let us define the error signal

$$e(n) = \frac{\mathbf{h}^H(n)\mathbf{x}(n)}{\|\mathbf{h}(n)\|} \quad (19)$$

where

$$\mathbf{x}(n) = [\mathbf{x}_1^T(n)\,\mathbf{x}_2^T(n)]^T \quad (20)$$

Note that minimizing the mean square value of $e(n)$ is equivalent to solving the above eigenvalue problem. Taking the gradient of $e(n)$ with respect to $\mathbf{h}(n)$ gives

$$\nabla e(n) = \frac{1}{\|\mathbf{h}(n)\|}\left(\mathbf{x}(n) - e(n)\frac{\mathbf{h}(n)}{\|\mathbf{h}(n)\|}\right) \quad (21)$$

and we obtain the gradient-descent constrained LMS algorithm:

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \mu e^*(n)\nabla e(n) \quad (22)$$

Substituting (19) and (21) into (22) gives

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \frac{\mu}{\|\mathbf{h}(n)\|}\left(\mathbf{x}(n)\mathbf{x}^H(n)\frac{\mathbf{h}(n)}{\|\mathbf{h}(n)\|} - |e(n)|^2\frac{\mathbf{h}(n)}{\|\mathbf{h}(n)\|}\right) \quad (23)$$

and taking statistical expectation after convergence, we get

$$\mathbf{R}\frac{\mathbf{h}(\infty)}{\|\mathbf{h}(\infty)\|} = E[|e(n)|^2]\frac{\mathbf{h}(\infty)}{\|\mathbf{h}(\infty)\|} \quad (24)$$

which is what is desired: the eigenvector $\mathbf{h}(\infty)$ corresponding to the smallest eigenvalue $E[|e(n)|^2]$ of the covariance matrix $\mathbf{R}_x$.

In practice, it is advantageous to use the following adaptation scheme

$$\mathbf{h}(n+1) = \frac{\mathbf{h}(n) - \mu e^*(n)\nabla e(n)}{\|\mathbf{h}(n) - \mu e^*(n)\nabla e(n)\|} \quad (25)$$

The algorithm (25) presented above is a little bit complicated and is very general to find the eigenvector corresponding to the smallest eigenvalue of any matrix $\mathbf{R}_x$. If the smallest eigenvalue is equal to zero, which is the case here, the algorithm can be simplified as follows:

$$e(n) = \mathbf{h}^H(n)\mathbf{X}(n) \quad (26)$$

and

$$\mathbf{h}(n+1) = \frac{\mathbf{h}(n) - \mu e^*(n)\mathbf{x}(n)}{\|\mathbf{h}(n) - \mu e^*(n)\mathbf{x}(n)\|} \quad (27)$$

Note that this algorithm can be seen as an approximation of the previous one by neglecting the terms is $|e(n)|^2$, which is reasonable since the smallest eigenvalue is equal to zero. In this application, the two algorithms (25) and (27) should have the same performance after convergence even with low SNRs.

IV. Simulation Results

Computer simulations were conducted to evaluate the performance of the proposed algorithm in comparison with existing algorithms. In all the simulations, two channels SIMO model is assumed. This means two times oversampling or two sensors at the receiver in real situation. The input signal is QAM. For simplicity of comparison, we assumed that the channel order L is known. The performance index is achieved by examination the root mean square error (RMSE) that is defined as [3].

$$\mathrm{RMSE} = \frac{1}{\|\mathbf{h}\|^2}\sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\|\mathbf{h}_{(i)} - \mathbf{h}\|^2} \quad (28)$$

where N_t is number of Monte Carlo trials, and $\hat{\mathbf{h}}_i$ is the estimate of the channels from the ith trial. For simulations, we used the real-measured channel that is a length-16 version of an empirically measured $T/2$-spaced digital microwave radio channel (M=2) with 230 taps, which we truncated to obtain a channel with L=7. The Microwave channel *chan1.mat* is founded at http://spib.rice.edu/spib/microwave.html. The shortened version is derived by linear decimation of the FFT of the full-length $T/2$-spaced impulse response and taking the IFFT of the decimated version (see [7] for more details on this channel). The channel coefficients are listed in Table 1. A total number of 50 independent trials was performed. All algorithms were initiated at $\mathbf{h}(0)=[1, 0, \ldots, 0, 1, 0, \ldots, 0]^T$ with the step size μ=0.01. Fig. 2 and Fig. 3 show the RMSE of the channel estimates from existing algorithms and the proposed algorithm for real-measured channel under

SNR=20dB and 10dB, respectively. From this figures, we can see that the proposed algorithm always performs better than others. Fig. 4 shows the 50 estimates of the channel.

IV. CONCLUSION

In this paper, a new and simple approach to adaptive blind channel identification has been presented. The method is based on adaptive eigenvalue decomposition. The eigenvector corresponding to the minimum eigenvalue of the covariance matrix of the received signals contains the channel impulse response. And we use a simple constrained LMS algorithm to estimate iteratively the eigenvector corresponding to the minimum eigenvalue. Simulation results have demonstrated the performance improvement of the proposed algorithm. In comparison with other algorithms, the proposed one seems to be more efficient in a low SNR channel and much more accurate. Our proposed algorithm is expected to use in the Viterbi algorithm for symbol detection for mobile communication system. Our future works include the extension to blind multi-input multi-output (MIMO) channel identification and the development of the constrained RLS algorithm for fast convergence.

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Table 1 Channel Coefficients.

	Real-measured Channel	
	i=1	i=2
$h_i(0)$	+0.2636-0.0113j	-0.0276+0.0073j
$h_i(1)$	-0.0186-0.0059j	+0.0350+0.0067j
$h_i(2)$	-0.0065+0.0039j	+0.0147+0.0020j
$h_i(3)$	+0.0236-0.0035j	+0.8760+0.0329j
$h_i(4)$	+0.7826+0.0113j	-0.2025-0.0015j
$h_i(5)$	+0.0754-0.0090j	-0.0225+0.0073j
$h_i(6)$	+0.0134+0.0010j	+0.0134-0.0023j
$h_i(7)$	+0.0042+0.0012j	+0.0042-0.0128j

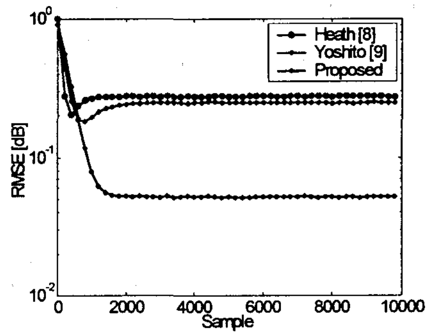

Fig. 2. RMSE comparison under SNR=20dB.

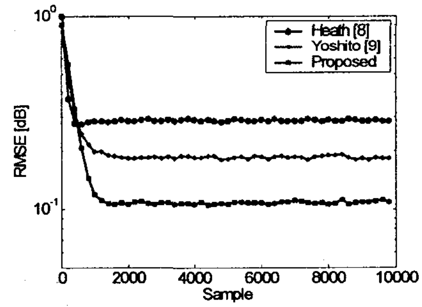

Fig. 3. RMSE comparison under SNR=10dB.

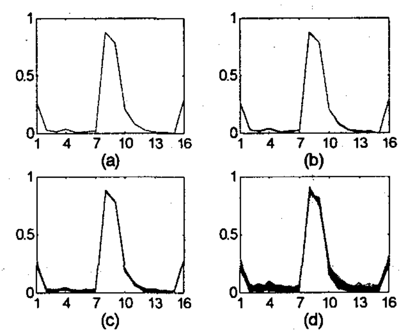

Fig. 4. 50 estimates of the channel, (a) original, (b) SNR=30dB, (c) SNR=20dB, and (d) SNR=10dB.