Performance Analysis of Double-layered ARQ

Yong Hun Uhm and Cheon Won Choi
Dankook University

Abstract In this paper, we propose a retransmission based error control scheme in which a stop-and-wait ARQ is simultaneously performed in two adjacent layers for the node-to-node error control. We develop an analytical numerical method to calculate the probability of error remains and the moments of the high layer message delay time at steady state. Using the analytical method, we investigate the performance of double-layered ARQ scheme with respect to the properties of the employed CRC codes and the characteristics of the involved channel.

1. Introduction

Automatic repeat request (ARQ) schemes are retransmission-based closed-loop error control methods [4][7]. Upon the reception of a data unit, the receiving node sends an acknowledgement to the transmitting node, if no error is detected in the received data unit. Such ARQ schemes are employed for the error control between adjacent nodes along a communication path and are also used for an end-to-end error control between source and destination nodes. For its proper operation, an ARQ scheme must be accompanied by an error detection function, which is usually implemented by a cyclic redundancy check (CRC) code (or shortened cyclic code or polynomial code in other words) [3][4][6][7].

For applications which pay relatively high cost at the end-to-end retransmission, (e.g., a file transfer over a wireless packet network in which a communication path for the end-to-end retransmission may be re-established due to terminal mobility), an accurate node-to-node error control is of necessity. A high accuracy in error control can be achieved by deploying a CRC code with a strong error detection capability. On the other hand, the error control accuracy can also be improved by provisioning ARQ schemes in two or more layers at adjacent nodes. (For example, ARQ schemes may be simultaneously used at data link and network layers in the OSI reference model.) However, such enhancement of node-to-node error control incurs a degradation of node-to-node delay performance. For a given channel, higher probability of error detection leads to an increase in retransmission number, which results in an inferior delay performance. A multi-layered ARQ also causes higher frequency of retransmission and consequently lower performance in node-to-node delay. Thus, node-to-node delay performance must be considered in the design of an node-to-node error control scheme aiming at high accuracy, and the delay performance must be quantitatively expressed for a compromise between control accuracy and delay performance (preferably by an analytical method).

In this paper, for the enhancement of node-to-node error control, we consider a double-layered stop-and-wait ARQ scheme in a packet-switched communication network. First, we present an approximation method for deriving probabilities of error remains for CRC codes. Secondly, using the approximate probability of error remains, we develop an analytical method for deriving the moments of delay time experienced by a high layer message at the transmitting node. Finally, we investigate the effect of channel characteristics, properties of CRC codes and structures of high and low layer messages on the average level of high layer message delay time at steady state.

In Section 2, we describe the double-layered ARQ scheme. In Section 3, we present an analytical numerical method to calculate the probability of error remains and the mean high layer message delay time at steady state. In Section 4, we investigate the effect of channel characteristics and properties of employed CRC codes on the probability of error remains and the mean high layer message delay time by using the analytical method.

2. Double-layered ARQ

Suppose that a session proceeds along a communication path involving many nodes. Let a EHLM (encoded high layer message) denote the basic data unit transported between adjacent nodes along the path. When an EHLM arrives at a node, the EHLM is disassembled into a number of ULLM's (uncoded low layer messages). Each ULLM is encoded with a CRC code and then transmitted to the receiving node via a BSC (binary symmetric channel). In such transmission, a stop-and-wait ARQ scheme is

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used. Upon the reception of an ELLM (encoded low layer message), a positive acknowledgement is sent back to the transmitting node, if no error is detected in the ELLM. When all the ULLM consisting of an EHLM arrive at the receiving node, the ULLM's are assembled into the EHLM and an error detection is carried out for the EHLM. (Note that the EHLM is already encoded with a CRC code.) If an error is detected in the EHLM, then the transmitting node repeats transmitting every ELLM of the EHLM.

3. Performance Analysis

3.1 Error Detection Capability

An (n, k) CRC code is known to detect $d_{\min}-1$ errors of any pattern, where $d_{\min}$ is the minimum distance between codewords [3][6][7]. Including Singleton's upper bound of $n-k+1$, there have been approximate bounds on the minimum distance, (e.g., Hamming, Plotkin, Elias upper bounds and Gilbert and Varsharmov lower bound) [3][6]. For simplicity, we set up an assumption as follows:

Assumption An (n, k) CRC code can only detect at most $n-k+1$ errors.

3.2 Probability of Error Remains

Under the assumption given in Section 3.1, we propose a method to calculate the probability of error remains, i.e., the probability that an UHLM (uncoded high layer message) at the receiving node still have errors.

Suppose that (N^+, K^+) and (N^-, K^-) CRC codes are used for encoding UHLM and ULLM, respectively. Set $\nu^+ = N^+ - K^+$ and $\nu^- = N^- - K^-$. Let ε be the bit error probability in the BSC. Let U denote the number of errors in an ELLM arriving at the receiving node. For $i = 0, \cdots, n$, set

$$\mathrm{Bino}(n, \varepsilon, i) = \binom{n}{i} \varepsilon^i (1-\varepsilon)^{n-i} \tag{1}$$

(i.e., $\mathrm{Bino}(n, \varepsilon, i)$ denotes the mass of the binomial distribution with parameters n and ε). Then, we have

$$h_U(j) = P(U=j) = \mathrm{Bino}(N^-, \varepsilon, j) \tag{2}$$

for $j = 0, \cdots, N^-$. Let V denote the number of errors in the ELLM which is last (re)transmitted to the receiving node. Note that the receiving node does not detect any error in such last ELLM. Thus, we have

$$h_V(j) = P(V=j) = \frac{\mathrm{Bino}(N^-, \varepsilon, j)}{\mathrm{Bino}(N^-, \varepsilon, 0) + \sum_{i=\nu^-+1}^{N^-} \mathrm{Bino}(N^-, \varepsilon, i)} \tag{3}$$

for $j = 0, \nu^-+1, \nu^-+2, \cdots, N^-$. Let W denote the number of errors in the payload of the last ELLM. Set

$$\mathrm{Hyper}(n, k, i, j) = \frac{\binom{k}{j}\binom{n-k}{i-j}}{\binom{n}{i}} \tag{4}$$

(i.e., $\mathrm{Hyper}(n, k, i, j)$ is the mass of the hypergeometric distribution with parameters n, k, i). Then, we have the mass of the distribution for W as follows:

$$\begin{aligned} h_W(0) &= P(W=0) \\ &= \mathrm{Hyper}(N^-, K^-, 0, 0) h_V(0) \\ &\quad + \sum_{i=\nu^-+1}^{N^- - K^-} \mathrm{Hyper}(N^-, K^-, i, 0) h_V(i) \\ h_W(j) &= P(W=j) \\ &= \sum_{i=\nu^-+1}^{N^-} \mathrm{Hyper}(N^-, K^-, i, j) h_V(i) \end{aligned} \tag{5}$$

for $j = 1, \cdots, K^-$. Suppose that an EHLM is disassembled into M ULLM's. Let X denote the number of errors in an EHLM arriving at the receiving node. Then,

$$X = W_1 + \cdots + W_M \tag{6}$$

where $W_1, \cdots, W_M$ are mutually independent and have the same distribution as W. Let h_X denote the mass of the distribution for X. Then, the mass can be expressed as

$$h_X(j) = P(X=j) = \sum_{C_j} h_W(i_1) \cdots h_W(i_M) \tag{7}$$

where $C_j = \{(i_1, \cdots, i_M) : i_1 + \cdots + i_M = j\}$. Let Y denote the number of errors in the EHLM which is last (re)transmitted to the receiving node. Note that the receiving node does not detect any error in such last EHLM. Thus, we have

$$h_Y(j) = P(Y=j) = \frac{h_X(j)}{h_X(0) + \sum_{i=\nu^++1}^{N^+} h_X(i)} \tag{8}$$

for $j = 0, \nu^++1, \nu^++2, \cdots, N^+$. Let Z denote the number of errors in the payload of the last EHLM. Then, we have

$$\begin{aligned} h_Z(0) &= P(Z=0) \\ &= \mathrm{Hyper}(N^+, K^+, 0, 0) h_Y(0) \\ &\quad + \sum_{i=\nu^++1}^{N^+ - K^+} \mathrm{Hyper}(N^+, K^+, i, 0) h_Y(i) \\ h_Z(j) &= P(Z=j) \\ &= \sum_{i=\nu^++1}^{N^+} \mathrm{Hyper}(N^+, K^+, i, j) h_Y(i) \end{aligned} \tag{9}$$

for $j = 1, \cdots, K^+$. Let η denote the probability of error remains, i.e., the probability that an UHLM at the receiving node still have errors. Then,

$$\eta = 1 - h_Z(0)\,. \qquad (10)$$

3.3 HLM Delay Analysis

Let δ denote the propagation delay time between the transmitting and receiving nodes. Set ζ to be the data rate. Let α denote the time elapsed from the moment an ELLM is transmitted to the moment the time-out period for listening an acknowledgement is over. Then,

$$\alpha = \frac{N^-}{\zeta} + 2\delta \qquad (11)$$

where the time to send an acknowledgement is ignored and the length of the time-out period is minimized. Let R^- denote the number of retransmissions of an ELLM. Then, R^- has a geometric distribution with parameter β, where

$$\beta = \sum_{j=1}^{\nu^-} h_U(j)\,. \qquad (12)$$

Define the completion time of a message to be the time elapsed from the moment the transmission of the message starts to the moment an acknowledgement for the message is received by the transmitting node. Let C^- denote the completion time of an ELLM. Then,

$$C^- = (R^- + 1)\cdot\alpha. \qquad (13)$$

Set $S = C_1^- + \cdots + C_M^-$, where $C_1^-, \cdots, C_M^-$ are independent and have the same distribution with C^-. Let R^+ denote the number of retransmissions of an EHLM. Then, R^+ has a geometric distribution with parameter γ, where

$$\gamma = \sum_{j=1}^{\nu^+} h_X(j)\,. \qquad (14)$$

Let C^+ denote the completion time of an EHLM. Then,

$$C^+ = \sum_{i=1}^{R^+ + 1} S_i \qquad (15)$$

where $S_1, S_2, \cdots$ are independent and have the same distribution with S. Suppose that the sequence of EHLM arrival times at the transmitting node is a Poisson process with parameter λ. Then, the transmitting node can be modeled as an M/G/1 queueing system with arrival rate λ and service time C^+ [1][2][5]. Let $\{D_n, n = 1, 2, \cdots\}$ be the sequence of EHLM delay times at the transmitting node. If $\lambda E(C^+) < 1$, then there exists a random variable D such that $D_n \xrightarrow{d} D$ as $n \to \infty$ [1]. Let Φ_D denote the Laplace_Stieltjes transform of the distribution function for D. Then,

$$\Phi_D(s) = \frac{[1 - \lambda E(C^+)]\, s\, \Phi_{C^+}(s)}{s - \lambda[1 - \Phi_{C^+}(s)]} \qquad (16)$$

where $\Phi_{C^+}(s)$ is the Laplace-Stieltjes transform of the distribution function for C^+ [1][2]. From the Laplace-Stieltjes transform in Equation (16), we calculate the moments of the EHLM delay time at steady state by differentiation.

4. Numerical Examples

Figures 1 and 3 illustrate the probability of error remains, and Figures 2 and 4 show the average EHLM delay time with respect to bit error probability. In these figures, the lengths of UHLM and ULLM are respectively fixed to 72 and 35 bits and the data rate is set to be 1 Mbps. The propagation delay time between transmitting and receiving nodes is also set to be 5 μsec. In this section, three CRC codes, identified as Codes 1, 2 and 3 are used, which are characterized by the generator polynomial orders of 3, 12 and 16, respectively. In Figure 2, we observe that the average delay time increases as the bit error probability increases up to a certain value. For the bit error probabilities greater than such value, however, the average delay time rather decreases. This phenomenon is explained as follows: As the bit error probability increases, the receiving node detects errors more frequently and the retransmission number is increased at the transmitting node. After the critical value, the bit error probability is too high so that the receiving node fails to detect errors. In Figures 1 and 2, we observe the performance trade-off between the probability of error remains and the average message delay time. In the same environment as in Figures 1 and 2, Figures 3 and 4 show the probability of error remains and the average message delay time under a single-layered ARQ scheme. From these figures, we observe that the double-layered ARQ scheme can effectively reduce the average delay level for given probability of error remains.

5. Conclusions

As a candidate scheme for improving accuracy in node-to-node error control, we proposed a double-layered ARQ scheme. For the performance evaluation of the scheme, we developed an analytical numerical method to calculate the probability of error remains and the average delay time of a high layer message at the transmitting node. Using the analytical method, we investigated the effect of channel characteristics and CRC codes' properties on the performance of the double-layered ARQ scheme. From numerical examples, we confirmed the trade-off between the probability of error remains and average message delay time and concluded the usefulness of the double-layered ARQ scheme under the constraint on average delay performance in comparison with a single-layered ARQ.

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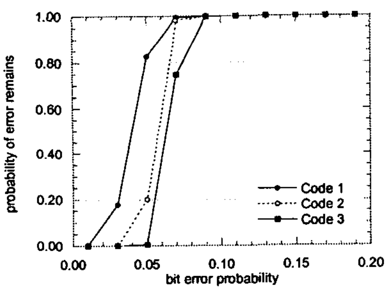

Figure 1 Probability of error remains under double-layered ARQ scheme (Code 1 is used for encoding high layer message.)

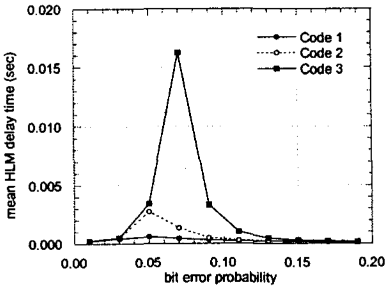

Figure 2 Average delay time of high layer message under double-layered ARQ scheme (Code 1 is used for encoding high layer message.)

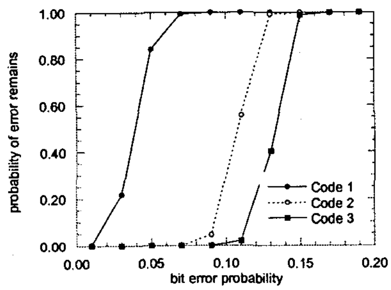

Figure 3 Probability of error remains under single-layered ARQ scheme (High layer message is uncoded.)

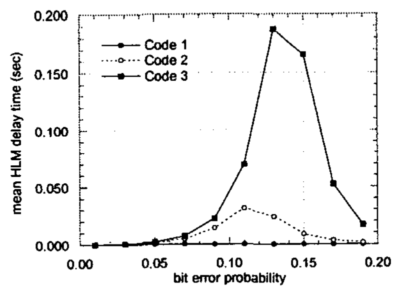

Figure 4 Average delay time of high layer message under single-layered ARQ scheme (High layer message is uncoded.)