

# A Efficient Channel Estimation of OFDM Using The Pilot Method

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## ABSTRACT

In this paper, we investigate a new approach pilot-symbol-aided channel estimation for orthogonal frequency division multiplexing (OFDM) systems. Until now, lots of channel estimation methods are proposed. The pilot symbol assisted modulation (PSAM) has good performance. But, our proposed Algorithm has more good performance than the conventional PSAM system. Our algorithm is that channel estimation performance using the less inserted pilot-symbols systems is almost same to the two times inserted pilot-symbols system. The proposed method is highly robust to fast Rayleigh fading channel. Simulation results are presented to demonstrate the performance of our proposed algorithm.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has recently been considered an efficient transmission method for broadband wireless communication systems. Also, this method is now being considered as one of the next generation of mobile communication systems. There are strong interests in an applying OFDM in wireless systems because of its various advantages in lessening the severe effects of frequency-selective fading [1][2][3]. In all of these applications, the multipath propagation causing both inter symbol interference (ISI) and inter carrier interference (ICI). To overcome these problems, we have to estimate the channel.

There is several channel estimation method in OFDM systems. One of them, pilot-symbol-aided method (PSAM) is good approach. PSAM uses the multiplexed pilot sub-carrier symbols in the transmitted signal to estimate the channel [4]-[7]. The scattered pilots can be used to obtain the samples of the channel frequency response. The estimate of the complete channel frequency response can then be computed via a two-dimensional interpolation in the time and frequency domains. The optimal interpolation filter design in the minimum mean square error (MMSE) sense requires the channel statistics that include the channel power delay profile and the Doppler frequency shift [8][9].

In this paper, we propose an improved channel estimation method for OFDM transmission over frequency selective fading channels using a small number of pilot sub-carriers. We proposed the channel estimation method which uses not only pilot symbol but also a pre-estimated data that will used in pilot autocorrelation. Our method has better robust feature in a fast fading environment than that of the conventional PSAM. One-dimensional (1-D), double 1-D, and two-dimensional (2-D) filtering algorithms have been proposed for pilot-symbol-aided estimation for OFDM systems in terrestrial audio and television broadcasting, fixed and mobile wireless communications. However, the filtering algorithms for pilot-symbol-aided estimation require channel statistics, such as the delay profile and the Doppler frequency, which are usually unknown in

wireless environments.

This paper is organized as follows. In section II, we describe the general OFDM systems and conventional channel estimation with PSAM. In Section III, we present our proposed estimation method. section IV presents some sample simulation results that demonstrate the potential of the proposed channel estimation method. And we conclude the paper in section V.

## II. SYSTEM DESCRIPTION

In this section, we briefly introduce the channel statistics and describe an OFDM system with a pilot-symbol-aided estimator.

### A. System Model

Fig.1 displays the OFDM baseband model used in this paper. We assume that the use of a cyclic prefix (CP) both preserves the orthogonality of the tones and eliminates interference (ISI) between consecutive OFDM symbols. Further, the channel is assumed to be slowly fading, so it is considered to be constant during on OFDM symbol. The number of tones in the system is  $N$  and the length of the CP is  $L$  samples.

Under these assumptions, we can describe the system as a set of parallel Gaussian channels with correlated attenuations  $h_k$ . The attenuations on each tone are given by

$$h_k = G\left(\frac{k}{NT_s}\right), \quad k = 0 \cdots N-1 \quad (1)$$

Where  $G(\cdot)$  is the frequency response of the channel  $g(\tau)$  during the OFDM symbol and  $T_s$  is the sampling period of the system. In matrix notation we describe the OFDM system as

$$y = Xh + n \quad (2)$$

Where  $y$  is the received vector,  $X$  is a diagonal matrix containing the transmitted signaling points,  $h$  is a channel attenuation vector, and  $n$  is a vector of independent identically distributed (i.i.d) complex zero-mean Gaussian noise with variance  $\sigma_n^2$ . The noise  $n$  is assumed to be uncorrelated with the channel  $h$ .

### B. Fading Channel Model

The fading channel output is given by

$$r_c(t) = c(t)s(t) + n_c(t) \quad (3)$$

where  $n_c(t)$  is zero mean, complex AWGN with power spectral density  $N_0$  in both real and imaginary components. The channel complex gain  $c(t)$  incorporates both fading and frequency offset:

$$c(t) = \exp(jw_0 t)g(t) \quad (4)$$

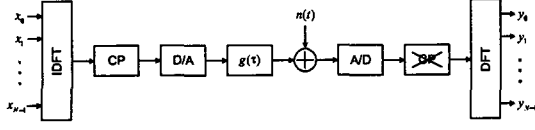


Fig. 1. OFDM baseband system with CP.

where  $w_0$  is the residual angular frequency offset after AFC, and  $g(t)$  is the complex Gaussian fading process with variance  $\sigma_g^2$  and Doppler spread  $f_D = w_0/2\pi$ . A wide-sense stationary uncorrelated scattering (WSS-US) mobile radio channel model can be expressed as

$$h(k, l) = \lim_{N \rightarrow \infty} \frac{1}{\sqrt{N}} \sum_{n=1}^N e^{j(\phi_n + 2\pi f_{D_n} T_s k + 2\pi \tau_n \Delta F l)} \quad (5)$$

with  $E[|h(k, l)|^2] = \sigma_h^2 = 1$ .  $N$  echoes superpose incoherently. Each path is characterized by a random phase  $\phi_n$ , a random Doppler shift  $f_{D_n}$  and a random delay  $\tau_n$ , where  $1 \leq n \leq N$ . The corresponding joint probability density function  $p_{\phi, f_D, \tau}(\phi, f_D, \tau)$  is needed to randomly choose  $\phi_n, f_{D_n}$  and  $\tau_n$ . The symbol duration,  $T_s$  is the spacing in the frequency domain.  $f_{D_{\max}} T_s$  and  $\tau_{\max} \Delta F$  are the normalized one-sided channel bandwidths in time and frequency domains, respectively.

Due to the assumption of uncorrelated scattering and for independent phases

$$p_{\phi, f_D, \tau}(\phi, f_D, \tau) = p_{\phi}(\phi) p_{f_D}(f_D) p_{\tau}(\tau) \quad (6)$$

where  $p_{f_D}(f_D)$  and  $p_{\tau}(\tau)$  are proportional to the so-called Doppler power spectrum and delay power spectrum, respectively [10].

### III. PROPOSED METHOD

In the following section we present a reduced-complexity LMMSE estimate of the channel attenuations  $\mathbf{h}$  from the received vector  $\mathbf{y}$  and the transmitted data  $\mathbf{X}$ . we assume that the received OFDM symbol contains data known to the estimator either training data or receiver decisions. The proposed method can also be used in pilot-symbol assisted modulation (PSAM) and two-dimensional Wiener filtering. Pilot-symbols are multiplexed into the transmitted data stream and channel estimation is performed by interpolation.

In conventional pilot based channel estimation methods, the estimate of pilot signals, based on least squares (LS) criterion, is given by

$$\begin{aligned} \hat{H}_{LS} &= [H_{LS}(0) \ H_{LS}(1) \ \dots \ H_{LS}(N-1)]^T \\ &= \mathbf{X}_p^{-1} \mathbf{Y}_p \\ &= \begin{bmatrix} Y(0) & Y(1) & \dots & Y(N-1) \\ X(0) & X(1) & \dots & X(N-1) \end{bmatrix} \end{aligned} \quad (7)$$

The LS estimate of  $\hat{H}_{LS}$  is susceptible to Gaussian noise and inter-carrier interference (ICI). Because the channel responses of data subcarriers are obtained by interpolating the channel responses of pilot subcarriers, the performance of OFDM system is highly dependent on the rigorousness of estimate of pilot signals. Thus an estimate better than the LS estimate is required.

The minimum mean square error (MMSE) estimation has a more good performance to the LS estimation for channel estimation in OFDM systems. The LMMSE channel estimation

$\mathbf{h}$  is given the received data  $\mathbf{y}$  and the transmitted symbols  $\mathbf{X}$ ,

$$\hat{H}_{LMMSE} = R_{HH} (R_{HH} + \sigma_n^2 (XX^H)^{-1})^{-1} \hat{H}_{LS} \quad (8)$$

Fig.1 Baseband model of OFDM system the covariance matrix is defined by

$$R_{HH} = E\{HH^H\} \quad (9)$$

A more generic solution is to average over the transmitted data, and a simplified linear MMSE estimator of pilot signals is obtained as:

$$\hat{H}_p = R_{HH} (R_{HH} + \frac{\beta}{SNR} I)^{-1} \hat{H}_{LS} \quad (10)$$

If the auto-correlation matrix  $R_{HH}$  and SNR are known in advance,  $R_{HH} (R_{HH} + \beta/SNR)^{-1}$  needs to be calculated only once.

The LMMSE method has very good channel estimation method but which is related with pilot's numbers. The number of pilot is inserted data is high, and then estimation performance is very good. On contrary the pilot is inserted into data small numbers then estimation performance is bad. But number of pilot is increased, and then the complexity is increased exponentially because we have to get the auto-correlation of pilot. There are some tradeoff between number of pilot and the complexity and performance.

First, we propose that using less pilot numbers get almost twice performance. So we can increase data efficient. After we can overcome our proposal's drawback with the low-rank estimators, singular value decomposition(SVD) method[11]. Our computational complexity is almost two times to the original PSAM.

The low-rank estimators will have an irreducible error floor due to the part of the channel that does not belong to the subspace. To eliminate this error floor up to a given SNR, we need to make sure that our estimator rank is sufficiently large. This prompts an analysis of the computational complexity of the rank-p estimator. In comparison with the estimator we have managed to reduce the number of multiplications from  $N$  to  $2p$  per tone. The smaller  $p$  is, the lower the computational complexity, but the larger the approximation error becomes. So we have to select optimal rank-p.

Optimal rank reduction is achieved by using the singular value decomposition (SVD). The SVD of the channel auto covariance matrix is

$$R_{HH} = U \Lambda U^H \quad (11)$$

where  $U$  is a unitary matrix containing the singular vectors and  $\Lambda$  is a diagonal matrix containing the singular values  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$  on its diagonal. It is shown that the optimal rank-p estimator is

$$\hat{h}_p = U \Lambda_p U^H \hat{h}_{LS} \quad (12)$$

where  $\Delta_p$  is a diagonal matrix with entries

$$\delta_k = \begin{cases} \frac{\lambda_k}{\lambda_k + \frac{\beta}{SNR}}, & k = 1, 2, \dots, p \\ 0, & k = p + 1, \dots, N \end{cases} \quad (13)$$

Interpreting the matrix  $U^H$  as a transform, the singular value

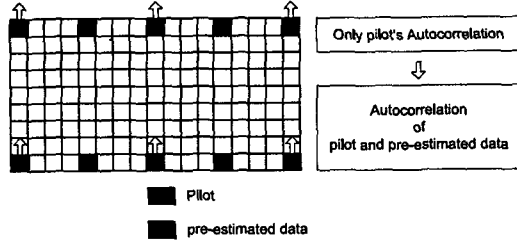


Fig. 2. Block diagram of proposed method.

$\lambda_k$  of  $R_{HH}$  is the channel power variance contained in the  $k$ -th transform coefficient after transforming the LS estimate  $\hat{h}_{LS}$ . The low-rank estimator can be interpreted as first projecting the LS estimates onto a subspace and then performing the estimation. If the subspace has a small dimension and can describe the channel well, the complexity of the estimator will be low while showing a good performance.

In the LMMSE method, we can get the  $h$  of LMMSE estimator with using orthogonal projection theory

$$E[(h - ky)y^H] = 0 \quad (14)$$

Then,

$$\hat{h}_{LMMSE} = R_{hp} R_{pp}^{-1} p \quad (15)$$

if the factor  $R_{hp}$  and  $R_{pp}$  have the detailed matrix values, channel estimation is correctness. But for the two factors makes more detailed matrix, we have to insert many pilot symbols. Our proposal explains that channel estimation is almost two times performance with new estimation method. The idea of the process of getting detailed correlation is depicted in Fig. 2.

First, we get the data that will be estimated previously than other data, we use it to the correlation factor. The data position that will be estimated first is the middle of adjacent two pilots. That makes correlation matrix is good. The next using the updated correlation matrix we can channel estimation the remained data.

In Original PSAM, inserted pilot position and the estimated channel and correlations are follows:

$$p = [1 \ T+1 \ \dots \ (N-1)T+1] \quad (16)$$

$$\hat{h} = R_{hp} R_{pp}^{-1} p \quad (17)$$

$N$  is a number of pilot used in correlation.  $R_{hp}$  is cross correlation of channel which will be estimated and pilot.  $R_{pp}$  is auto correlation of pilots with  $N$  by  $N$  matrix. To the start, we estimate only several data which will be used in updated correlation. Updated correlation is made of above estimated data and pilots. Numerical expression are follows:

$$\hat{u} = \left[ 1 + \frac{T}{2} \quad \frac{3T}{2} + 1 \quad \dots \quad \frac{(2N-1)T}{2} + 1 \right] \quad (18)$$

$$\hat{p} = p + \hat{u}$$

$$\hat{R}_{pp} = [(2N-1) \times (2N-1)] \quad (19)$$

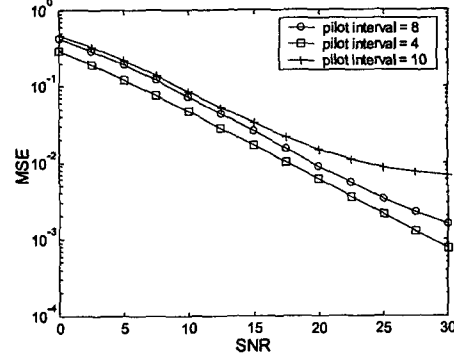


Fig. 3. The original PSAM with pilot interval variation in Doppler frequency is 180Hz.

$\hat{R}_{pp}$  is a updated correlation. For use this, we can get good channel estimation with new correlation like as :

$$\tilde{h}_{LMMSE} = \hat{R}_{hp} \hat{R}_{pp}^{-1} p \quad (20)$$

$\tilde{h}_{LMMSE}$  is estimated channel well with updated auto-correlation  $\hat{R}_{hp}$  and  $\hat{R}_{pp}$ . But if channel is very stable, our proposed method has bad performance than the original PSAM. In real fading environments, Because the mobile channel which is similar to Rayleigh fading channel is changed very fast. We will show that our proposed method is very good compared with conventional PSAM on the SNR in next section.

#### IV. SIMULATION RESULTS

In this section, we use computer simulation to show the performance of conventional methods and the proposed method. The channel model used in this paper as follows;

$$G(f; t) = \frac{1}{M} \sum_{k=1}^M e^{j(\theta_n + 2\pi F_{Dn} t + 2\pi f \tau_n)} \quad (21)$$

where  $\theta_n$  is a phase,  $\tau_n$  is  $n$ -th path delay time and  $F_{Dn}$  is Doppler frequency. Rayleigh fading by Jakes spectrum model is applied to this channel model. The system in this paper has a 5MHz bandwidth, with 1024 by 513 data frames and  $\tau_{rms} = 12.8\mu s$ . We have the same channel correlation in each subsystem as we have in the 128-tone scenario in this paper ( $L/N=8/128$ ). By estimating the channel attenuation  $h$  in each subsystem independently, we neglect the correlation between tones in different subsystems, but obtain the same mean squared error (MSE) performance as in our 128-tone scenario. The advantage is a significant reduction of complexity. Channel correlation is divided into time and frequency domain. The correlation matrix for the attenuation vector  $h$

$$R_{hh} = E\{hh^H\} = [r_{m,n}] \quad (22)$$

The uniform channel correlation between the attenuation  $h_m$  and  $h_n$  in this system is ;

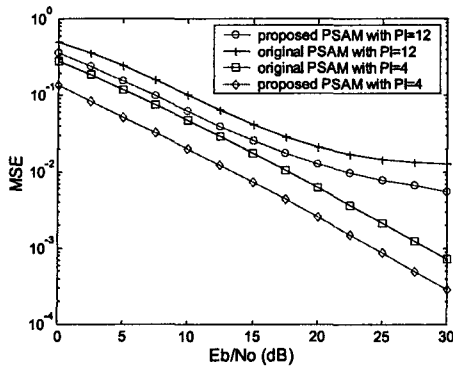


Fig. 4. The MSE of the conventional PSAM and proposed PSAM and Doppler frequency is 180Hz. (PI means pilot interval)

$$r_{m,n} = \begin{cases} 1, & \text{if } m = n \\ \frac{1 - e^{-j2\pi L \frac{m-n}{N}}}{j2\pi L \frac{m-n}{N}}, & \text{if } m \neq n \end{cases} \quad (23)$$

Fig. 3 shows estimation performance with pilot interval. As pilot interval is larger than 8, MSE is decreased very quickly. For certain performance we have to maintain pilot interval suitably. Fig. 4 explains our proposed method is superior to original PSAM method. Our method has a more complexity of computation compare with the original PSAM. We overcome this drawback with SVD method. The low rank estimator presented in this paper is based on frequency correlation only, but the time correlation of the channel can also be used. The two dimensional LMMSE estimator can be simplified using the same technique with rank reduction. Fig. 5 presents SVD method, which uses our proposed PSAM, is very good performance in MSE. This explains our method has much better performance with same complexity of PSAM.

## V. CONCLUSION

In this paper, we have studied the PSAM with updating correlation parameters. Especially our method can significantly improve the performance in fast fading environments. The proposed PSAM with SVD is significantly efficient estimation. The computational complexity of the proposed channel estimator is studied and seeks the complementary measures when channel is changed quickly. In conclusion, the proposed channel estimator provides practical channel estimation for OFDM systems based on PSAM with updating auto- and cross-correlation of channel and pilots.

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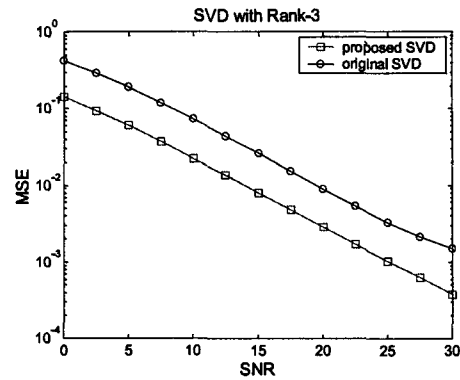


Fig. 5. proposed method is lowering the complexity by SVD algorithms compared with original PSAM in same environments.

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