Joint Blind Data/Channel Estimation Based on Linear Prediction

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ABSTRACT

Blind identification and equalization of communication channel is important because it does not need training sequence, nor does it require a priori channel information. So, we can increase the bandwidth efficiency. The linear prediction error method is perhaps the most attractive in practice due to the insensitive to blind channel estimator and equalizer length mismatch as well as for its simple adaptive algorithms. In this paper, we propose method for fractionally spaced blind equalizer with arbitrary delay using one-step forward prediction error filter from second-order statistics of the received signals for SIMO channel. Our algorithm utilizes the forward prediction error as training sequences for data estimation and desired signal for channel estimation.

I. INTRODUCTION

Multipath propagation appears to be a typical limitation in mobile digital communication where it leads to severe intersymbol interference (ISI). The classical techniques to overcome this problem use either periodically sent training sequence or blind techniques exploiting higher order statistics (HOS). Adaptive equalization using training sequence wastes the bandwidth efficiency but in blind equalization, no training is needed and the equalizer is obtained only with the utilization of the received signal. Since the seminal work by Tong *et al.* the problem of estimating the channel response of multiple FIR channel driven by an unknown input symbol has interested many researchers in the signal processing areas and communication fields[2][3][4].

For the most part, algebraic and second-order statistics (SOS) techniques have been proposed that exploit the structural techniques (Hankel, Toeplitz matrix, *et al.*) of the single-input multiple-output (SIMO) channel or data matrices. The information on channel parameters or transmitted data is typically recovered through subspace decomposition of the received data matrix (deterministic method) or that of the received data correlation matrix (stochastic method). Although very appealing from the conceptual and signal processing techniques point of view, the use of the aforementioned techniques in real world applications faces serious challenges. Subspace-based techniques lay in the fact that they relay on the existence of numerically well-defined dimensions of the noise-free signal or noise subspaces. Since these dimensions are obviously closely related to the channel length, subspace-based techniques are extremely sensitive to channel order mismatch[5].

The prediction error method (PEM) offer an alternative to the class of techniques above. PEM, which were first introduced by Slock *et al.* and later refined by Meraim *et al.*, exploited the i.i.d. property of the transmitted symbols and apply a linear prediction error filter on the received data. The PEM offers great practical advantages over most other proposed techniques. First, channel estimation using the PEM remains consistent in the presence of the channel length mismatch.. This property guarantees the robustness of the technique with respect to the difficult channel length estimation problem. Another significant advantage of the PEM is that it lends itself easily to a low-cost adaptive implementation such as adaptive lattice filters. But the decision delay cannot be controlled with existing one-step prediction error method[5][6][7].

In this paper, we propose method for blind equalizers with arbitrary decision delay using one-step forward prediction error filter (FPEF) and channel estimation. We utilize the forward prediction error (FPE) as training signal for symbol estimation and desired signal for channel estimation. Also, we derive an adaptive algorithm for proposed method.

II. PROBLEM STATEMENT

Let $x(t)$ be the continuous-time signal at the output of a noisy communication channel

$$x(t)=\sum_{k=-\infty}^{\infty}s(k)h(t-kT)+v(t) \tag{1}$$

where $s(k)$ denotes the transmitted symbol at time kT, $h(t)$ denotes the continuous-time channel impulse response, and $v(t)$ is additive noise. The fractionally spaced discrete-time model can be obtained either by time oversampling or by the sensor array at the receiver. The oversampled single-input single-output (SISO) model results SIMO model as in Fig. 1. The corresponding SIMO model is described as follows

$$x_i(n)=\sum_{k=0}^{L-1}s(k)h_i(n-k)+v_i(n),\ i=0,1,\cdots,P-1 \tag{2}$$

where P is the number of subchannel, and L is the maximum order of the P subchannel.

Let

$$\begin{aligned}\mathbf{x}(n)&=[x_0(n)\cdots x_{P-1}(n)]^T\\ \mathbf{h}(n)&=[h_0(n)\cdots h_{P-1}(n)]^T\\ \mathbf{v}(n)&=[v_0(n)\cdots v_{P-1}(n)]^T\end{aligned} \tag{3}$$

We represent $x_i(n)$ in a vector form as

$$\mathbf{x}(n)=\sum_{k=0}^{L-1}s(k)\mathbf{h}(n-k)+\mathbf{v}(n) \tag{4}$$

Stacking N received vectors samples into an $(NP\times1)$-vector, we can write a matrix equation as

$$\mathbf{x}_N(n)=\mathbf{H}\mathbf{s}(n)+\mathbf{v}_N(n) \tag{5}$$

where $\mathbf{H}$ is a $NP\times(N+L)$ block Toeplitz matrix, $\mathbf{s}(n)$ is $(N+L)\times1$, $\mathbf{x}_N(n)$, and $\mathbf{v}_N(n)$ are $NP\times1$ vectors.

$$\begin{aligned}\mathbf{s}(n)&=[s(n)\cdots s(n-L-N+1)]^T\\ \mathbf{x}_N(n)&=[\mathbf{x}^T(n)\cdots\mathbf{x}^T(n-N+1)]^T\\ \mathbf{v}_N(N)&=[\mathbf{v}^T(n)\cdots\mathbf{v}^T(n-N+1)]^T\end{aligned} \tag{6}$$

$$\mathbf{H}=\begin{bmatrix}\mathbf{h}(0)&\cdots&\mathbf{h}(L-1)&\cdots&\mathbf{0}\\ \vdots&\ddots&\vdots&\ddots&\vdots\\ \mathbf{0}&\cdots&\mathbf{h}(0)&\cdots&\mathbf{h}(L-1)\end{bmatrix} \tag{7}$$

We assume the following throughout in this paper.

A1) The input sequence $s(n)$ is zero-mean and white with variance σ_s^2.

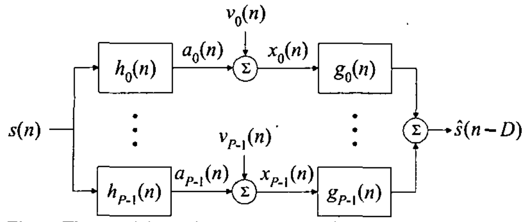

Fig. 1. The multichannel representation of a T/P-spaced equalizer.

A2) The additive noise $v(n)$ is stationary with zero mean and white with variance σ_v^2.
A3) The sequences $s(n)$ and $v(n)$ are uncorrelated.
A4) The matrix $\mathbf{H}$ has full rank, i.e., the subchannels $h_i(n)$ have no common zeros to satisfy the Bezout equation.
A5) The dimensions of $\mathbf{H}$ obey $NP>L+N$.

Consider an FIR linear ZF- or MMSE-equalizer shown in Figure 1, where $g_i(n)$ for i=0,1,..., P–1 is the order N equalizer of the ith subchannel. The equalizer impulse response in vector form is

$$\mathbf{g}(n)=[g_0(n),\cdots,g_{P-1}(n)]^T \tag{8}$$

A D-delay equalizer vector of length NP is given as

$$\mathbf{g}_D=[\mathbf{g}^T(0),\cdots,\mathbf{g}^T(N-1)]^T \tag{9}$$

and the symbol is estimated from

$$\hat{s}(n-D)=\mathbf{g}_D\mathbf{x}_N(n) \tag{10}$$

The output of the equalizer approaches s_{n-D} for some decision delay D. Then this equalizer is known as the D-delay ZF-equalizer. According to (5)-(7), $\mathbf{x}_N(n)$ has nonzero correlation with only $s(n),\ldots,s(n-N-L+1)$. Therefore, decision delay D is usually in the interval [0, N+L–1]. For finite SIMO channels, blind equalizer of the finite length can be found if assumption A4) holds and the equalizer length $N\geq L(P-1)$ [7].

III. Data and Channel Estimation

A. Multichannel Linear Prediction

Consider the following multichannel one-step forward prediction problem

$$\begin{aligned}\mathbf{f}_N(n)&=\mathbf{x}(n)-[\mathbf{p}_1\mathbf{x}(n-1)+\cdots+\mathbf{p}_N\mathbf{x}(n-N)]\\&=[\mathbf{I}_P\quad -\mathbf{P}_N]\mathbf{x}_{N+1}(n)\end{aligned} \tag{11}$$

where $-\mathbf{p}_k$ for k=1 ..., N are $P\times P$ matrices of a FPEF of order N.

The FPEF coefficients are selected such that mean square value of $\mathbf{f}_N(n)$, i.e., $E[\|\mathbf{f}_N(n)\|^2]$, is minimized. Therefore, for any set of FPEF coefficients $\mathbf{p}_k$

$$\frac{\partial E[\mathbf{f}_N(n)\mathbf{f}_N^H(n)]}{\partial \mathbf{p}_k^H}=0,\ \text{for}\ 1\leq k\leq N \tag{12}$$

We obtain as following

$$\begin{bmatrix}\mathbf{r}(0) & \mathbf{r}(1) & \cdots & \mathbf{r}(N-1)\\ \mathbf{r}^H(1) & \mathbf{r}(0) & \cdots & \mathbf{r}(N-2)\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{r}^H(N-1) & \mathbf{r}^H(N-2) & \cdots & \mathbf{r}(0)\end{bmatrix}\begin{bmatrix}\mathbf{p}_1\\ \mathbf{p}_2\\ \vdots\\ \mathbf{p}_N\end{bmatrix}=\begin{bmatrix}\mathbf{r}(1)\\ \mathbf{r}(2)\\ \vdots\\ \mathbf{r}(N)\end{bmatrix} \tag{13}$$

where $\mathbf{r}(i-j)=E[\mathbf{x}(n-i)\mathbf{x}^H(n-j)]$.

When the FPEF is optimum in the sense of MSE, the input signal vector $\mathbf{x}_{N+1}(n)$ and the prediction error $\mathbf{f}_N(n)$ are orthogonal. Therefore,

$$E[\mathbf{x}_{N+1}(n)\mathbf{f}_N^H(n)]=0 \tag{14}$$

As shown in [5] and [6], we obtain

$$\mathbf{f}_N(n)=\mathbf{h}(0)s(n) \tag{15}$$

To achieve fast convergence, we can use the RLS algorithm to update the FPEF as following:

• Compute output:

$$\hat{\mathbf{x}}(n)=\mathbf{P}_N(n)\mathbf{x}_N(n-1) \tag{16}$$

• Compute FPE:

$$\mathbf{f}_N(n)=\mathbf{x}(n)-\hat{\mathbf{x}}(n) \tag{17}$$

• Compute Kalman gain:

$$\mathbf{K}(n)=\frac{\lambda^{-1}\mathbf{Q}(n-1)\mathbf{x}_N(n-1)}{1+\lambda^{-1}\mathbf{x}_N^H(n-1)\mathbf{Q}(n-1)\mathbf{x}_N(n-1)} \tag{18}$$

• Update inverse of the correlation matrix:

$$\mathbf{Q}(n)=\lambda^{-1}\mathbf{Q}(n-1)-\lambda^{-1}\mathbf{K}(n)\mathbf{x}_N^H(n-1)\mathbf{Q}(n-1) \tag{19}$$

• Update FPEF coefficients:

$$\mathbf{P}_N(n)=\mathbf{P}_N(n-1)+\mathbf{f}_N(n)\mathbf{K}^H(n) \tag{20}$$

The term λ ($0\leq\lambda\leq 1$) is intended to reduce the effect of past values on the statistics when the filter operates in nonstationary environment. It affects the convergence speed and the tracking accuracy of the algorithm[1]. The FPEF coefficients can also be computed by an LMS algorithm. In a simple manner, the FPE can be computed by (16) and (17), and the FPEF coefficients can be updated by

$$\mathbf{P}_N(n)=\mathbf{P}_N(n-1)+\mu\,\mathbf{f}_N(n)\mathbf{x}_N^H(n-1) \tag{21}$$

where μ, the adaptation step-size, is a positive constant.

B. Best Delayed Symbol Estimation and Channel Estimation

Let us first consider MMSE-equalization. A zero-delay MMSE-equalizer can be obtained as shown in [6] and [7]

$$\mathbf{g}_0^{\text{MMSE}}=\sigma_s^2\mathbf{h}^H(0)\mathbf{F}^{-1}[\mathbf{I}_P\ -\mathbf{P}_N] \tag{22}$$

where $\mathbf{F}$ is the covariance matrix of $\mathbf{f}_N(n)$ as following

$$\mathbf{F}=E[\mathbf{f}_N(n)\mathbf{f}_N^H(n)]=\sigma_s^2\mathbf{h}(0)\mathbf{h}^H(0) \tag{23}$$

A zero-delay ZF-equalizer can also be obtained from FPEF as shown in [6] and [7]

$$\mathbf{g}_0^{\text{ZF}}=\frac{\mathbf{h}^H(0)}{\|\mathbf{h}(0)\|^2}[\mathbf{I}_P\ -\mathbf{P}_N] \tag{24}$$

where $\|\mathbf{h}(0)\|$ is the Euclidean norm of $\mathbf{h}(0)$.

When additive noise $\mathbf{v}(n)$=0, the MMSE- and ZF-equalizer become equivalent. But in noisy environment, MMSE-equalizer has better performance than ZF-equalizer generally[2][7].

As described in [13], multistep prediction has been suggested as a solution to the arbitrary decision delay equalization problem. The multistep prediction error can be modeled as an output of a truncated channel with no additive noise. The equalization of [12] is proposed a combination of two multistep FPEF. The equalization of [7] consists of a cascade of a multistep FPEF and one-step BPEF. Multistep prediction-based method require two prediction error filters and need to estimate channel coefficient corresponding to decision delay[9].

It is obvious that multistep prediction-based methods are required more computational complexity than one-step FPEF-based methods and, moreover, needed to channel identification procedure before equalization. But one-step FPEF-based method are needed to first channel coefficient, $\mathbf{h}(0)$ only. A feasible solution for estimation of the $\mathbf{h}(0)$ is given in [5], where the additive noise ignored. More accurate method is eigen-pair tracking using the covariance matrix of PE in (23)[6][11].

We propose new method for arbitrary decision delay blind equalizer based on one-step FPEF. As described in (15), FPE contains transmitted symbol and it can be used as a training sequence. Proposed method consists of four functional blocks. First part functions FPEF to produce FPE, second part estimates channel coefficient, $\mathbf{h}(0)$, third part choose best decision delay, and fourth part is fractionally-spaced linear equalizers (FSLE).

From the covariance matrix of FPE in (23), its estimation of adaptive manner is given by

$$\mathbf{F}(n)=\lambda\mathbf{F}(n-1)+\mathbf{f}_N(n)\mathbf{f}_N^H(n) \tag{25}$$

Compared with (23), $\mathbf{h}(0)$ is the column of $\mathbf{F}(n)$ with the largest norm. We can use either of the following equations to obtain the ZF- and MMSE-equalizer outputs[6][7].

$$\begin{aligned}\hat{s}_{\text{MMSE}}(n)&=\sigma_s^2\mathbf{h}^H(0)\mathbf{F}^{-1}(n)\mathbf{f}_N(n)\\ \hat{s}_{\text{ZF}}(n)&=\frac{\mathbf{h}^H(0)}{\|\mathbf{h}(0)\|^2}\mathbf{f}_N(n)\end{aligned} \tag{26}$$

It should be noted that the MMSE equalizer is designed for transmitted symbol recovery at specific decision delay. Thus, different decision delay can result in different performance. A recursive form to get best decision delay is discussed in [8], [10], and [14]. To get best decision delay choice, [8] and [10] propose the minimizing MSE is given by

$$J_{\mathrm{MSE}}(D)=1-\mathbf{H}^{H}(D)\mathbf{R}^{+}\mathbf{H}(D) \qquad (27)$$

where $\mathbf{H}(D)$ is the $(D+1)$th block column of the channel convolution matrix $\mathbf{H}$ and $\mathbf{R}$ is the autocorrelation matrix of oversampled received signal. But it is not very useful because $\mathbf{H}$ is unknown. If the transmitted symbols have constant modulus (CM), which is practical case in digitally modulated signal such as QAM or PSK, the best decision delayed blind equalizer can be determined by the following CM index [14]

$$J_{\mathrm{CM}}(D)=\sum(|\mathbf{g}_{D}^{H}\mathbf{x}_{N}(n)|^{2}-1)^{2} \qquad (28)$$

The blind equalizer having the smallest J_{MSE} or J_{CM} value will be considered as the best decision delayed blind equalizer. In many practical channels, it has been observed [2] that selecting $D\approx(N+L)/2$ results in good performance. For given MMSE- or CM-sense optimized decision delay D, we can get training-like sequence, $t(n)$, as following

$$\begin{aligned} t_{\mathrm{ZF}}(n)&=\hat{s}_{\mathrm{ZF}}(n-D)\\ t_{\mathrm{MMSE}}(n)&=\hat{s}_{\mathrm{MMSE}}(n-D) \end{aligned} \qquad (29)$$

The output of FSLE is given by

$$y(n)=\sum_{k=0}^{P-1}\mathbf{c}_{k}^{H}(n)\mathbf{x}_{k}(n) \qquad (30)$$

where $\mathbf{c}_k(n)$ is the equalizer coefficients of the kth subchannel and $\mathbf{x}_k(n)$ is the input vector of the kth subchannel

$$\begin{aligned} \mathbf{c}_{k}(n)&=[c_{k,0}(n),c_{k,1}(n),\cdots,c_{k,N-1}(n)]^{T}\\ \mathbf{x}_{k}(n)&=[x_{k}(n),x_{k}(n-1),\cdots,x_{k}(n-N+1)]^{T} \end{aligned} \qquad (31)$$

We can use the LMS algorithm to update the equalizer coefficients as following

$$\mathbf{c}_{k}(n+1)=\mathbf{c}_{k}(n)+\mu e^{*}(n)\mathbf{x}_{k}(n),\ \text{for } k=0,\cdots,P-1 \qquad (32)$$

where $e(n)$ is either $t_{\mathrm{ZF}}(n)-y(n)$ for ZF criterion or $t_{\mathrm{MMSE}}(n)-y(n)$ for MMSE criterion.

Using estimated symbol, we regard the estimated symbol as the desired signal for basic adaptive identification block. We estimated the delay-optimized symbol, $\hat{s}(n-D)$. For this applications of the adaptive channel estimation, the adaptation algorithm can be implemented only with a delay in the coefficient update. This so-called delay LMS (DLMS) algorithm is given by

$$\begin{aligned} \mathbf{w}_{k}(n+1)&=\mathbf{w}_{k}(n)+\mu e_{k}^{*}(n-D)\mathbf{x}_{k}(n-D)\\ e_{k}(n-D)&=\hat{s}(n-D)-\mathbf{w}_{k}^{H}(n-D)\mathbf{x}_{k}(n-D) \end{aligned} \qquad (33)$$

where

$$\mathbf{w}_{k}(n)=[w_{k}(n),\cdots,w_{k}(n-L+1)]^{T},k=0,\cdots,P-1 \qquad (34)$$

IV. Simulation Results

In this section, we use computer simulations to evaluate the performance of the proposed algorithm. The source symbols are drawn from a 16-QAM constellation with a uniform distribution. The noise is drawn from a white Gaussian distribution at a varying SNR. As shown in Figure 1, we can define the SNR as follows

$$\mathrm{SNR}=E\left[\sum_{j=0}^{P-1}|a_{j}(n)|^{2}\right]\Bigg/E\left[\sum_{k=0}^{P-1}|v_{k}(n)|^{2}\right] \qquad (35)$$

As a performance index, we estimate the MSE, which is defined in [6].

All results concerning MSE are ensemble averages of 50 independent Monte Carlo runs. Algorithm initialization parameters are $\delta=10^{-3}$, $\lambda=0.995$, and $\mu=0.005$. The number of subchannels is set to $P=2$. For all simulations, we use the RLS algorithm for updating FPEF coefficient matrix and use MSE criteria in (29) and (32). The simulated channel is a length-16 version of an empirically measured $T/2$-spaced digital microwave radio channel ($P=2$) with 230 taps, which we truncated to obtain a channel with $L=8$. The Microwave channel *chan1.mat* is founded at http://spib.rice.edu/spib/microwave.html. The shortened version is derived by linear decimation of the FFT of the full-length $T/2$-spaced impulse response and taking the IFFT of the decimated version (see [16] for more details on this channel). In Fig. 2, we show the $J_{\mathrm{MSE}}(D)$ and $J_{\mathrm{CM}}(D)$ versus decision delay D under SNR=30dB. The equalized received signal constellation plots are shown in Fig. 3 for an SNR of 20dB and 30dB. We set FPEF length to 8, equalizer length N=8, and choose the optimum delay, D=7 for this simulation. We

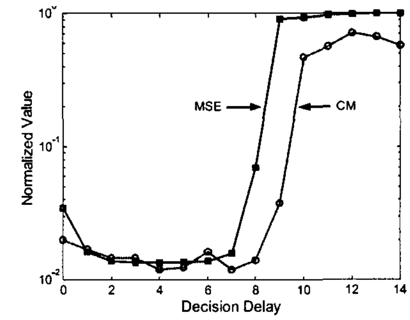

Fig. 2. The best decision delay choice rule.

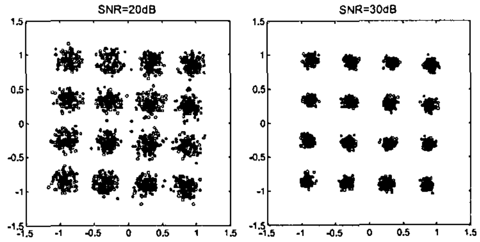

Fig. 3. Scatter plots after equalization.

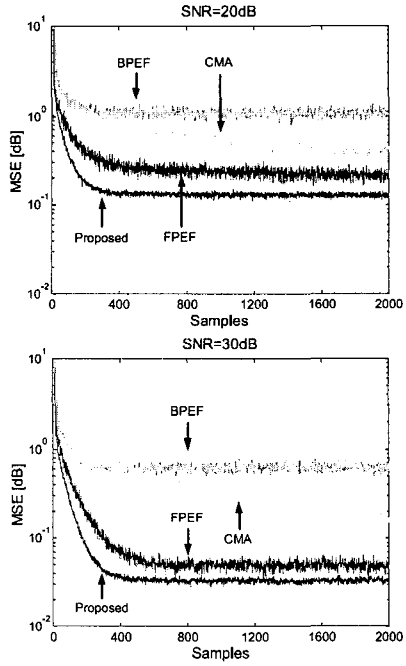

Fig. 4. The MSE comparison under SNR=20dB and SNR=30dB.

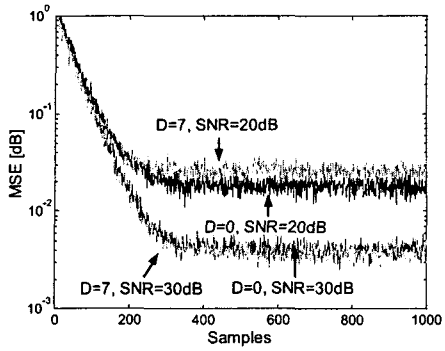

Fig. 5. The Mean Square estimation Error.

compare the performance of the proposed algorithm with some existing algorithms: the constant modulus algorithm in [1] (denotes CMA), the one-step FPEF-based algorithm in [6] and [7] (denotes FPEF), and the one-step BPEF-based algorithm in [6] and [7] (denotes BPEF). We set both FPEF length and FSLE order to 8 for proposed algorithm. Let the equalizer order be N=18 for CMA, N=8 for FPEF, and N=8 for BPEF. Fig. 4 shows the MSE curves for the proposed algorithm and existing algorithms under SNR=20 dB and 30dB, respectively. For channel estimation, we set estimator order be L=8. Fig. 5 shows the MSE curves for channel estimation using DLMS algorithm with D=7 and D=0. Fig. 6 shows 50 estimates of the channel under SNR=20dB. In this figures, solid line denotes the original channel, dotted line denotes the averaged estimates±standard deviation, and the circle symbol represents the mean value of the 50 estimates.

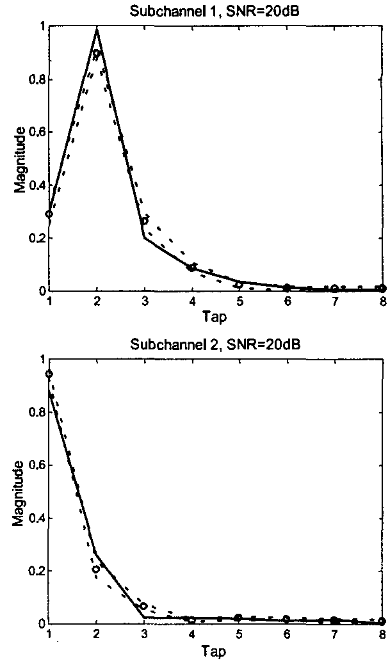

Fig. 6. The estimated channel under SNR=20dB.

V. CONCLUSION

We have developed adaptive blind equalization based on one-step FPEF with arbitrary decision delay control and channel estimation. Our proposed method ensures flexible decision delay control and provides flexibility for a practical implementation since various well-known adaptive algorithms, including RLS and LMS algorithm, can be used to implement the proposed method. We consider FPE as training sequence and utilize it for arbitrary decision delay blind equalization and channel estimation. For symbol estimation, compared with HOS-based algorithm such as CMA or cumulant algorithm, proposed method is based on SOS; thus faster convergence can be achieved with little computational complexity. The weakness of the proposed method lies as well; the magnitude of the first channel coefficient, **h**(0), should be sufficiently large. Further research on the effect of this fact is needed. This aspect faces also to previous PEF-based blind equalization problem. For channel estimation, the proposed method seems to be more efficient in a low SNR channel and much more accurate.

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