# MATHEMATICAL MODELING OF VSB-BASED DTV CHANNELS

*Hyoung-Nam Kim, Yong-Tae Lee, and Seung Won Kim*

Broadcasting System Research Department
Electronics and Telecommunications Research Institute (ETRI)
e-mail: kimhn@etri.re.kr

## ABSTRACT

We analyze mathematically a VSB (vestigial side-band) transceiver system for the Advanced Television Systems Committee (ATSC) digital television standard and extract a near-baseband equivalent VSB channel model. This model shows the multi-path fading effect of the quadrature component on the in-phase component. Also, we obtain a simplified model of the VSB transceiver system, which is represented by convolution of the transmission signal (before modulation) and the VSB channel. This simplified model is efficiently used for simulation of VSB systems to improve its performances, especially in an equalization part. Applying the DTV channel specifications tested by the Advanced Television Test Center (ATTC) to the channel model, we obtain an equivalent VSB channel and show the equalization result by using the conventional decision-feedback equalizer (DFE).

## I. INTRODUCTION

In the terrestrial digital television (DTV) standard of the Advanced Television Systems Committee (ATSC), 8-vestigial sideband (VSB) has been adopted as a modulation method [1][2]. Compared to the coded orthogonal frequency division multiplexing (COFDM) modulation of the European and Japanese DTV standard, the VSB transmission system has an advantage of wide coverage area but suffer from multi-path fading [3]. To solve the problem, there have been many efforts in various ways. One of the major things is to develop an efficient equalization method [4].

Since the VSB modulation has an asymmetrical property, the VSB-modulated signal includes quadrature components as well as in-phase ones. This fact yields that the in-phase and quadrature components affect each other under multi-path fading channels. To compromise such channel effects, the equalizer has to suppress both the in-phase and quadrature multi-path fading effects appeared in the in-phase signal.

In this paper, we analyze mathematically the 8-VSB transceiver system for the ATSC standard and extract a near-baseband equivalent channel model.

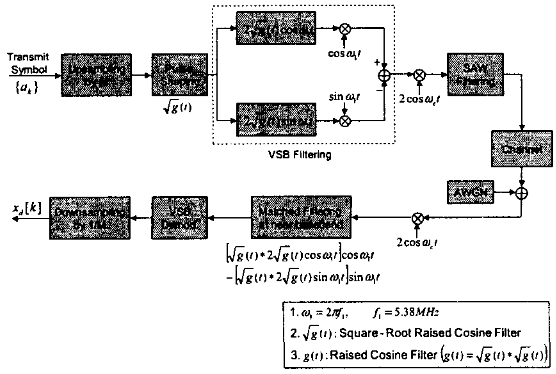



Figure 1: VSB simulation model.

This is efficiently used for simulations of the VSB system to improve its performances, especially in an equalization part. Applying some channel information specified by the Advanced Television Test Center (ATTC) to our mathematical channel model, we obtain an equivalent VSB channel. Then, the channel output signal is equalized by the conventional decision-feedback equalizer (DFE) and the results are shown.

## II. SYSTEM DESCRIPTION AND MODELING OF THE 8-VSB SYSTEM

A functional block diagram of an VSB system is shown in Fig. 1. In this section, we explain the role of each block and simplify the VSB transceiver channel model.

### II-A. Pulse shaping

Letting $\{a_k\}$ and $s(t)$ be an information symbol sequence and a continuous-time signal comprised of the information symbols, respectively, we obtain

$$s(t) = \sum_{k=0}^{\infty} a_k \delta(t - kT), \qquad (1)$$

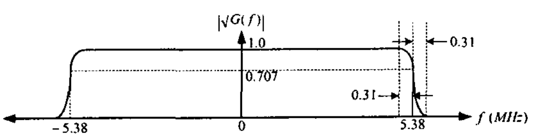



Figure 2: Spectrum of a square-root-raised-cosine filter.

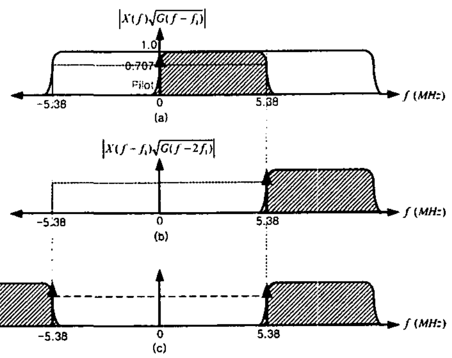



Figure 3: VSB filtering and up-converting (a) VSB filtering $(x(t) * \sqrt{g}(t)e^{j\omega_1 t}$, where $\omega_1 = 2\pi \times 5.38$ MHz), (b) up-converting to near-baseband frequency $([x(t) * \sqrt{g}(t)e^{j\omega_1 t}]\, e^{j\omega_1 t})$, and (c) extracting the real part for transmission $(2\mathrm{Re}\left\{[x(t) * \sqrt{g}(t)e^{j\omega_1 t}]\, e^{j\omega_1 t}\right\})$.

where $t = \frac{nT}{M}$ $(n = 0, 1, 2, \cdots)$[1]. In order to avoid inter-symbol interference (ISI), pulse-shaping filters (PSF) satisfying the Nyquist criterion are used at the transmitter [5]. For such a PSF, we use a raised-cosine (RC) filter comprised of a pair of square-root-raised-cosine (SQRC) filters at the transmitter and the receiver [2]. Note that the SQRC filter at the receiver has a role in matched-filtering. Let $g(t)$ and $\sqrt{g}(t)$ denote a RC filter and the corresponding SQRC filter whose spectrum is shown in Fig. 2. These two functions have the relationship of

$$g(t) = \sqrt{g}(t) * \sqrt{g}(t), \tag{2}$$

where $*$ denotes a convolution operator. The pulse-shaped signal $x(t)$ represented by

$$x(t) = s(t) * \sqrt{g}(t). \tag{3}$$

### II-B. VSB filtering and Up-converting

The pulse-shaped signal is modulated by the VSB filter and then up-converted with the near-baseband frequency of 5.38 MHz. Fig. 3 shows the procedure of VSB filtering and up-converting in frequency domain. The resulting signal $x_v(t)$ can be obtained as

$$\begin{aligned} x_v(t) &= \mathrm{Re}\left\{\left[x(t) * 2\sqrt{g}(t)e^{j\omega_1 t}\right] e^{j\omega_1 t}\right\} \\ &= x_I(t)\cos\omega_1 t - x_Q(t)\sin\omega_1 t, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \omega_1 &= 2\pi f_1 = 2\pi \times 5.38(\mathrm{Mhz}), \\ x_I(t) &= x(t) * 2\sqrt{g}(t)\cos\omega_1 t, \\ x_Q(t) &= x(t) * 2\sqrt{g}(t)\sin\omega_1 t. \end{aligned} \tag{5}$$

[1]We use a discrete-time model for analog representation to simulate transceiver systems. To achieve it, the information symbols are over-sampled by inserting $M-1$ zero symbols. In that case $M$ becomes the over-sampling factor of the symbol frequency

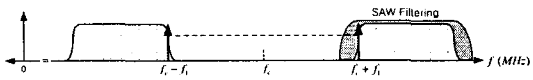



Figure 4: Up-converting to passband and SAW filtering

### II-C. Up-converting to passband and SAW filtering

The VSB modulated signal is up-converted with the carrier frequency of $\omega_c$ by multiplying $2\cos\omega_c t$. It follows that a SAW filter extracts only upper-sideband components of the up-converted signal. The process is shown in Fig. 4 and is described as follows:

$$\begin{aligned} x_s(t) &= F_{SAW}\left\{x_v(t) \times 2\cos\omega_c t\right\} \\ &= x_I(t)\cos(\omega_c + \omega_1)t - x_Q(t)\sin(\omega_c + \omega_1)t, \end{aligned} \tag{6}$$

where $F_{SAW}\{\cdot\}$ denotes a passband filtering with the SAW filter.

### II-D. Multi-path Fading Channel

Consider linear and time-invariant FIR channels. Multi-path fading is the major cause of channel distortion. We consider three factors for the multi-path fading effect, such as:

- $\alpha_i$ : attenuation constant at $i$th path
- $\theta_i$ : phase shift at $i$th path
- $\tau_i$ : relative time delay at $i$th path

The output of the channel is given by

$$\begin{aligned} x_c(t) &= \sum_{i=-N_1}^{N_2} \alpha_i x_I(t-\tau_i)\cos\left[(\omega_c+\omega_1)(t-\tau_i)+\theta_i\right] \\ &\quad - \sum_{i=-N_1}^{N_2} \alpha_i x_Q(t-\tau_i)\sin\left[(\omega_c+\omega_1)(t-\tau_i)+\theta_i\right] \\ &\quad + w(t) \\ &= \sum_{i=-N_1}^{N_2} \alpha_i\left[x_I(t-\tau_i)\cos\phi_i - x_Q(t-\tau_i)\sin\phi_i\right] \end{aligned}$$

$$\times \cos(\omega_c + \omega_1)t$$
$$- \sum_{i=-N_1}^{N_2} \alpha_i \left[x_Q(t-\tau_i)\cos\phi_i + x_I(t-\tau_i)\sin\phi_i\right]$$
$$\times \sin(\omega_c+\omega_1)t$$
$$+ w(t), \quad (7)$$

where

$$\phi_i = \theta_i - (\omega_c + \omega_1)\tau_i, \quad (8)$$

$N_1$ and $N_2$ are the numbers of pre-cursors and post-cursors of the channel, respectively, and $w(t)$ is a white Gaussian noise process with power spectral density of $\sigma_n^2$.

### II-E. Down-converting to near baseband

At the receiver, the received signal distorted by the channel is first down-converted to near baseband prior to matched filtering. Let $\varepsilon$ be a phase difference between the carrier frequency and the oscillator frequency of the synchronous detector. Using the oscillator, we obtain the near-baseband received signal $x_r(t)$ given by

$$\begin{aligned}
x_r(t) &= x_c(t)\cdot 2\cos(\omega_c t - \varepsilon) \\
&= \sum_{i=-N_1}^{N_2} \alpha_i \cos(\phi_i + \varepsilon) \\
&\quad \times \left[x_I(t-\tau_i)\cos\omega_1 t - x_Q(t-\tau_i)\sin\omega_1 t\right] \\
&\quad - \sum_{i=-N_1}^{N_2} \alpha_i \sin(\phi_i+\varepsilon) \\
&\quad \times \left[x_Q(t-\tau_i)\cos\omega_1 t + x_I(t-\tau_i)\sin\omega_1 t\right] \\
&\quad + w(t)\cdot 2\cos(\omega_c t - \varepsilon) \qquad (9)
\end{aligned}$$

### II-F. Matched Filtering

Under white noise condition, matched filtering maximizes the SNR (signal to noise ratio) of an input signal [5]. We compute the output of the matched filter at near-baseband. The impulse response of the filter is given by

$$\begin{aligned}
\sqrt{g}_M(t) &= (\sqrt{g}(t) * 2\sqrt{g}(t)\cos\omega_1 t)\cos\omega_1 t \\
&\quad - (\sqrt{g}(t) * 2\sqrt{g}(t)\sin\omega_1 t)\sin\omega_1 t.
\end{aligned}$$

Using (3) and (4), it can be easily shown that the output of the matched filter becomes

$$\begin{aligned}
x_m(t) &= x_r(t) * g_M(t) \\
&= \sum_{i=-N_1}^{N_2} \alpha_i \cos(\phi_i+\varepsilon) \\
&\quad \times [\{s(t-\tau_i) * g_{MI}(t)\}\cos\omega_1 t \\
&\quad - \{s(t-\tau_i) * g_{MQ}(t)\}\sin\omega_1 t] \\
&\quad - \sum_{i=-N_1}^{N_2} \alpha_i \sin(\phi_i+\varepsilon) \\
&\quad \times [\{s(t-\tau_i) * g_{MQ}(t)\}\cos\omega_1 t \\
&\quad + \{s(t-\tau_i) * g_{MI}(t)\}\sin\omega_1 t] \\
&\quad + w(t)\cdot 2\cos(\omega_c t-\varepsilon) * \sqrt{g}_M(t), \qquad (10)
\end{aligned}$$

where

$$\begin{aligned}
g_{MI}(t) &= g(t) * 2g(t)\cos\omega_1 t, \qquad (11)\\
g_{MQ}(t) &= g(t) * 2g(t)\sin\omega_1 t. \qquad (12)
\end{aligned}$$

### II-G. VSB demodulation

The matched filtered output is demodulated by down-conversion to the baseband and low-pass filtering. The demodulated signal is

$$\begin{aligned}
x_d(t) &= x_m(t)\cdot 2\cos\omega_1 t \\
&= \sum_{i=-N_1}^{N_2} \alpha_i\cos(\phi_i+\varepsilon)\cdot[s(t-\tau_i) * g_{MI}(t)] \\
&\quad - \sum_{i=-N_1}^{N_2} \alpha_i \sin(\phi_i+\varepsilon)\cdot[s(t-\tau_i) * g_{MQ}(t)] \\
&\quad + w(t) * \sqrt{g}(t) * 2\sqrt{g}(t)\cos\omega_1 t, \qquad (13)
\end{aligned}$$

where we have used the approximation of

$$w(t)\cdot 2\cos(\omega_c t - \varepsilon) \approx w(t)$$

because $w(t)$ is a white process.

Letting

$$\beta_i = \alpha_i\cos(\phi_i+\varepsilon) \quad \text{and} \quad \gamma_i = \alpha_i \sin(\phi_i+\varepsilon),$$

using (11) and the relationship of

$$s(t-\tau_i) = s(t) * \delta(t-\tau_i),$$

and then rearranging in (13), we obtain

$$\begin{aligned}
x_d(t) &= \sum_{i=-N_1}^{N_2} [\beta_i h_I(t-\tau_i) - \gamma_i h_Q(t-\tau_i)] \\
&\quad * g(t) * s(t) \\
&\quad + w(t) * \sqrt{g}(t) * 2\sqrt{g}(t)\cos\omega_1 t, \qquad (14)
\end{aligned}$$

where $\delta(t)$ is the Dirac-Delta function and

$$\begin{aligned}
h_I(t) &= 2g(t)\cos\omega_1 t \qquad (15)\\
h_Q(t) &= 2g(t)\sin\omega_1 t. \qquad (16)
\end{aligned}$$

So, we can find the real impulse response of the baseband equivalent VSB channel model $h_R(t)$ by rewrit ing (14) as

$$x_d(t) = h_R(t) * g(t) * s(t) + \sqrt{g}(t) * 2\sqrt{g}(t)\cos\omega_1 t * w(t), \qquad (17)$$

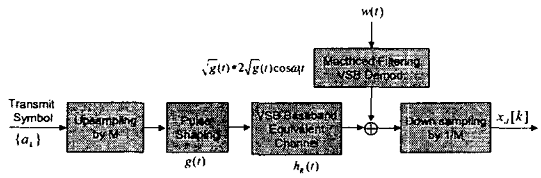



Figure 5: Baseband equivalent VSB simulation model.

where

$$h_R(t) = \sum_{i=-N_1}^{N_2} [\beta_i h_I(t - \tau_i) - \gamma_i h_Q(t - \tau_i)] . \quad (18)$$

The discrete-time form of the equivalent channel is obtained by uniformly sampling $h_R(t)$ at an integer fraction of the symbol period, $T/M$ and is given by

$$h_R[n] = \sum_{i=-N_1}^{N_2} (\beta_i h_I[n - d_i] - \gamma_i h_Q[n - d_i]) . \quad (19)$$

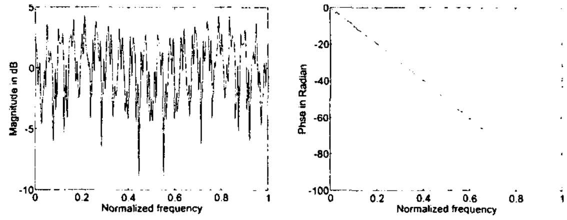
Fig. 5 shows the simplified block diagram based on the equivalent VSB model.

## III. SIMULATION RESULTS

We present a simulation result using the equivalent VSB model whose channel is obtained by introducing channel information specified by the ATTC [6]. The channel is an ensemble of five echoes with delays, amplitudes, and phases and is shown in Table 1. Fig. 6 gives the amplitude and phase responses of the channel. Note that the channel exhibits deep fading frequencies. The equalizer input SNR for the 8-VSB scheme is 20 dB. We use a symbol-spaced DFE(64,192) (64 forward taps and 19 feedback taps) in the training mode (2-VSB). The eye-diagrams before and after equalization are shown in Fig. 7. Though the DFE opens an eye, the margin is not small enough to make error-free decisions. This means that more efficient equalization schemes are required to improve the performance of 8-VSB receivers for severe channels.

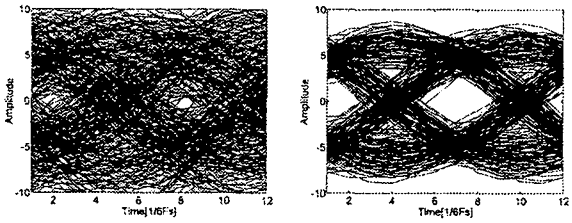
Table 1: Static Multiphth used by ATTC

| Delay ($\mu s$) | Amplitude (dB) | Phase (degree) |
|---|---|---|
| -1.75 | -20 | 45 |
| +0.197 | -20 | 167 |
| +1.80 | -10 | 25 |
| +5.75 | -14 | 66 |
| +17.95 | -18 | 225 |




(a) (b)

Figure 6: (a) Amplitude response. (b) Phase response.




(a) (b)

Figure 7: Eye diagram (a) Equalizer input signal. (b) Equalizer output signal.

## IV. CONCLUSION

The equivalent VSB channel model has been derived by mathematically analyzing the VSB transceiver system for the ATSC DTV starndard. Since the VSB is an asymmetrical modulation method, though channel information such as delays, amplitudes, and phases is given, the impulse response of the corresponding channel cannot be directly shown. In this paper, however, we show that with the derived model it can be easily obtained from such channel information. Therefore, the equivalent VSB model is very useful for simulations of the VSB system to improve its performances, especially in an equalization part.

## REFERENCES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