Pseudo-Distance Map Based Watersheds for Robust Region Segmentation

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Abstract

In this paper, we present a robust region segmentation method based on the watershed transformation of a pseudo-distance map (PDM). A usual approach for the segmentation of a gray-scale image with the watershed algorithm is to apply it to a gradient magnitude image or the Euclidean distance map (EDM) of an edge image. However, it is well known that this approach suffers from the oversegmentation of the given image due to noisy gradients or spurious edges caused by a thresholding operation. In this paper, we show that applying the watershed algorithm to the PDM, which is a regularized version of the EDM and is directly computed from the edge-strength function (ESF) of the input image, significantly reduces the oversegmentation, and the final segmentation results obtained by a simple region-merging process are more reliable and less noisy than those of the gradient- or EDM-based methods. We also propose a simple and efficient region-merging criterion considering both boundary strengths and inner intensities of regions to be merged. The robustness of our method is proven by testing it with a variety of synthetic and real images.

1. Introduction

Image segmentation is one of the most important processes for image understanding and analysis such as recognition, visualization, and object-based compression. Therefore, many segmentation techniques have been proposed. Among them, the *watershed transformation* is known as a very powerful segmentation tool in many applications [2], [5]. A typical approach for segmenting a gray-scale image with the watershed transformation is to make use of its gradient image as an input to the transformation since high gradients constitute watershed lines that correspond to the region boundaries of the gray-scale image. This method is called the *gradient watershed.* In another approach [5], the *Euclidean distance map* (EDM) constructed from an edge image is utilized since the watershed algorithm has a characteristic of separating connected or overlapping blobs when applied to the corresponding EDM. However, the above approaches usually give rise to a serious *oversegmentation* problem due to noisy gradients or spurious edges caused by the thresholding operation.

In this paper, we propose a new segmentation method based on the watershed transformation of a *pseudo-distance map* (PDM), which provides initial segmentation results better than the conventional methods. A PDM can be thought of as a regularized version of an EDM. While the EDM is obtained from an edge image, the PDM is directly computed from the *edge-strength function* (ESF) of a given gray-scale image without thresholding it. The value of the PDM is almost equal to zero where edge strength is relatively large, and it has nearly constant slopes at the points with small edge strength except at the positions where two opposite slopes meet, which in fact, correspond to the skeleton of the input image. Initial segmentation results are obtained by applying the watershed transformation to the inverted PDM. Due to the full utilization of edge strength information and the regularization effect of the variational formulation, the watershed transformation of the inverted PDM usually produces more reliable and less noisy initial segmentation results than the gradient- or EDM-based methods while preserving the advantage of the EDM-based method. Through several experiments, we show that our region-merging method together with our initial segmentation method gives robust and reliable final segmentation results.

2. Pseudo-Distance Map

In our previous work [4], we proposed a pseudo-distance map for extracting skeletons from gray-scale images without region segmentation or edge detection. In this subsection, we briefly introduce how to compute a PDM from a given ESF.

2.1. One dimensional formulation

We begin by introducing an energy functional that is minimized to obtain a PDM in one dimension. Assuming that an ESF $v(x)$ is given, which ranges from 0 to 1 and monotonically increases as an edge at x gets stronger, the functional is given by

$$E(f) = \int \underbrace{\alpha v[f^2 + f_x^2 + (b - f_{xx})^2]}_{(A)} + \underbrace{\beta(a^2 - f_x^2)^2}_{(B)} dx, \tag{1}$$

where α, β, a, and b are positive constants and f is the PDM to be computed. The functional works as follows: If v is small, the minimization of E will be dominated by the term (B), which means that the slope of f will approach a or $-a$. If v is large, the term (A) will also affect the minimization of E and the constraints on f will work; that is, the magnitudes of f and f_x should become zero and f_{xx} should approach the positive constant b. These constraints make f have local minima of nearly zero at the positions where v is large. Therefore, the overall shape of f will become similar to that of an EDM after the minimization of E.

The function f that minimizes the functional E can be computed by the variational method if the initial form of f is appropriately given. Note that f_{xx} in Equation (1) causes the fourth-order derivative of f in the corresponding PDE, which tends to make the PDE noise-sensitive. To avoid the use of f_{xx}, we introduce a new function g, which approximates f_x, and incorporate it with Equation (1):

$$\begin{aligned} E(f,g) &= \int \alpha v[f^2 + f_x^2 + (b - g_x)^2] + \beta(a^2 - f_x^2)^2 \\ &+ (g - f_x)^2 \, dx. \end{aligned} \tag{2}$$

Equation (2) gives rise to two PDEs

$$\begin{aligned} f_t &= \alpha[(vf_x)_x - vf] + 2\beta f_{xx}(3f_x^2 - a^2) \\ &+ (f_{xx} - g_x), \end{aligned} \tag{3}$$

$$g_t = \alpha[(vg_x)_x - bv_x] + (f_x - g). \tag{4}$$

We implemented the above PDEs using central finite-difference approximations and solved them with the initial value of f set to $1 - v$.

2.2. Extension to two dimensions

The extension of Equation (2) to two dimensions is straightforward:

$$\begin{aligned} E(f,g,h) &= \int \alpha v[f^2 + f_x^2 + f_y^2 + (b - g_x)^2 \\ &+ \frac{1}{2}(g_y + h_x)^2 + (b - h_y)^2] \\ &+ \beta(a^2 - ||\nabla f||^2)^2 \\ &+ (g - f_x)^2 + (h - f_y)^2 \, dxdy, \end{aligned} \tag{5}$$

where

$$g \approx f_x \quad \text{and} \quad h \approx f_y. \tag{6}$$

Consequently,

$$f_{xx} \approx g_x, \; f_{yy} \approx h_y, \text{ and } f_{xy} \approx \frac{1}{2}(g_y + h_x). \tag{7}$$

The corresponding PDEs are

$$\begin{aligned} f_t &= \alpha(\nabla \cdot v\nabla f - vf) \\ &+ 2\beta[(||\nabla f||^2 - a^2)\nabla^2 f \\ &+ 2(f_x^2 f_{xx} + 2f_x f_y f_{xy} + f_y^2 f_{yy})] \\ &+ (\nabla^2 f - g_x - h_y), \end{aligned} \tag{8}$$

$$\begin{aligned} g_t &= \alpha\{2[v(g_x - b)]_x + [v(g_y + h_x)]_y\} \\ &+ 2(f_x - g), \end{aligned} \tag{9}$$

$$\begin{aligned} h_t &= \alpha\{[v(g_y + h_x)]_x + 2[v(h_y - b)]_y\} \\ &+ 2(f_y - h). \end{aligned} \tag{10}$$

To speed up the convergence and avoid falling into the local extrema, we add a new term $\gamma\phi\nabla^2 f$ to Equation (8), where ϕ is given by

$$\phi = \begin{cases} 1 & \text{if } \{f < \epsilon_1\} \text{ or } \{|\det(\mathbf{H})| < \epsilon_2, \\ & \quad \text{trace}(\mathbf{H}) > \epsilon_3, v < \epsilon_4\}. \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where $\epsilon_1 \leq 0$, $\epsilon_2 > 0$, $\epsilon_3 > 0$, $\epsilon_4 > 0$, and their absolute values are very small. The matrix $\mathbf{H}$ is a Hessian matrix of f, which is given by

$$\mathbf{H} = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{bmatrix} \approx \begin{bmatrix} g_x & \frac{1}{2}(g_y + h_x) \\ \frac{1}{2}(g_y + h_x) & h_y \end{bmatrix}. \tag{12}$$

During the iterative computation of f, the new term activates only when $f(x,y)$ has a negative value at (x,y) or has a valley shape (i.e., local minimum) where $v(x,y)$ is very small. This term makes f escape from the two cases by smoothing it out where they occur. Note that the two eigenvalues λ_1 and λ_2 ($|\lambda_1| \leq |\lambda_2|$) of the Hessian matrix of $f(x,y)$ correspond respectively to the minimum and maximum second-order directional derivatives (i.e., curvatures) of f at (x,y). Therefore, at the valleys of f, the following condition is usually satisfied:

$$\det(\mathbf{H}) = \lambda_1 \cdot \lambda_2 \approx 0 \text{ and } \text{trace}(\mathbf{H}) = \lambda_1 + \lambda_2 \approx \lambda_2 > 0. \tag{13}$$

At first glance, our method seems to involve many parameters that need to be adjusted. However, considering the role of each parameter, one can see that the parameters γ, ϵ_1, ϵ_2, ϵ_3, ϵ_4, a, and b are independent of the given ESFs. Therefore, once the parameter values selected are proven to be suitable for convergence, they are also suitable for other ESFs. Note that just as in the case of the ESF, we can control the smoothness of the resulting PDM by adjusting α and β. As α/β gets smaller, the more the details of the PDM are smoothed out.

3. Robust Image Segmentation Based on a Pseudo-Distance Map

In this section, we introduce the overall procedure of our region segmentation method together with our new region-merging criterion. The procedure consists of the following four steps:

1. An ESF is computed from an input gray-scale image using the Ambrosio and Tortorelli's method [1].
2. A PDM is computed from the ESF by numerically solving the PDEs introduced in Section 2.
3. An initial region segmentation result is obtained by applying the watershed transformation to the inverted PDM. We used Vincent and Soille's watershed algorithm [5].
4. Region-merging is carried out with our new merging criterion.

For region-merging, we developed a new dissimilarity function appropriately combining the edge strength and region intensity information. Let R_i be the set of pixels belonging to a region i, and let Γ_i and Γ_{ij} represent the set of boundary pixels of the region i and the set of pixels belonging to the common boundary between the regions i and j, respectively. If the two regions i and j are adjacent to each other and $||R_i|| \leq ||R_j||$, where $||R||$ represents the cardinality of a set R, the proposed function measuring the dissimilarity between the regions i and j is given by

$$\delta(i,j) = \kappa \frac{||\Gamma_{ij}||}{||\Gamma_i||} E(\Gamma_{ij}) + \left(1 - \frac{||\Gamma_{ij}||}{||\Gamma_i||}\right) |\mu(R_i) - \mu(R_j)|, \quad (14)$$

where $E(R)$ and $\mu(R)$ are the average edge strength and intensity values of pixels belonging to R, and κ is a scaling constant.

4. Experimental Results

To show the usefulness of our method, we compared the performance of our method with that of the method proposed by Haris *et al.* [3]. Their method consists of four stages. The first stage is the reduction of the noise corrupting the original image while preserving its *meaningful structures*, which is based on the homogeneity/heterogeneity assumption for the image regions. At the second stage, Gaussian gradients are calculated and their magnitudes below a certain threshold are set to zero. At the next stage, the resulting thresholded gradient image is fed into the watershed algorithm, which produces an initial segmentation result. At the final stage, an iterative region-merging algorithm is applied to the watershed regions to produce a final segmentation result. For a fair comparison between the two methods, we applied the same noise reduction technique proposed by Haris *et al.* to an input image before obtaining the ESF or Gaussian gradients. We also used the same watershed algorithm proposed by Vincent and Soille [5] and the same dissimilarity function of Equation (14), where we normalized the gradient magnitudes to use the dissimilarity function in

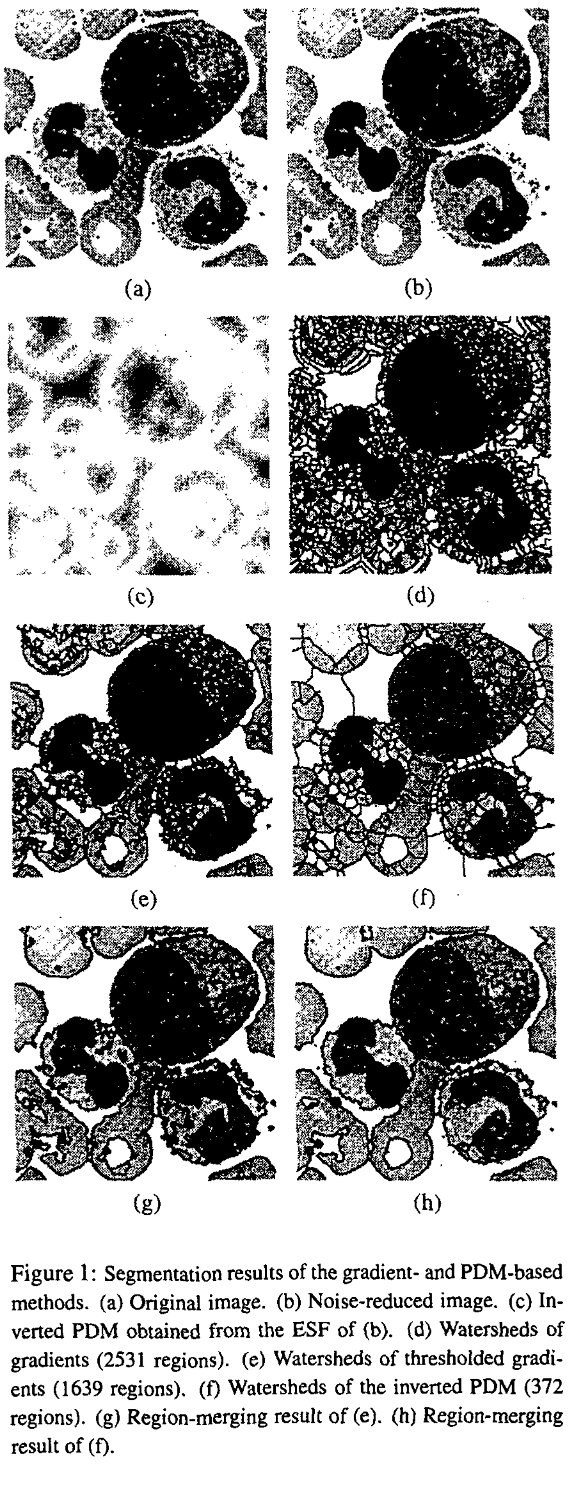

Figure 1: Segmentation results of the gradient- and PDM-based methods. (a) Original image. (b) Noise-reduced image. (c) Inverted PDM obtained from the ESF of (b). (d) Watersheds of gradients (2531 regions). (e) Watersheds of thresholded gradients (1639 regions). (f) Watersheds of the inverted PDM (372 regions). (g) Region-merging result of (e). (h) Region-merging result of (f).

the Haris *et al.* method. Therefore, the biggest difference between the methods is the input to the watershed transformation. In our method, the PDM is used instead of the

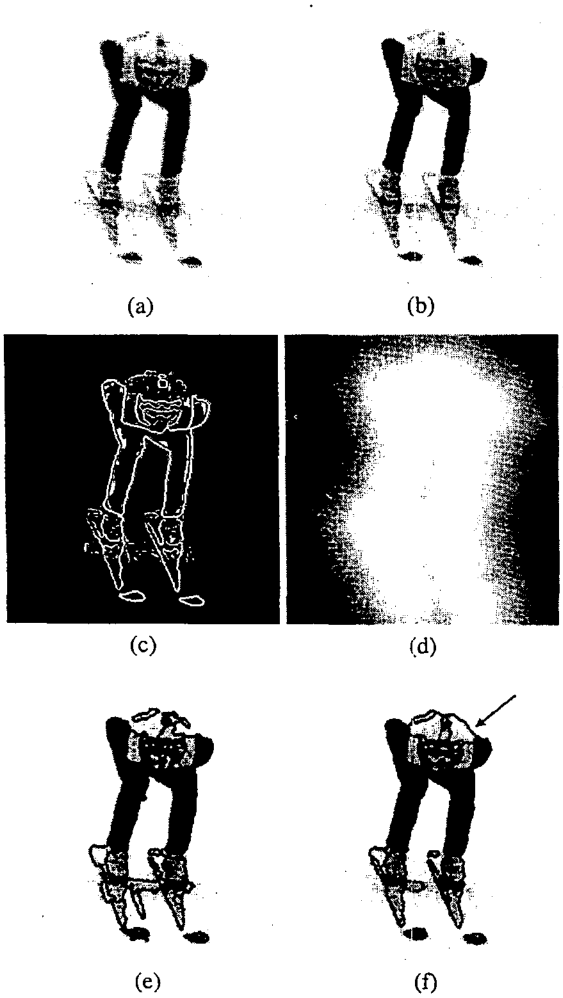

Figure 2: Other segmentation results of the gradient- and PDM-based methods. (a) Original image. (b) Noise-reduced image. (c) ESF obtained from (b). (d) Inverted PDM obtained from (c). (e) Final segmentation result based on thresholded gradients. (f) Final segmentation result based on the PDM.

thresholded gradient image.

From the results of Figure 1, one can see that although the Haris *et al.* method significantly reduces the number of initial partitions, it still produces a large number of partitions when compared to our method. In addition, the final segmentation results show that the proposed PDM-based segmentation method provides region boundaries that are less noisy and more accurate than the method using the thresholded gradients.

Figure 2 shows other segmentation results of the gradient and the PDM-based methods. The ESF of Figure 2(c) was obtained by Ambrosio and Tortorelli's method, and the PDM of Figure 2(d) was obtained from the ESF with $\alpha = 0.6$ and $\beta = 1.0$. The final segmentation results of Figures 2(e) and (f) were obtained by iteratively merging adjacent regions whose dissimilarity is less than 15.0. Comparing the two figures, one can easily recognize that the proposed method produced a more accurate and less noisy segmentation result. Note that the part pointed by an arrow in Figure 2(f) clearly shows that our method can provide well-completed region boundaries even if some parts of the boundary have nearly zero edge strength values.

5. Conclusions

In this paper, we proposed a new region segmentation method based on the watershed transformation of a pseudo-distance map. Since the PDM is a regularized version of a Euclidean distance map and is directly computed from an edge-strength function, its watershed transformation usually produces more reliable and less noisy initial segmentation results than the gradient- or EDM-based methods while preserving the useful property of the EDM-based method. We also proposed an efficient region-merging criterion based on edge strengths and region intensities. Through several experiments, we showed that the proposed region-merging criterion together with the PDM-based method yielded final segmentation results better than the conventional methods.

The main drawback of our method is that it requires a long computation time. Although this drawback can be overcome by computing the solutions of the PDEs in parallel, we plan to develop a computationally more efficient algorithm in the future.

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