Square-root 형 등리플 파형성형 필터의 간단한 설계

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Simple Design of Equiripple Square Root Pulse Shaping Filter

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Abstract

In this paper, we introduce a simple design method for root-squared type raised cosine filter with equiripple characteristics. Through some design examples, we show that the proposed filter has much better performance in ripple than the conventional SRCF at the expense of small increasing of ISI. In addition, the proposed filter is compatible with conventional SRCF. Finally, we designs the filter for W-CDMA which uses RRC (Root Raised Cosine) with a=0.22, in 12bit finite precision.

I. Introduction

In digital communication, transmission filter has the role of pulse shaping for reducing inter symbol interference (ISI) and limiting the bandwidth. Raised Cosine Filter (RCF) and Square root Raised Cosine Filter (SRCF) are widely used as a pulse-shaping filter because those can be designed from closed-form equation and have smaller ISI [1]. Since they show less attenuation near the stopband edge than the other stopband frequencies as shown in Fig. 1, it causes larger spurious emission in adjacent channel and then requires more exhaustive guard band or longer filter [2].

In this paper we propose simple method for designing equiripple square root pulse shaping filter. The proposed method uses conventional equiripple filter design algorithm, such as remez exchange [3], with specific ripple weighting ratio. Since the proposed filter is designed with equiripple criterion, that may have equal and minimum ripples, and shows better attenuation near the band edges than conventional SRCF. In addition, the proposed filter is compatible with conventional SRCF. In other words, the cascade of the proposed filter and SRCF has much smaller ripple at the expense of slight increase of ISI than the pair of SRCFs does.

II. Square Root Raised Cosine Filter

Impulse response of RCF with symbol rate T and roll-off factor α is given by [1]

$$p(t)=\left(\frac{\sin(\pi t/T)}{\pi t/T}\right)\left(\frac{\cos(\alpha\pi t/T)}{1-(2\alpha t/T)^2}\right) \tag{1}$$

and its frequency response is

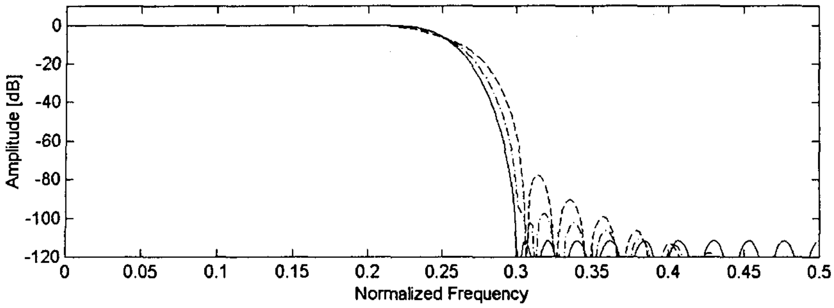

Fig. 1. Frequency Responses of the SRCF(dash), the proposed filter(solid), and the SRCF-proposed pairs(dashdot)

$$P(\omega)=\begin{cases} T & 0\leq|\omega|\leq(1-\alpha)\pi/T \\ \frac{T}{2}\left(1-\sin\left[\frac{T}{2\alpha}(|\omega|-\pi/T)\right]\right) & (1-\alpha)\pi/T\leq|\omega|\leq(1+\alpha)\pi/T \\ 0 & |\omega|>(1+\alpha)\pi/T \end{cases} \quad (2)$$

For digital RCF, $p(n) = p(t)|_{t=nT/Ts}$, where T_s is the sampling interval.

For matched filter system, overall response of transmission filter $G_T(\omega)$ and receiving filter $G_R(\omega)$ should be $p(t)$ as follows.

$$|G_T(\omega)G_R(\omega)|=|G_T(\omega)G_R^*(\omega)|=|P(\omega)| \quad (3)$$

Since each filter in transmitter and receiver is symmetry and has real-value coefficients, $g_T(t)=g_R^*(t)$ and $g(t)$ becomes the square root of RCF in (1) and then

$$g(t)=\frac{4\alpha}{\pi\sqrt{T}}\cdot\frac{\cos((1+\alpha)\pi t/T+T\sin((1-\alpha)\pi t/T)/(4\alpha t)}{1-(4\alpha t/T)^2} \quad (4)$$

It is noted that digital SRCF also can be obtained by $t=nT/T_s$.

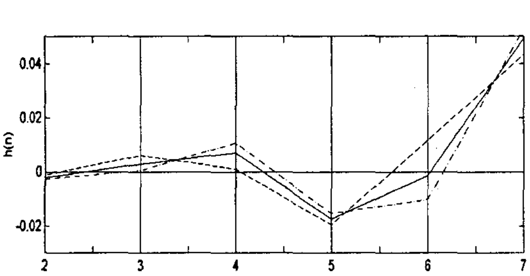

Fig. 2. Impulse responses of h(n) with various ripple weights δ_s/δ_p=0.1(dash), 1(solid), and 10 (dashdot).

III. Design of Equiripple Square Root Pulse Shaping Filter

Conventional equiripple filters can be designed with parameters of band edge frequencies, the number of taps, and relative ripple weighting ratio of each band. It is noted that the zero crossing points of filter coefficients, which affect ISI, are varied with ripple weights as shown in Fig. 2. For zero-ISI, h(n) in Fig. 2 should be zero for n=2, 4, 6 when T_s=0.5T.

Now we consider the relation between ISI and ripple weighting ratio W. Since pulse shaping filter is always lowpass filter, let $W = \delta_s/\delta_p$. In case of nominal pulse shaping filter, not square root type, we find that the filter designed with $W\approx1$ has minimum ISI through lots of design examples. However, W of square root pulse shaping filter may be proportional to the number of taps because the ripple of passband and that of stopband in (3) have the following relation.

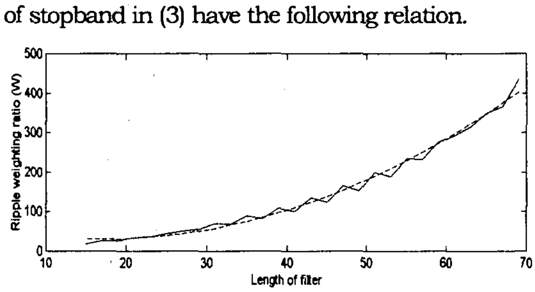

Fig. 3. Ripple weighting ratio W as a function of filter length (solid) and its 2nd order fitting (dash)

$$1+\delta_p^G=\sqrt{1+\delta_p^F},\quad \delta_s^G=\sqrt{\delta_s^F} \qquad (5)$$

where δ^G_p and δ^F_p are passband ripples of $G(\omega)$ and $F(\omega)$, respectively, and δ_s's are stopband ripples. δ^G_s is a square root of δ^F_s as expected. However δ^G_p is equal to or less than a half of δ^F_p because $(1+x)^q = 1 + qx + q(q-1)x^2/2! + \ldots \approx 1 + qx + \varepsilon(x^2)$ for $x<<1$. For example, if $\delta^F_p = \delta^F_s = 0.01$ ($W=1$), then δ^G_p and δ^G_s are 0.005 and 0.1 ($W=20$), respectively. If both of $\delta^F_p = \delta^F_s$ are reduced to 0.0001, then δ^G_p and δ^G_s are 0.00005 and 0.01 ($W=200$). Therefore W is increased when the ripple of $F(\omega)$ is decreased. In other words, if we increase the length of filter, L for reducing the ripple, then W of square root PSF is also increased.

In Fig. 3, we show Ws with minimum ISI for various L's when the filter is designed as a equiripple type with oversampling ratio=2 and α=0.2, by using remez exchange algorithm. Using 2nd order polynomial fitting in least square sense, W can be represented as a function of the length of filter.

$$W(L)=0.138L^2-4.690L+70.625 \quad \text{for } 15\le L\le 70 \quad (6)$$

It is noted that equiripple square root pulse shaping filter can be designed by using any conventional equiripple filter design method having adjustable ripple weight. Through the paper, all filters are designed by using remez exchange algorithm in MATLAB [4].

***Example I*:** Design the square root pulse shaping filter with 45 tap, 2 times oversampling, and roll off factor α=0.2. Then f_p and f_s are $(1-\alpha)/4=0.2$ and $(1+\alpha)/4=0.3$ normalized frequency, respectively, and W is 139.025 from (6). Figure 1 shows the frequency responses of the proposed filter and conventional SRCF. The stopband ripples of the proposed and the SRCF are −112dB and −55.1dB, respectively. ISI of the proposed filter is 0.00329 that is sufficiently small and comparable to 0.00271 of SRCF. It is noted that the pair of the proposed filter and the conventional SRCF also shows better attenuation than SRCF as shown Fig. 1. ■

Table I. Coefficients of Example 2

n	h(n)	n	h(n)	N	h(n)
0, 66	-2	12,	-12	24,	55
1, 65	2	13,	-9	25,	23
2, 64	3	14,	-1	26,	-42
3, 63	-1	15,	13	27,	-97
4, 62	-2	16,	19	28,	-103
5, 61	-2	17,49	14	29,	-24
6, 60	-3	18,	-2	30,	131
7, 59	0	19,	-26	31,	323
8, 58	6	20,	-34	32,	476
9, 57	7	21,	-20	33	540
10,	3	22,	17		
11,	-6	23,	48		

The proposed filter is more useful for wireless system at the expense of slight increasing of ISI because smaller ripple near the stopband edge induces less adjacent channel interference. In addition, since the proposed method can be applied to finite-precision filter design algorithms [5]-[6], the proposed filter may show smaller ISI than the filter designed by simple rounding of SRCF.

***Example II*:** Design the pulse shaping filter for W-CDMA with α=0.22 and Tc=0.26042μs. Then f_p and f_s are 0.195 and 0.305 normalized frequency, respectively, and W is 139.025 from (6). By using MILP (Mixed Integer Linear Programming), we design 67 tap SRCF with 12 bit coefficients. The proposed filter is shown in Tab. I. In Table II, we show the comparison of overall

Table II. ISI Comparison of the Conventional SRCF and the proposed filter

Tx \ Rx		71 tap SRCF	177 tap SRCF	The proposed
71 tap SRCF	Ripple(dB)	-50.75	-74.41	-90.60
	ISI	0.00123	0.000725	0.00289
177 tap SRCF	Ripple(dB)	-	-88.07	-94.64
	ISI	-	0.000273	0.00267
The proposed	Ripple(dB)	-	-	-99.49
	ISI	-	-	0.00158

performance between the proposed filter and the conventional SRCF. It is noted that the efficient implementation of this filter, which use shift-and-add architecture, can be found in [7] ■

IV. Concluding Remarks

The proposed filter shows much better performance in ripple than the conventional SRCF at the expense of small increasing of ISI. In addition, the proposed filter is compatible with conventional SRCF. Finally, since the proposed method can be applied to any conventional equiripple filter design algorithm, that can be easily extended to finite precision filter.

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