검사 체적 유한요소법을 이용한 압출 다이 내의 유동 해석

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Flow Analysis within a Extrusion Die by Using Control Volume Finite-Element Method

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INTRODUCTION

Extrusion is one of the main processing methods for thermoplastics. Extrusion is used to manufacture pipes, films, fibers, cables, wires, and so on. The screw and die are the most important parts in an extruder. In this study we are interested in the flow behavior in the die with continuously varying cross-section. Because the polymer melt has a low thermal conductivity, the convective contribution is predominant in the flow. In solving such an convection-diffusion problem, CVFEM is an widely used technique. It can handle irregular-shaped and multiply connected domains and can also give accurate solutions over all range of Peclet numbers, while the conventional Galerkin finite element methods could produce oscillatory solutions at high Peclet numbers.

THEORETICAL MODELING

For the analysis of the flow field within the die following assumptions were used. The gravitational force is neglected, the flow within the die is in a steady state and the inertial force is much smaller than the viscous force. The polymer melt flow and the heat transfer are governed by the following differential equations.

Continuity equation

$$\nabla \cdot \mathbf{v} = 0 \qquad (1)$$

Momentum equation

$$-\nabla p + \nabla \cdot \boldsymbol{\tau} = 0 \qquad (2)$$

Energy equation

$$\rho C_p \mathbf{v} \cdot \nabla T = \nabla \cdot (k \nabla T) + \eta \dot{\gamma}^2 \qquad (3)$$

where ρ is the density of the fluid, C_p heat capacity, η the viscosity of the fluid, k thermal conductivity, p hydraulic pressure, T temperature, and u, v, and w are the velocity components in the x, y, and z directions respectively. We employed the 5-constant modified Cross model as the constitutive equation.

$$\eta = \frac{\eta_0(T,p)}{1+\left(\eta_0 \dot{\gamma} / \tau^*\right)^{1-n}} \qquad (4)$$

$$\eta_0(T,p) = B \exp\left(\frac{T_b}{T}\right) \exp(\beta p) \qquad (5)$$

When the energy equation is solved, the temperature distribution in the solid die must be known to use it as a wall boundary condition. The temperature distribution in the die is calculated by the finite element method.

NUMERICAL ANALYSIS

Control Volume Finite Element Method

To avoid checkerboard-type pressure fields, present study is based on the equal-order CVFEM [3]. The four-node tetrahedron shown in Fig. 2 is used as the basic discretization element.

Control-Volume Conservation Equations

An integral formulation can be obtained by applying the conservation principle for a fixed control volume V. When the principals are applied to the polyhedral control volume surrounding the node 1 of the tetrahedral element shown in Fig. 2, the resulting integral conservation equation can be cast in the following form.

$$\left[\int_{atos}\mathbf{J}\cdot\mathbf{n}ds+\int_{ctor}\mathbf{J}\cdot\mathbf{n}ds+\int_{dsor}\mathbf{J}\cdot\mathbf{n}ds-\int_{atcdsor}S_\phi dV\right]+\begin{bmatrix}\text{similar constributions from other}\\ \text{elements associated with node 1}\end{bmatrix} \quad (6)$$
$$+\left[\text{boundary contributions, if applicable}\right]=0$$

The integral equations that express the conservation of momentum can be obtained from eq. (6) by replacing **J** with ρ**v**.

Formulation for the Flow within the Die

The fluid flow problem is governed by the conduction-type equation [3]. For the conduction-type problem, the general form of flux is written as the following form:

$$\mathbf{J}=-\Gamma_\phi\nabla\phi \quad (7)$$

The interpolation function in x-direction is given as follows:

$$u=Ax+By+Cz+D-\frac{1}{\eta}\frac{\partial p}{\partial x}\left[x-\frac{1}{4}\left(y^2+z^2\right)\right] \quad (8)$$

Using Gaussian quadrature approximation, the resulting discretization equation of u for node i can be expressed as follows:

$$a_i^u u_i=\sum_n a_n^u+d_i^u+\sum_n \Omega_n^u p_n \quad (9)$$

The interpolation functions in x- and y- direction and discretization equations of v and w can be reduced similarly to the eqs (7) and (8) respectively. The pressure and velocity values obtained by the above equations are corrected by a SIMPLEC procedure.

Formulation for the Heat Transfer

The heat transfer problem is governed by eq. (3). Because this problem is a convection-diffusion type, the local flow-oriented Cartesian coordinate (X, Y, Z) is used. The heat flux and the source term can be expressed as follows:

$$\mathbf{J}=\left(\rho C_p UT-k\frac{\partial T}{\partial X}\right)\mathbf{i}+\left(\rho C_p VT-k\frac{\partial T}{\partial Y}\right)\mathbf{j}+\left(\rho C_p WT-k\frac{\partial T}{\partial Z}\right)\mathbf{k} \quad (10)$$

The interpolation function for temperature that satisfies the convection-diffusion equation is the following:

$$T = A^T\xi + B^TY + C^TZ + D + S_T\left[\frac{X}{N\rho C_pU_{av}} - \frac{1-1/N}{4k}\left(Y^2+Z^2\right)\right] \quad (11)$$

Using Gaussian quadrature, the resulting temperature discretization equations for node i can be expressed as follows:

$$a_i^TT_i = \sum_n a_n^TT_n + b_i^T \quad (12)$$

where the summation is taken over all neighbors of node i. The under-relaxation method is employed to avoid divergence in the iterative procedure.

Boundary Conditions

It is assumed that the velocity is fully developed in the inlet of the head. At the wall boundary, no-slip condition is used for the momentum equation and isothermal condition is used for the energy equation. The pressure difference between the inlet and the outlet is assumed to be an arbitrary value at first, and then flow rate is calculated from that result. This procedure is repeated until the obtained flow rate is equal to the experimentally measured flow rate.

RESULTS AND DISCUSSION

A commercial polypropylene is used as the material for the analysis. Material properties are given in Table 1. Constants of the viscosity model are given by reference [1]. The geometry of the L-shaped profile extrusion die used in the simulation and the experiment is shown in Fig. 3. In order to compare the results with the previous work, the conditions are set equal to those of the previous work [1]. The temperature of the head is 473K. 2406 nodes and 10729 elements are used for the simulation of the flow in the die. The locations of the pressure sensors in the experiment are shown in Fig. 3. In the case that the mass flow rate is 1.24 g/s, the calculated pressure profiles are shown in Fig. 4. The simulation results obtained by the 3-D CVFEM agree well with the experimental measurements and are more accurate than those of the 2-D cross-sectional method. The velocities and the temperature in some cross-sections of the die were shown in Fig. 5.

CONCLUSION

Three-dimensional non-isothermal numerical simulation is performed to determine the flow behavior within an extrusion die. Pressure distribution given by the 3-D CVFEM agrees well with the experimental measurements and is more accurate than those obtained by 2-D cross-sectional method. The results can be used for design of the extrusion die and study of the extrudate swell.

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REFERENCES

1. D. J. Seo, J. R. Youn, *Fibers and Polymers*, (1999) 1 p. 103-110.
2. B. R. Baliga and S. V. Patankar, Elliptic systems: Finite-element method II, in W. J. Minkowycz, E. M. Sparrow, R. H. Pletcher, and G. E. Schneider (eds), "*Handbook of Numerical Heat Transfer*", Wiley, New York, (1988) p. 421-461.
3. C. Prakash, *Numerical heat transfer*, (1986) 9 p. 253-276.
4. S. V. Patankar, "*Numerical heat transfer and fluid flow*", Hemisphere, Washington, D.C., (1980).

Table 1. Material properties and five constants of the viscosity model.

	polypropylene
Density (kg /m^3)	770
Heat capacity (J / kg ·K)	3060
Heat conductivity (W / m ·K)	0.151
N	0.342
τ^* (Pa)	7.52E+3
B (Pa ·s)	2.36E-2
T_b (K)	5.236E+3
β (Pa^{-1})	1.5E-8

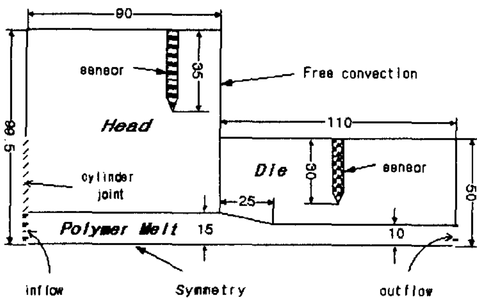

Figure 1. Schematic diagram of the die head and die channel.

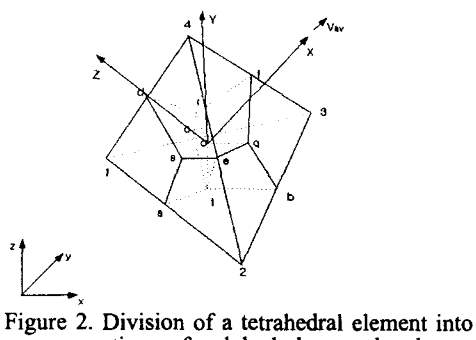

Figure 2. Division of a tetrahedral element into portions of polyhedral control volumes and local flow-oriented X, Y, Z and global x, y, z coordinate system

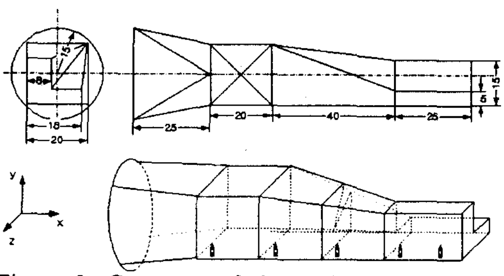

Figure 3. Geometry of the L-shaped profile extrusion die.

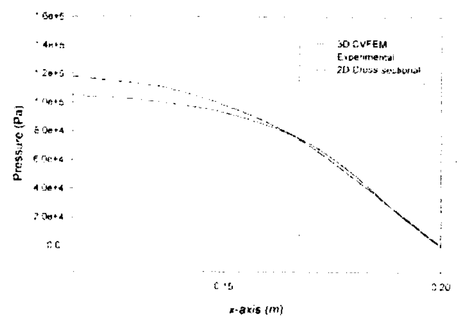

Figure 4. Pressure variation of the polymer melt along the boundary of the channel, where die temperature is 473 K and the flow rate is 0.84 g/s.

(a) (b)

Figure 5. The contour plots for die temperature of 473 K and mass flow rate of 1.24 g/s at x=0.195m.

(a) velocity in the x-direction (m/s)
(b) temperature (K)