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제어 체적 유한요소법을 이용한 이형 압출 다이 내의 삼차원 유동 해석

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Three dimensional flow analysis within a profile extrusion die by using control volume finite-element method

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1. Introduction

Three-dimensional flow analysis was performed by using the control volume finite-element method for design of a profile extrusion die. Because polymer melt is complicated and cross-sectional shape of the profile extrusion die is changing continuously, the fluid flow within the die must be analyzed three-dimensionally. CVFEM is used for three dimensional simulation and a commercially available polypropylene is used for theoretical and experimental investigations. The 5-constant modified Cross model is used for the numerical analysis. Simulations are performed for conditions of three different screw speeds and three different die temperatures. Predicted pressure distribution is compared with the experimental measurements and the results of the previous two-dimensional study.

2. Theoretical Modeling

In the Cartesian coordinate system, the fluid flow problem and the heat transfer problem are governed by the following differential equations.

Continuity equation:

$$\nabla \cdot \mathrm{v} = 0 \qquad (1)$$

momentum equation

$$-\nabla p + \nabla \cdot \tau = 0 \qquad (2)$$

energy equation

$$\rho C_p \frac{dT}{dt} - k\nabla^2 T = \Phi \qquad (3)$$

3. Numerical Analysis

Steady, three dimensional, elliptic convection-diffusion phenomena are governed by differental equations that can be cast in the following general form.

$$div(\,\mathrm{J}) = S_\phi, \qquad \mathrm{J} = \rho \mathrm{v}\phi - \Gamma_\phi \nabla\phi \qquad (4)$$

An integral formulation corresponding to equation (4) can be obtained by applying the conservation principle for ϕ to a control volume. The resulting integral conservation equation, when applied to the polyhedral control volume surrounding to node 1 of the tetrahedral element shown in fig. 1, can be cast in the following form:

$$\left[\int_{atos}\mathrm{J}\cdot\mathrm{nds} + \int_{ctor}\mathrm{J}\cdot\mathrm{nds} + \int_{dsor}\mathrm{J}\cdot\mathrm{nds} - \int_{atcdsor}\mathrm{S}_\phi\mathrm{dV}\right]$$

$$+[\text{similar contributions from other elements associated with node 1}] \qquad (5)$$

$$+[\text{ boundary contributions, if applicable}] = 0$$

Interpolation function is defined with respect to a local flow-oriented Cartesian coordinate system (X, Y, Z).

$$\phi = \mathrm{A}^\phi\xi + \mathrm{B}^\phi\mathrm{Y} + \mathrm{C}^\phi\mathrm{Z} + \mathrm{D}^\phi + \mathrm{S}_\phi\left[\frac{\mathrm{X}}{\mathrm{N}\rho\mathrm{U}_{av}} - \frac{1-1/\mathrm{N}}{4\Gamma_\phi}(\mathrm{Y}^2+\mathrm{Z}^2)\right] \qquad (6)$$

$$\xi = \frac{\Gamma_\phi}{\rho\mathrm{U}_{av}}\left\{\exp\left[\frac{\mathrm{Pe}_\Delta(\mathrm{X}-\mathrm{X}_{max})}{\mathrm{X}_{max}-\mathrm{X}_{min}}\right]-1\right\} \qquad (7)$$

$$\mathrm{Pe}_\Delta = \rho\mathrm{U}_{av}\left(\frac{\mathrm{X}_{max}-\mathrm{X}_{min}}{\Gamma_\phi}\right) \qquad (8)$$

where N is a user-specified index. In most elliptic problems, N=2 has been found to be most satisfactory. The discretization equations for equations (1)-(3) can be finally cast in the following forms:

$$\mathrm{a}_i^u\mathrm{u}_i = \sum_n \mathrm{a}_n^u\mathrm{u}_n + \mathrm{d}_i^u + \sum_n \Omega_n^u\mathrm{p}_n \qquad (9)$$

$$\mathrm{a}_i^v\mathrm{v}_i = \sum_n \mathrm{a}_n^v\mathrm{v}_n + \mathrm{d}_i^v + \sum_n \Omega_n^v\mathrm{p}_n \qquad (10)$$

$$\mathrm{a}_i^w\mathrm{w}_i = \sum_n \mathrm{a}_n^w\mathrm{w}_n + \mathrm{d}_i^w + \sum_n \Omega_n^w\mathrm{p}_n \qquad (11)$$

$$e_i p_i' = \sum_m e_m p_m' + f_i \qquad (12)$$

$$\mathrm{a}_i^T\mathrm{T}_i = \sum_n \mathrm{a}_n^T\mathrm{T}_n + \mathrm{b}_i^T \qquad (13)$$

4. Results and Discussion

The geometry of the L-shape profile extrusion die used for the simulations and the experiment. For the screw speed, 30, 60, and 90 rpm were used, and for the die temperature, 453, 473, and 493K were used. The head temperature was assumed to be 473K. A total of 2406 nodes and 10729 elements were used for the die. When the flow rate is 1.24 g/s, the pressure profiles at different die temperatures are shown in figures 2 to 4. Two lines in figures 2 to 4 indicate two numerical results, i.e. numerical prediction by using three-dimensional CVFEM and by using two-dimensional cross-sectional method, and the other line shows the experimental result. It is clear that the pressure difference is smaller as the die temperature is increased.

5. Conclusion

The velocity, pressure, and temperature distributions of the fluid flow within the profile extrusion die are calculated by using the control-volume finite-element method (CVFEM). An equal-order CVFEM for three-dimensional fluid flow problem and the CVFEM for three-dimensional heat convection-diffusion problem considering viscous heating are formulated. Pressure distributions given by the numerical calculations are compared with the experimental measurement and the two-dimensional numerical calculation for the same profile extrusion die. Simulation results obtained by the three dimensional CVFEM agree well with the experimental measurements and are more accurate than those obtained by using two dimensional cross-sectional method. The results can be utilized for design of the extrusion die and study of the extrudate swell.

6. Reference

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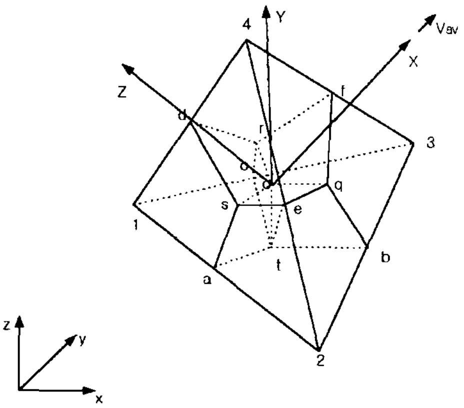

Fig. 1. Division of a tetrahedral element into portions of polyhedral control volumes and local flow-oriented X, Y, Z and global x, y, z coordinate systems.

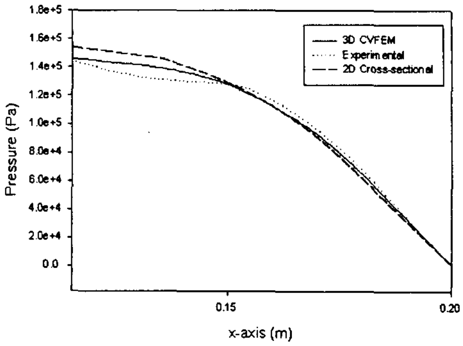

Fig. 2. Pressure variation of the polymer melts along the boundary of the channel in the case of die temperature at 453 K and flow rate of 1.24 g/s.

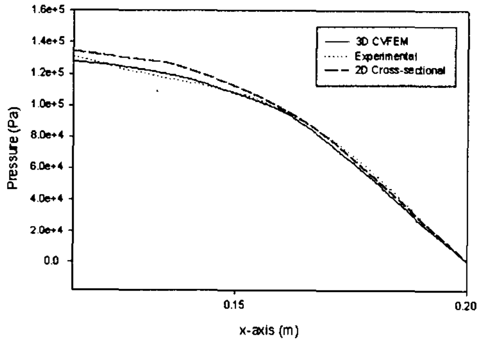

Fig. 3. Pressure variation of the polymer melts along the boundary of the channel in the case of die temperature at 473 K and flow rate of 1.24 g/s.

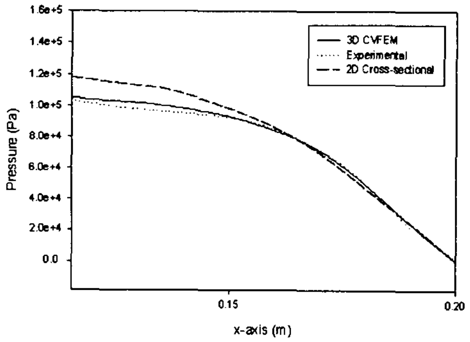

Fig. 4. Pressure variation of the polymer melts along the boundary of the channel in the case of die temperature at 493 K and flow rate of 1.24 g/s.