생존도를 고려한 WDM 망의 경로설정 및 파장할당
Routing and Wavelength Assignment in Survivable WDM Networks

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Abstract

We consider the routing and wavelength assignment problem in survivable WDM transport network without wavelength conversion. We assume the single-link failure and a path protection scheme in optical layer. When a physical network and a set of working paths are given, the problem is to select a link-disjoint protection path for each working path and assign a wavelength for each working and protection path. We give an integer programming formulation of the problem and propose an algorithm to solve it based on column generation technique and variable fixing. We devise a branch-and-price algorithm to solve the column generation problem. We test the proposed algorithm on some randomly generated data and test results show that the algorithm gives very good solutions.

1. Introduction

Survivability has been an important issue in designing the fiber-optic based telecommunication network. In WDM network, several lightpaths pass a link and the failure of a network component such as a fiber link can lead to the failure of all those lightpaths. Because each lightpath is expected to operate at a rate of several Gbps, such a failure can lead to a severe disruption in the network's traffic. Optical layer protection schemes in WDM network can be classified in much the same manner as SONET/SDH protection schemes. Refer [2] and [5] for detailed characteristics and comparisons of those schemes.

Many studies on the design problems in WDM network, which is mainly in terms of determining routing and wavelength assignment to each optical channel, has been performed [3,6,7] but the problem in survivable network has not been studied intensively. Recently, studies on survivable WDM network are performed [4,8,9, 10,11,12]. All the studies assume a failure scenario and a protection scheme, respectively. They determine the working and(or) protection paths against the assumed network failure.

In this paper, we consider the routing and wavelength assignment problem in survivable WDM transport network without wavelength conversion. We assume the single-link failure scenario and a path protection scheme in optical layer. When a failure occurs, the event is quickly disseminated to all the pertinent nodes, which set up the backup paths for the failed lightpaths and switch data to them. The scheme is being implemented in novel OXC (Optical Cross-Connect) which has the capability to change the routing patterns at a node [13]. Due to time constraints, the scheme is typically based on predetermined backup paths.

In wavelength assignment for protection paths, there are two possible methods [10,11]. One method assigns the same wavelength to the protection paths as its corresponding working path (Method-1). The other method assigns an arbitrary wavelength for each protection path (Method-2). In this paper, we assume method-1.

2. Problem Formulation

As mentioned in the previous section, we consider the routing and wavelength assignment problem in survivable WDM network. We assume the single-link failure and the following path protection scheme. When a working path is established, a protection path for the working path is also determined and the protection path is activated when the working path fails. The predetermined protection path for each working path is independent to the location of the failure. In other words, a working path and the corresponding protection path are link-disjoint.

When a physical network and a set of working paths are given, we must select a link-disjoint protection path for each working path and assign a wavelength for working and protection paths. In wavelength assignment, a working path and corresponding protection path must be assigned the same wavelength. We will call this problem survivable routing and wavelength assignment (SRWA) problem. We assume that no wavelength conversion is permitted. Thus, wavelength reuse is very important for the efficiency and the objective is to minimize the number of the used wavelengths to maximize the wavelength reuse.

First, we introduce some notation.

$G=(V,E)$: physical network
P : set of given working paths
o_p, d_p : two end nodes of working path $p \in P$
$R(p)$: set of all possible protection paths for working path $p \in P$
$R = \cup_{p \in P} R(p)$
$E(p)$: set of links used by path p

Let's define a routing configuration as a set of paths in P and R. We call routing configuration c as *identical survivable independent routing configuration* (ISIRC) if all the paths in c can be established using one wavelength and it has following characterisics. If working path p is contained to an ISIRC, then a corresponding protection path $r \in R(p)$ must be contained to it because they use the same wavelength. A 'closed trail' on a graph is a closed walk that traverses each link at most once. Because p and r are link-disjoint, those two paths form a closed trail. In other words, an ISIRC consists of several closed trails and each closed trail is divided into a working path and a corresponding protection path. Note that no two protection paths in an ISIRC are activated at the same time because the corresponding working paths must be in the ISIRC and they don't share any link. Thus, if a link is used by a working path, then the link can not be used by any other working or protection path. Otherwise, the link can be used by several protection paths. We introduce some more notation.

$H(p)$: set of all possible closed trails on G which contain working path $p \in P$

$H = \cup_{p \in P} H(p)$

$E(h)$: set of links used by closed trail $h \in H$

C_I : set of all ISIRC's.

Note that a closed trail in $H(p)$ is consist of working path p and a protection path $r \in R(p)$. In other words, $E(h) = E(p) \cup E(r)$. Then, an ISIRC can be represented by a binary vector $a_c \in B^{|P|}$. The p^{th} element of a_c , denoted as a_{pc} is 1 if a closed trail in $H(p)$ is contained in c, otherwise, $a_{pc} = 0$.

Then, we can formulate SRWA as the following integer program.

$$(MP) \quad \min \sum_{c \in C_I} z_c$$

$$s.t. \quad \sum_{c \in C_I} a_{pc} z_c \geq 1, \text{for all } p \in P \qquad (1)$$

$$z_c \in \{0,1\} \text{ for all } c \in C_I$$

Each decision variable $z_c = 1$, $c \in C_I$ if ISIRC c is established, otherwise, $z_c = 0$. Constraints (1) ensure that at least one closed trail for each working path must be selected. It also means that all given working paths and a protection path for each working path should be established. We can easily obtained the wavelength assignment by assigning the same wavelength to paths contained in an ISIRC. Then, each working and corresponding protection path use the same wavelength.

Let MLP be the LP relaxation of MP. MLP has exponentially many variables. However, we can solve MLP by using the column generation technique [1]. First, we assume that a subset C'_I of C_I is given. Then we construct a restricted LP relaxation RMLP by replacing C_I by C'_I in MLP. Let α_p be the dual variable corresponding to the p^{th} constraint in (1). We can solve RMLP by the simplex method and let z^* be an optimal solution to RMLP and α^* be the values of the dual variables returned by the simplex method in RMLP. Then, z^* is an optimal solution to MLP if the following condition is satisfied because the reduced cost of all variables are nonnegative [1].

$$\sum_{p \in P} \alpha^*_p a_{pc} \leq 1 \quad \text{for all } c \in C_I \backslash C'_I \quad (2)$$

Note that SP is the problem to find a maximum weight identical survivable independent routing configuration, where closed trail $h \in H(p)$ has weight α^*_p.

3. Algorithm for Column Generation Problem

As noted in the previous section, column generation problem is find a maximum weight SIRC. Then, it can be formulated as follows.

$$(SP)$$

$$\max \sum_{p \in P} \sum_{r \in H(p)} \alpha^*_p x_h$$

$$s.t. \quad \sum_{h \in H(p)} d^r_{eh} x_h - (1 - y_e) \leq 0,$$

$$\text{for all } p \in P \text{ and } e \in E \quad (3)$$

$$\sum_{p \in P} \sum_{h \in H(p)} d^p_{eh} x_h - y_e \leq 0, \quad \text{for all } e \in E \quad (4)$$

$$x_h, y_e \in \{0,1\} \text{ for all } h \in H \text{ and } e \in E$$

A closed trail is consist of a working path and a corresponding protection path. The coefficient $d^r_{eh} = 1$ if the protection path in closed trail h pass link e, otherwise $d^r_{eh} = 0$. Similarly, $d^p_{eh} = 1$ if the working path in closed trail h pass link e, otherwise $d^p_{eh} = 0$. Each decision variable $x_h = 1$, $h \in H$ if closed trail h is selected, otherwise, $x_h = 0$. Decision variable $y_e = 1$, $e \in E$ if link e is used by a working path, otherwise, $y_e = 0$. Constraints (3) ensure that link e may be used by multiple protection paths if the link is not used by any working path. Constraints (4) ensure that at most one working path can pass on a link. Thus, (3) and (4) satisfy the restriction for an ISIRC and feasible solution to SP is an ISIRC.

The number of variables x_h in SP is exponentially many. But, we can solve the LP relaxation of SP by column generation technique.

3.1 Column generation for SP

Let SLP be the LP relaxation of SP and let SLP' be a restricted LP relaxation obtained by replacing $H'(p) \subset H(p)$, for all $p \in P$. Let β_{pe}, for all $p \in P$ and $e \in E$ and ρ_e , for all $e \in E$, be the nonnegative dual variables associated with constraints (3) and (4), respectively. We can solve

SLP' by the simplex method and let (x^*,y^*) be the obtained optimal solution to SLP' and let be $(\overline{\beta},\overline{\rho})$ be the corresponding optimal dual solution. Then, the reduced cost of each closed trail $h \in H(p)$, denoted c_h, is as follows.

$$c_h = \alpha_p^* - \sum_{e \in E(r)} \overline{\beta_{pe}} - \sum_{e \in E(p)} \overline{\rho_e}$$

Note that α_p^* and $\sum_{e \in E(p)} \overline{\rho_e}$ are determined values for a working path p because $E(p)$ is known. Thus, we only find a protection path which has maximum weight where link e has weight $\overline{\beta_{pe}}$. Then, the column generation problem is to find a shortest path between o_p and d_p on a given network $G=(V,E \setminus E(p))$ with the link weight $\overline{\beta_{pe}}$. Because the link weights are nonnegative, we can solve the problem in polynomial time.

3.2 Branch-and-price procedure for SP

After solving SLP, we perform branch-and-price procedure to get an optimal solution to SP. In the branch-and-price procedure, a branching rule is required such that the column generation is possible after branching.

Now, we present our branching rule. When optimal solution (x^*,y^*) to SLP' is obtained, denote $U(x^*,y^*) \subseteq P$ as the set of links such that corresponding y^* has fractional value. If $|U(x^*,y^*)|>0$, we branch on y_{e^*} such that $e^* = \arg\max_{e \in U(x^*,y^*)} y_e^*$. We make two new branches, one of them forces y_{e^*} to be 1, the other forces y_{e^*} to be 0. In those branches, the column generation problem is not changed.

If $|U(x^*,y^*)|=0$, it is determined whether each link is used by any working path or not. Then, we define $P(x^*,y^*;e) \subset P$ for all $e \in E$ such that working path $p \in P(x^*,y^*;e)$ if and only if there exists a closed trail $h \in H(p)$ with $x_h^*>0$ such that $e \in E(p)$. Clearly, if (x^*,y^*) is an integral solution, then $|U(x^*,y^*)|=0$ and $|P(x^*,y^*;e)| \leq 1$. Converse is not always true. However, we can derive the following positive result if we use the simplex method. We omit the proof of them because the length of the paper.

Proposition 1. Suppose an optimal solution (x^*,y^*) to SLP' is obtained by the simplex method. When $|U(x^*,y^*)|=0$ and $|P(x^*,y^*;e)| \leq 1$ for all $e \in E$ then (x^*,y^*) is integral.

For a given solution (x^*,y^*) such that $|U(x^*,y^*)|=0$, we get $P(x^*,y^*;e)$ for all $e \in E$. Then, we choose e^* with the lowest index such that $e^* = \arg\max_{e \in E} |P(x^*,y^*;e)|$. If $|P(x^*,y^*;e^*)| =1$, we get an integral solution by proposition 1. Otherwise, we divide $P(x^*,y^*;e^*)$ into two disjoint sets $P_1(x^*,y^*;e^*)$ and $P_2(x^*,y^*;e^*)$, such that $P_1(x^*,y^*;e^*)=\{p^*\}$ and $P_2(x^*,y^*;e^*)=P(x^*,y^*;e^*) \setminus P_1(x^*,y^*;e^*)$, where $p^* = \arg\max_{p \in P(x^*,y^*;e^*)} \sum_{h \in H(p)} x_h^*$ with the lowest index. Then we make two branches in the branch and bound tree such that any closed trail for $p \in P_1(x^*,y^*;e^*)$ can not be selected in the first node and any closed trail for $p \in P_1(x^*,y^*;e^*)$ can not be selected in the second node.

In the first node, we require $x_h=0$, for all $h \in H(p)$ such that $p \in P_1(x^*,y^*;e^*)$. Moreover, for each $p \in P_2(x^*,y^*;e^*)$, we need not to generate column in the first node. For the second node, we apply the same scheme. Thus, the column generation is still possible in the branch.

4. Overview of the algorithm for SRWA

In this section, we present the overall algorithm to solve MP. As mentioned before, MLP has exponentially many variables. However, we can solve MLP efficiently by using the column generation technique. In the previous section, we explained the algorithm to solve the column generation problem SP. After solving MLP, if the obtained optimal solution to MLP is not integral, we perform a variable fixing procedure to get an integral solution to SRWA.

First, we select a variable which has maximum value among the variables having fractional value in the last RMLP and then fixed the value of the variable to 1. After fixing, we solve RMLP and we perform the column generation procedure until no more column is generated. If the obtained solution is integral then we have found an integral solution to SRWA. Otherwise, we select another variable which has fractional value and fixed it to 1 and then generate columns again. We repeat above steps until we get an integral solution. The procedure does not guarantee to find an optimal solution to SRWA. But the last RMLP may contain many columns that are part of the optimal solution because the most profitable columns are generated and we generate more columns in the variable fixing procedure. Thus, we can expect to find a good solution. We can check the quality of our solution by comparing it with the lower bound obtained from the optimal value of MLP. Computational results in the next section show that our solution is very good.

5. Computational Result

We tested our algorithm on the NSFNET shown in figure 1. For the test, we randomly generated 1 or 2 working paths for all pairs of two nodes with the same probability. We use the shortest path routing for working path. We tested 20 randomly generated problem instances. The tests were run on a pentium PC(700MHz) and we used CPLEX callable library as an LP solver.

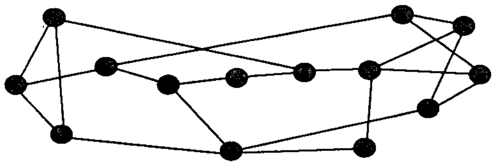

Figure 1. National Science Foundation Network

Test results are summarized in Table 1. In the table, the heading 'col1' and 'col2' refer to the number of generated columns until MLP is solved to optimality and the number of generated columns in the variable fixing procedure after solving MLP, respectively. The heading 'fix' refers to the number of fixed variables in the variable fixing procedure. Z_{MLP} refers to the value obtained by rounding up the optimal objective value of MLP which provides a lower bound on the optimal objective value of SRWA. Z refers to the objective value obtained by our algorithm. Dif is defined as Dif = $Z - Z_{MLP}$ and it gives an upper bound on the difference between the optimal solution value and obtained solution value. The time to solve the problem is reported under the heading of Time.

Test results show that our algorithm gives optimal solutions for all test problem instances and MLP gives a very tight lower bound on the optimal objective value of SRWA.

Table 1. Computational Results

	col1	col2	fix	Z_{MLP}	Z	Dif	Time(sec)
1	97	59	8	28	28	0	323.02
2	250	53	11	25	25	0	2606.32
3	172	65	13	27	27	0	1328.04
4	270	83	4	23	23	0	2131.21
5	189	70	6	29	29	0	1616.34
6	233	68	12	25	25	0	999.15
7	218	84	3	29	29	0	1489.64
8	185	69	8	25	25	0	773.52
9	146	44	7	26	26	0	941.47
10	196	106	15	21	21	0	1731.36
11	144	3	2	25	25	0	725.72
12	346	51	9	29	29	0	3061.27
13	186	67	5	24	24	0	1485.41
14	186	128	14	24	24	0	1790.79
15	222	53	4	25	25	0	1435.70
16	238	4	1	28	28	0	1337.87
17	93	15	2	29	29	0	241.73
18	149	17	3	24	24	0	858.04
19	218	20	1	23	23	0	914.18
20	116	74	10	26	26	0	896.66

6. Conclusions

In this paper, we consider the routing and wavelength assignment problem on survivable WDM network under the single-link failure. We proposed an integer programming formulation and an algorithm based on column generation and variable fixing. We tested our algorithm on randomly generated data and test results showed that our algorithm provided very good solutions.

In this paper, we considered a path protection scheme. Considering other protection schemes could be good research works. We assumed that working paths are given, but the problem which decide the routing of working and protection paths together may be worth consideration.

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