Performance Analysis of Gas Lubricated Flexure-Pivot Tilting Pad Journal Bearings

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Abstract – A numerical analysis for the gas lubricated flexure-pivot tilting pad journal bearing has been accomplished. The film pressure are obtained by Newton-Raphson method and the dynamic coefficients are evaluated by the pad assembly method. The effects of the pivot position of the pad on the static and dynamic characteristics are presented for three pads journal bearing with LBP. The optimum pivot positions for the static performance is different from that of the dynamic performance.

Key Words – flexure pivot, tilting pad bearing, offset, preload, rotational stiffness of pad, compressibility parameter

1. Introduction

Hydrodynamic gas bearings, in comparison to other bearings lubricated with oils, have a great benefit on the frictional loss and the maintenances. Recently, many concerns for gas lubricated bearings, especially the hydrodynamic gas bearing, are increasing as the shaft rotating speed of turbomachinery is getting higher. But the gas bearings should be designed more carefully by using a rotordynamic analysis because their lower damping can make some instability problems at the critical speeds[1,2,3].

Present work is to develop the program for performance analysis of gas lubricated flexure pivot tilting pad bearings. The program includes considering all pivot flexibility such as the rotational, radial and tangential stiffness in the dynamic coefficient reduction. The code was proved by compared with the results of [4]. In this paper the offset effects on the static and dynamic performance are investigated for a given bearing dimension.

2. Governing Equations

2.1 Film thickness

Figure 1 describes the coordinate system for a three pad flexure pivot pad bearing. The film thickness for each pad is

$$h = c_p - x_e \cos\theta - y_e \sin\theta - (c_p - c_b)\cos(\theta - \theta_p) - (\delta R - \xi)\sin(\theta - \theta_p) \tag{1}$$

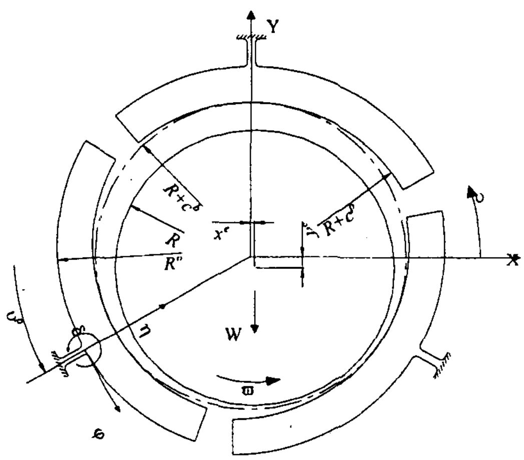

Figure 1 Coordinate system of a flexure pivot pad bearing

where c_p and c_b are the pad machined clearance and the assembled bearing clearance, respectively. In the flexure pivot pad bearings, the pivot displacement(ξ) in the tangential direction can be occurred by tangential reaction forces, and this effect on the bearing performance will be significant in fluid film bearings rather than gas bearings.

2.2 Reynolds Equation

Reynolds equation for compressible fluid is presented as follows.

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\rho h^3\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(\rho h^3\frac{\partial p}{\partial z}\right)=6\mu\omega\frac{\partial(\rho h)}{\partial\theta}+12\mu\frac{\partial(\rho h)}{\partial t} \tag{2}$$

And the state equation for a perfect gas is

$$p=\rho\Re T \tag{3}$$

By substituting the Eq.(3) into the Eq.(2), and then Reynolds equation in non-dimensional form is

$$\frac{\partial}{\partial\theta}\left(PH^3\frac{\partial P}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\varsigma}\left(PH^3\frac{\partial P}{\partial\varsigma}\right)=\Lambda\frac{\partial(PH)}{\partial\theta}+2\Lambda\frac{\partial(PH)}{\partial\tau} \tag{4}$$

where the dimensionless parameters are

$$\varsigma=z/(L/2),\ P=p/p_a,\ H=h/c_{po},$$
$$\tau=\omega t,\ \Lambda=\frac{6\mu\omega}{p_a}\left(\frac{R}{c_p}\right)^2$$

The film thickness in dimensionless form is

$$H=\bar{c}_p-X\cos\theta-Y\sin\theta-m\bar{c}_p\cos(\theta-\theta_p)$$
$$-(\bar{\delta}-\bar{\xi})\sin(\theta-\theta_p) \tag{5}$$

$$\bar{c}_p=c_p/c_{po},\ X,Y=(x_e,y_e)/c_{po},$$
$$m=(c_p-c_b)/c_p,\ \bar{\delta}=\delta(R/c_{po})$$
$$\bar{\xi}=\xi/c_{po}$$

where c_{po} represents an initial value for the pad machined clearance before the thermal and mechanical deformations. Pressure boundary conditions for Reynolds equation are P=1.0 at all solid boundaries of the pad. This condition allows a negative film pressure that is under the ambient pressure.

2.3 Stiffness and Damping equations

The film thickness can be described as below if the shaft and the pad are whirling as $\mathrm{Re}\{\Delta Xe^{j\nu t},\Delta Ye^{j\nu t},\Delta\bar{\delta}e^{j\nu t}\}$ about the operating eccentricity.

$$H=H_o-\Delta Xe^{j\nu t}\cos\theta-\Delta Ye^{j\nu t}\sin\theta$$
$$-\Delta\bar{\delta}e^{j\nu t}\sin(\theta-\theta_p) \tag{6}$$

Therefore the pressure in the film will also be perturbed as

$$P=P_o+P_x\Delta Xe^{j\nu t}+P_y\Delta Ye^{j\nu t}+P_\delta\Delta\bar{\delta}e^{j\nu t} \tag{7}$$

Where H_o and P_o are the film thickness and the film pressure under the static equilibrium, and P_x, P_y, and P_δ are the perturbed pressures that has complex values. The real and imaginary part of these perturbed pressures will contribute at the stiffness and damping of the bearing.

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial P_o^2}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\varsigma}\left(H^3\frac{\partial P_o^2}{\partial\varsigma}\right)=\Lambda\frac{\partial(P_oH_o)}{\partial\theta} \tag{8a}$$

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial P_oP_x}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\varsigma}\left(H^3\frac{\partial P_oP_x}{\partial\varsigma}\right)=\Lambda\frac{\partial}{\partial\theta}(H_oP_x-P_o\cos\theta)$$
$$+\frac{3\Lambda\cos\theta}{H_o}\frac{\partial(P_oH_o)}{\partial\theta}+3P_oH_o^3\frac{\partial P_o}{\partial\theta}\frac{\partial}{\partial\theta}\left(\frac{\cos\theta}{H_o}\right)+j(2\gamma\Lambda)(H_oP_x-P_o\cos\theta) \tag{8b}$$

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial P_oP_y}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\varsigma}\left(H^3\frac{\partial P_oP_y}{\partial\varsigma}\right)=\Lambda\frac{\partial}{\partial\theta}(H_oP_y-P_o\sin\theta)$$
$$+\frac{3\Lambda\sin\theta}{H_o}\frac{\partial(P_oH_o)}{\partial\theta}+3P_oH_o^3\frac{\partial P_o}{\partial\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{H_o}\right)+j(2\gamma\Lambda)(H_oP_y-P_o\sin\theta) \tag{8c}$$

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial P_oP_\delta}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\varsigma}\left(H^3\frac{\partial P_oP_\delta}{\partial\varsigma}\right)=\Lambda\frac{\partial}{\partial\theta}(H_oP_\delta-P_o\sin(\theta-\theta_p))$$
$$+\frac{3\Lambda\sin(\theta-\theta_p)}{H_o}\frac{\partial(P_oH_o)}{\partial\theta}+3P_oH_o^3\frac{\partial P_o}{\partial\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin(\theta-\theta_p)}{H_o}\right)$$
$$+j(2\gamma\Lambda)(H_oP_\delta-P_o\cos(\theta-\theta_p)) \tag{8d}$$

where γ is the whirl ratio(ν/ω).

The reaction forces on the shaft are

$$\begin{Bmatrix}F_x\\F_y\end{Bmatrix}=\sum^{npads}\int_{-L/2}^{+L/2}\int_{\theta_s}^{\theta_e}(p-p_a)\begin{Bmatrix}-\cos\theta\\-\sin\theta\end{Bmatrix}Rd\theta dz \tag{9}$$

Therefore the static reaction forces in dimensionless form are:

$$\begin{Bmatrix} \overline{F}_{xo} \\ \overline{F}_{yo} \end{Bmatrix} = \sum^{npads} \int_0^1 \int_{\theta_s}^{\theta_e} (P_o - 1) \begin{Bmatrix} -\cos\theta \\ -\sin\theta \end{Bmatrix} d\theta d\varsigma \qquad (10)$$

and the non-dimensional stiffness and damping coefficients are

$$\begin{Bmatrix} K_{xx} \\ K_{yx} \end{Bmatrix} = \sum^{npads} \int_0^1 \int_{\theta_s}^{\theta_e} \mathrm{Re}(P_x) \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} d\theta d\varsigma \qquad (11)$$

$$\begin{Bmatrix} C_{xx} \\ C_{yx} \end{Bmatrix} = \frac{1}{\gamma} \sum^{npads} \int_0^1 \int_{\theta_s}^{\theta_e} \mathrm{Im}(P_x) \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} d\theta d\varsigma \qquad (12)$$

Other terms can be described as the similar forms with above. And the non-dimensional reaction forces and dynamic coefficients are defined by

$$\overline{F}_x = \frac{F_x}{p_a LR}, \quad K_{xx} = \frac{c_{po} k_{xx}}{p_a LR}, \quad C_{xx} = \frac{c_{po} \omega c_{xx}}{p_a LR}$$

1. Numerical Approach

1.1 finite difference equations

Reynolds equation(8a) can be described in finite difference form

$$\begin{aligned} f_{i,j} = {} & \frac{H_o{}^3{}_{i-1/2,j}}{2\Delta\theta^2}\left(P_o{}^2{}_{i-1,j} - P_o{}^2{}_{i,j}\right) + \frac{\Lambda H_{oi-1/2,j}}{2\Delta\theta}\left(P_{oi-1,j} + P_{oi,j}\right) \\ & + \frac{H_o{}^3{}_{i+1/2,j}}{2\Delta\theta^2}\left(P_o{}^2{}_{i+1,j} - P_o{}^2{}_{i,j}\right) - \frac{\Lambda H_{oi+1/2,j}}{2\Delta\theta}\left(P_{oi+1,j} + P_{oi,j}\right) \\ & \left(\frac{D}{L}\right)^2 \frac{H_o{}^3{}_{i,j-1/2}}{2\Delta\varsigma^2}\left(P_o{}^2{}_{i,j-1} - P_o{}^2{}_{i,j}\right) + \left(\frac{D}{L}\right)^2 \frac{H_o{}^3{}_{i,j+1/2}}{2\Delta\varsigma^2}\left(P_o{}^2{}_{i,j+1} - P_o{}^2{}_{i,j}\right) = 0 \end{aligned} \qquad (13)$$

The film pressure can be obtained from nonlinear Eq.(13) directly. But it may take place the problems in convergence of the film pressure. In present work, we use the Newton-Raphson method as follows to compute the film pressure without convergence problems.

$$\begin{aligned} f^{(n+1)}{}_{i,j} = {} & f^{(n)}{}_{i,j} + \frac{\partial f^{(n)}{}_{i,j}}{\partial P_{oi-1,j}} \Delta P_{oi-1,j} + \frac{\partial f^{(n)}{}_{i,j}}{\partial P_{oi,j}} \Delta P_{oi,j} \\ & + \frac{\partial f^{(n)}{}_{i,j}}{\partial P_{oi+1,j}} \Delta P_{oi+1,j} + \frac{\partial f^{(n)}{}_{i,j}}{\partial P_{oi,j-1}} \Delta P_{oi,j-1} + \frac{\partial f^{(n)}{}_{i,j}}{\partial P_{oi,j+1}} \Delta P_{oi,j+1} = 0 \end{aligned} \qquad (14)$$

Then the film pressure is

$$P_o{}^{(n+1)} = P_o{}^{(n)} + \Delta P_o{}^{(n)} \qquad (15)$$

By introducing the ADI method to solve the equation(14) and then the Eq.(14) for j-th column can be written in a matrix form as

$$\{\Delta P_{oj}\} = [A_j]^{-1}\{b_j\}$$

$$P_o{}^{(n+1)}{}_j = P_o{}^{(n)}{}_j + \Delta P_o{}^{(n)}{}_j \qquad j = 1,2,\Lambda\ \Lambda\ ,N \qquad (16)$$

where,

$$\{\Delta P_{oj}\} = \{\Delta P_{o1,j}, \Delta P_{o2,j}, \Delta P_{o3,j}, \Lambda\ \Lambda\ , \Delta P_{oM-1,j}, \Delta P_{oM,j}\}^T$$

$$\{b_j\} = -\{f^{(n)}{}_{1,j}, f^{(n)}{}_{2,j}, f^{(n)}{}_{3,j}, \Lambda\ \Lambda\ , f^{(n)}{}_{M-1,j}, f^{(n)}{}_{M,j},\}$$

$$[A_j] = \begin{bmatrix} d_1 & c_1 & 0 & \Lambda & \Lambda & 0 \\ a_2 & d_2 & c_2 & 0 & \Lambda & 0 \\ 0 & a_3 & d_3 & c_3 & \Lambda & M \\ M & M & O & O & \Lambda & M \\ 0 & \Lambda & \Lambda & a_{M-1} & d_{M-1} & c_{M-1} \\ 0 & 0 & \Lambda & \Lambda & a_M & d_M \end{bmatrix}$$

$$d_i = \left(\frac{\partial f_{i,j}}{\partial P_{oi,j}}\right)^{(n)}, \quad a_i = \left(\frac{\partial f_{i,j}}{\partial P_{oi-1,j}}\right)^{(n)}, \quad c_i = \left(\frac{\partial f_{i,j}}{\partial P_{oi+1,j}}\right)^{(n)}$$

1.2 Equilibrium positions

The moment due to the film pressure and the rotational stiffness of pivot is

$$M = \int_{-L/2}^{+L/2}\int_{\theta_s}^{\theta_e}(r_\xi f_\eta - r_\eta f_\xi)Rd\theta\,dz - k_\theta(\delta - \delta_o) \quad (17)$$

where,

$$\begin{aligned} r_\xi &= R\sin(\theta-\theta_p) \\ r_\eta &= R_o - R\cos(\theta-\theta_p) \\ f_\xi &= (p-p_a)\sin(\theta-\theta_p)+\tau_\theta\cos(\theta-\theta_p) \\ f_\eta &= -(p-p_a)\cos(\theta-\theta_p)+\tau_\theta\sin(\theta-\theta_p) \end{aligned} \quad (18)$$

Where R_o represents the radius from the center of pad to the pivot and τ_θ is the shear stress on the pad surface due to the fluid viscosity and can be neglected in general tilting pad bearings. The second term in Eq.(17) is the reaction moment by the rotational stiffness of pivot and δ_o is initial machined tilt-angle. Then the Eq.(17) is rearranged as below.

$$M = \int_{-L/2}^{+L/2}\int_{\theta_s}^{\theta_e} R_o(p-p_a)\sin(\theta-\theta_p)Rd\theta\,dz - k_\theta(\delta-\delta_o) \quad (19)$$

This moment must be zero to maintain the pad under the equilibrium position. The tilt angle can be found and Newton-Raphson iteration.

$$\delta^{(n+1)} = \delta^{(n)} - \frac{\Delta M}{k_{\delta\delta}+k_\theta} \quad (20)$$

Where ΔM is a moment balance error about the pivot and $k_{\delta\delta}$ is the rotational stiffness of the pad by fluid film. In the gas bearings, the effective rotational stiffness of the pad, $k_{\delta\delta}+k_\theta$ can be negative value since the negative pressure is allowed, this may occur an instability on the pad.

The equilibrium position of journal can be also determined by a similar way above.

$$x_e^{(n+1)} = x_e^{(n)} - \frac{k_{yy}\Delta F_x - k_{xy}\Delta F_y}{k_{xx}k_{yy}-k_{xy}k_{yx}}$$

$$y_e^{(n+1)} = y_e^{(n)} - \frac{k_{xx}\Delta F_y - k_{yx}\Delta F_x}{k_{xx}k_{yy}-k_{xy}k_{yx}} \quad (21)$$

Where ΔF_x, ΔF_y are horizontal and vertical force balance errors.

4. Results and Discussions

For the comparisons of the offset effects, all results are obtained for the bearing geometry as follows.

$$d = 20\,mm,\ L = 18\,mm,\ c_p = 25\,\mu m$$
$$m = 0.5,\ p_a = 10135\,Pa,$$
$$\theta_e - \theta_s = 100^\circ\ (3\,pads,\ LBP)$$

The flexure-pivot pad bearing has a rotational stiffness at the pivot, unlike the conventional tilting

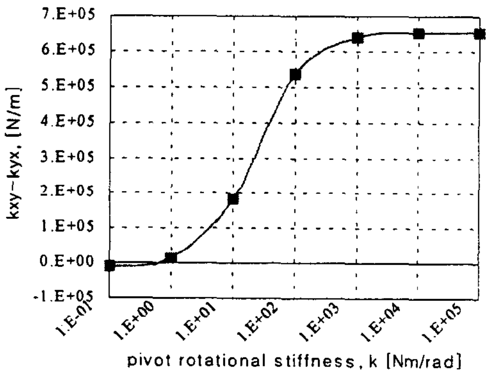

Fig. 2 Pivot rotational stiffness effects on cross couple stiffness

pad bearings. The cross couple terms in the dynamic coefficients is generated by the rotational

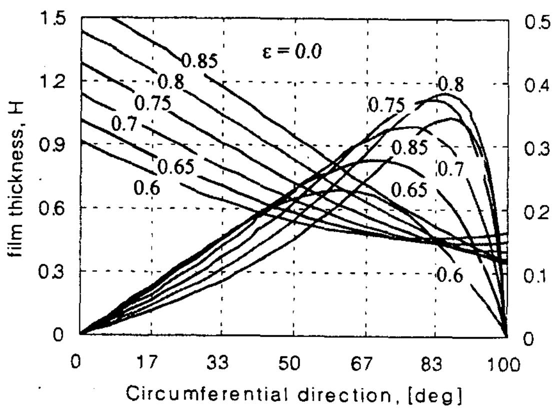

Fig. 3 Film pressure and film thickness (50,000rpm)

stiffness of pad at the web of flexure-pivot pad. Figure 2 shows the effects of the pivot rotational stiffness on the cross couple stiffness. In general, the oil whip does not occur until the value of $k_{xy} - k_{yx}$ reach up to about a half value of that of

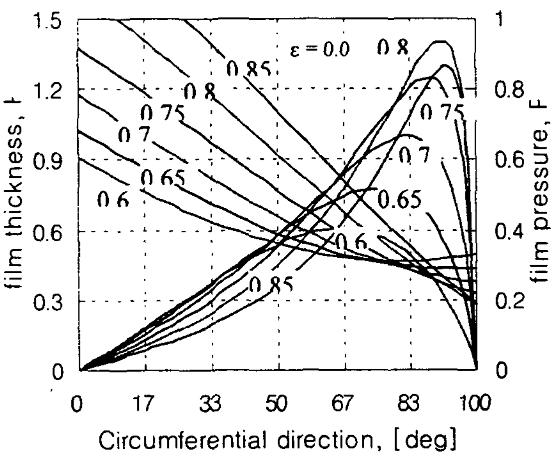

Fig. 4 Film pressure and film thickness (100,000rpm)

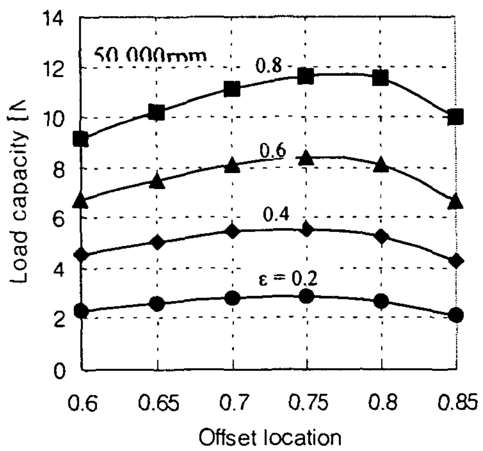

Fig. 5 Effects of offset location on load capacity

infinite pivot rotational stiffness. Therefore, in this paper, the rotational stiffness sets as 8 Nm/rad which is equivalent to about 20% of the saturation value.

Figures 3 and 4 present the film pressure and the film thickness for 50,000rpm and 100,000rpm, respectively. The film pressure considerably varies with the offset, the maximum film pressure increases until the offset is 0.8, unlike the oil

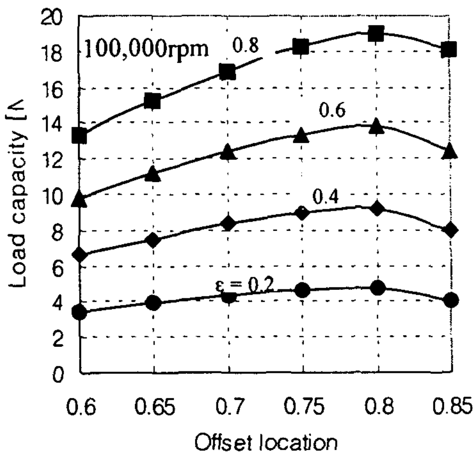

Fig. 6 Effects of offset location on load capacity

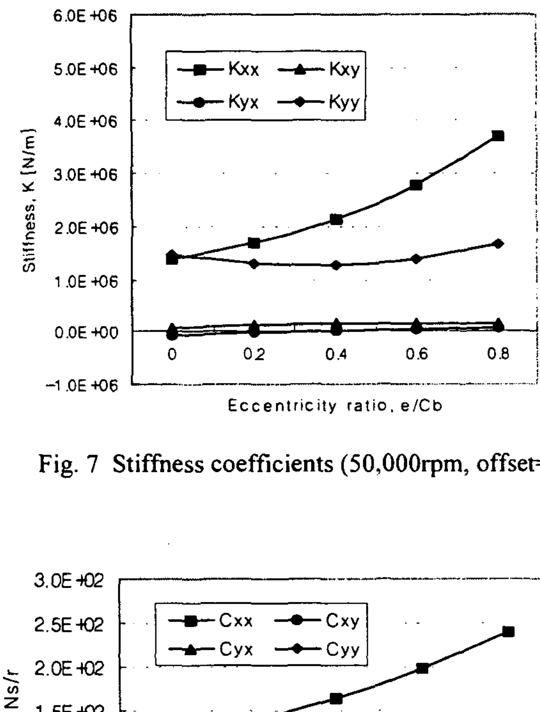

Fig. 7 Stiffness coefficients (50,000rpm, offset=0.65)

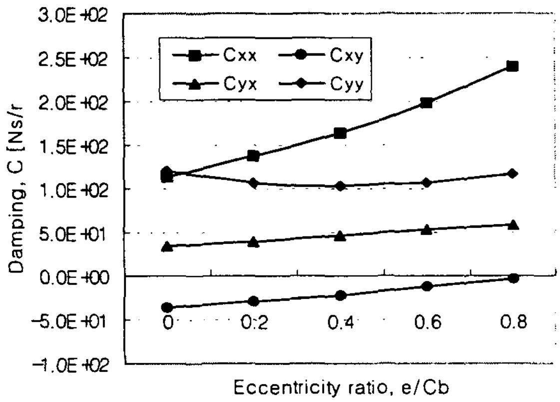

Fig. 8 Damping coefficients (50,000rpm, offset=0.65)

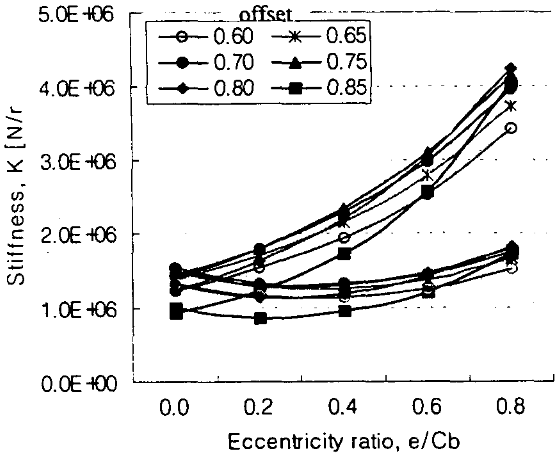

Fig. 9 Offset effect on the stiffness coefficient (50,000rpm)

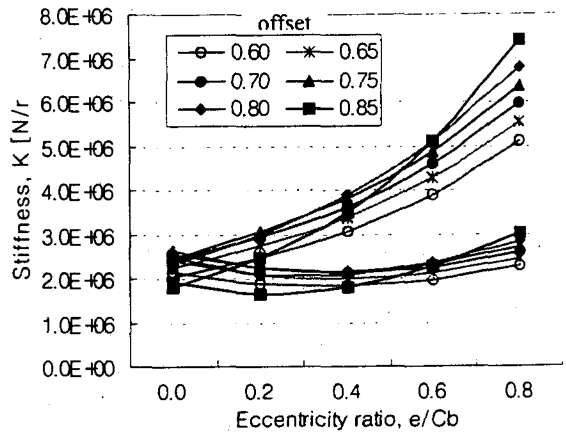

Fig. 11 Offset effect on the stiffness coefficient (100,000rpm)

lubricated tilting pad journal bearings. That means the optimum offset for the static performance could be a higher value in the gas bearings. The film thickness ratio is increased with the offset.
These tendencies are more severe at higher rotating speed as shown in Fig.4. That is the optimum offset for the static performance increases with the compressibility parameter, Λ.

Figures 5 and 6 describe the load capacity according to the offset location. The optimum offset location for the load capacity varies with the eccentricity ratio and the rotating shaft speed (the compressibility parameter). It is shown that the optimum offset location of the static performance has a higher value with the compressibility parameter.

Figs. 7 and 8 are shown the dynamic coefficients for the offset of 0.65. The cross couple terms in the coefficients are not vanished due to the pivot rotational stiffness of 8Nm/rad. But their magnitude is not high in comparison of the direct terms. In the direct coefficients, the horizontal coefficients are increased with the eccentricity ratio, but the vertical coefficients decrease with the eccentricity ratio and increase after an eccentricity ratio again. And the cross couple damping coefficients are not small compared with the direct terms.

Fig. 9 shows the offset effect on the stiffness coefficients for 50,000 rpm. The stiffness also

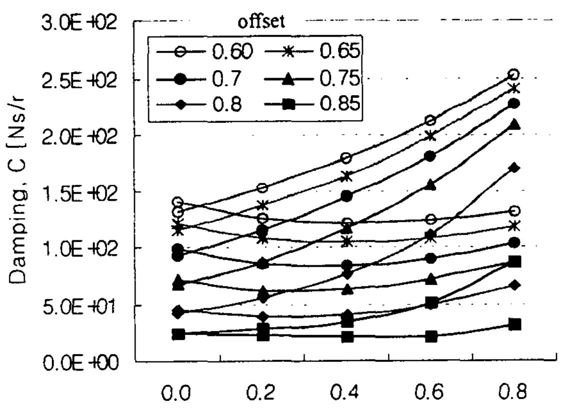

Fig. 10 Offset effect on the damping coefficient (50,000rpm)

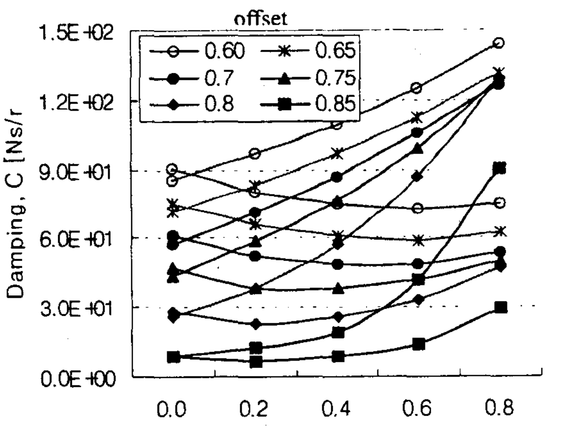

Fig. 12 Offset effect on the damping coefficient (100,000rpm)

varies with the offset at a given eccentricity ratio. The offset for maximum stiffness is a similar value with the static case. But the damping is very different from the stiffness cases, as shown in Fig. 10. The damping decreases deeply with the offset. For the damping, it is better to use a lower offset. Since higher damping is very important in the gas lubricated tilting pad bearing, to decide the offset from just the static or stiffness results is not a good way. The effect of the offset on the stiffness and the damping coefficients are shown in Figs. 11 and 12. The stiffness increases with the rotating speed at given eccentricity ratio and the offset value. To get a higher stiffness coefficient the offset should be larger at higher speed. The damping coefficients decrease with the rotating speed. This reason is that the average film thickness become higher because the film thickness ratio increase with the rotating speed.

5. Conclusions

The performance analysis of gas lubricated flexure-pivot tilting pad journal bearing has been performed. The offset effects on the static and the dynamic characteristics of the bearing are investigated for a given bearing dimensions.

The optimum offset for the static performance can be up to 0.8 dependent on the compressibility parameter. The optimum offset for the stiffness coefficients is also very similar with the situation of the static case. But for the damping the offset should be a lower value. In the hydrodynamic gas bearings, the damping is very important to avoid the larger imbalance response at the critical speeds. Therefore the offset would be better to decide by consideration of the damping coefficient.

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