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수면에 설치된 2차원 이중 연직면에 의한 파랑 반사의 양함수 해법

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An Explicit Solution to the Reflection and Transmission of Wav Surface Vertical Barrier

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Key Word : Gravity wave(중력파), Reflection(반사), Transmission(투과), Surface vertical

Abstract

단일 소파막에 대한 파도 반사문제와 투과, 또 이중으로 설치된 소파막에 대한 반사와 투과문제에 대한 양함수 해를 도출하였다. 단일 막에 대한 파도의 반사가 에어리 파의 해당 벽에 의해서만 입자반사로서 일어난다고 보고 이를 원거리 파도에너지와 일치시켜 반사 파고를 얻었다. 이중 막에 대해서는 각각의 파도가 독립적으로 단일 막에 의해서 반사 또는 투과가 되고 이들이 중첩되어 전체 파고가 형성되어지는 것으로 간주하여 전체 반사계수와 투과계수를 결정하였다. 단일 막에 대한 투과계수의 결과는 정성적으로 다른 계산결과와 비슷한 경향을 보이며, 실험결과보다는 높게 예측되었다. 이중 단일 막에 대한 결과는 낮은 주파수에서 실험결과와 유사한 경향을 보이고, 중간 주파수에서 완만한 감소를 보였다. 계산이 양함수 해로 얻어지므로 개념설계와 현장에서 유용하게 활용될 수 있으리라 기대된다.

1. Introduction

Wave reflection and transmission has been a hydrodynamic problem that is im design of the dam, wave barrier, offshore structure, and wave energy extra structures are used for wave barriers. The vertical surface barrier is a sim structure for applications. Ursell and Wiegel[1)] suggest different ways to pre reflection and transmission of the vertical barrier, 森平[2] has performed expe prediction give qualitative trends, but still remains discrepancy quantitat experiments.

Double barrier problem has more difficulties compared to the single barrier approximate solution is required in preliminary design, the fast and efficient provided yet. In this paper an explicit method is suggested that shows the system variable.

2. Reflection and Transmission through a Single Barrier

The reflection and transmission waves for an undercut vertical barrier with zero thickness are calculated by the related theories[1]. A derivation for the refraction is described based on the wave maker theory.

For coordinates (x, z) is assigned to describe two dimensional water waves. The incoming wave potential with height H_i is given as

$$\Phi_{i1} = \frac{H_i}{2}\frac{g}{\sigma}\frac{\cosh k(z+h)}{\cosh kh}\sin(\sigma t - kx) \quad (1)$$

where h is the depth of the water, k the wave number, g gravity acceleration, and σ the circular frequency. The wave on the surface is then

$$\eta_i = \frac{1}{g}\frac{\partial \Phi}{\partial t}\bigg|_{z=0} = \frac{H_i}{2}\sin(\sigma t - kx) = \frac{H_i}{2} Re\ ie^{i(\sigma t - kx)} \quad (2)$$

The incident wave propagates along the x-direction and meets the water barrier. Let the reflection potential in general form be

$$\Phi_r = A_p \cosh k(z+h)\sin(\sigma t + kx) + (Ax + B) + Ce^{-k_s x}\cos k_s(z+h)\cos \sigma t \quad (3)$$

where k and k_s are the solutions of the equation $\sigma^2 = gk\tanh kh$ and $\sigma^2 = -gk_s \tan k_s h$ respectively, that result from the kinematic free surface condition. When there is no uniform flow, A is zero and B is set to be zero without loss of generality. The wave maker potential for the flunger or flap type is written as with the kinematic boundary condition

$$u(0,z,t) = -\frac{\partial \Phi}{\partial x}(0,z,t) = -A_p k\cosh k(z+h)\cos \sigma t + \sum_{s=1}^{\infty} C_s k_s \cos k_s(h+z)\cos \sigma t \quad (4)$$

where u is the horizontal velocity. The velocity condition to the reflection wave potential is given by the zero sum of the incident and reflection wave horizontal velocities as

$$u(0,z,t) = -\frac{kg}{\sigma}\frac{\cosh k(z+h)}{\cosh kh}\frac{H_i}{2}\cos \sigma t \quad (5)$$

on the interval $-d_1 < z < 0$. From Eqs. (4) and (5), the equality

$$-\frac{kg}{\sigma}\frac{\cosh k(z+h)}{\cosh kh}\frac{H_i}{2} U_s(z+d_1) = -A_p k\cosh k(z+h) + \sum_{s=1}^{\infty} C_s k_s \cos k_s(h+z) \quad (6)$$

holds on $-h < z < 0$, where $U_s(z+d)$ is a unit step function. The set of functions $\{\cosh k(z+h), \cos k_s(z+h), s = 1, 2, \ldots, \infty\}$ on the interval $-h < z < 0$ comprises a complete harmonic series of orthogonality functions and thus any continuous function can be expanded in terms of them. The orthogonality expansion gives

$$A_p = \frac{\frac{kg}{\sigma}}{\cosh kh}\frac{H_i}{2}\frac{\int_{-d_1}^{0}\cosh^2 k(z+h)dz}{k\int_{-h}^{0}\cosh^2 k(z+h)dz} \quad (7)$$

or

$$A_p = \frac{g}{\sigma}\frac{H_i}{2}\frac{(\cosh kd_1 \sinh kd_1 + kd_1)}{\cosh kh(\cosh kh \sinh kh + kh)} \quad (8)$$

The reflection wave potential is then

$$\Phi_r = \frac{g}{\sigma}\frac{H_i}{2}\frac{(\cosh kd_1 \sinh kd_1 + kd_1)}{\cosh kh(\cosh kh \sinh kh + kh)}\cosh k(z+h)\sin(\sigma t + kx) \quad (9)$$

neglecting the fast decaying terms. The reflected wave coefficient is then

$$R_w \equiv \frac{H_r}{H_i} = \frac{\cosh k(2h-d)\sinh kd + kd}{\cosh kh \sinh kh + kh} \quad (10)$$

The transmission coefficient is obtained by the relation

$$\chi \equiv \frac{H_t}{H_i} = \sqrt{1 - R_w^2} \quad (11)$$

or

$$\chi = \sqrt{\frac{(2\sinh kh\cosh kh - \sinh k(h-d)\cosh k(h-d) + kh + kd)}{(\sinh k(h-d)\cosh k(h-d) + kh - kd)/(\sinh kh\cosh kh + kh)}} \qquad (12)$$

Here the phase difference is zero. These coefficients are used in prediction of the waves around the double barrier. These coefficients can be determined other ways and used.

3. Waves Prior to, In Between, and Aft the Double Barrier

When there exists the double barrier, rigorous analysis requires complex model dynamics. In this paper an approximation method is presented to avoid the modeling and analysis. Assumptions are that the barriers are separated e transient transmission and reflection through one barrier not to be affected from each wave profile transmits and is reflected independently. Then the result merely the linear sum of each transmitted and reflected wave.

The incoming gravity wave with height H_{1i} approaches from the left side to the as shown in Fig. 1. A part of the incoming wave H_{1r} is reflected in the region I part H_{2i} transmits through the first barrier into the region II. This tran propagates further and hits the second barrier. Here a part of the wave H_{2r} is the region II and the remainders H_{3i} propagates further away in the region III.

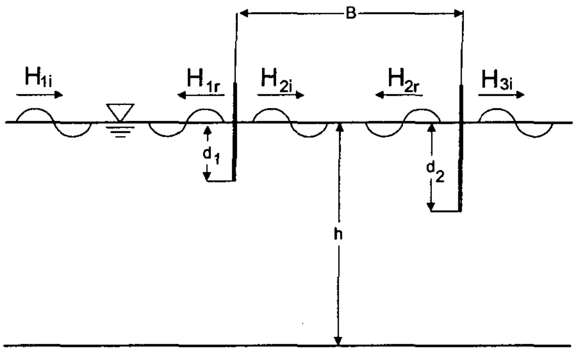

Fig. 1 Barrier profile and definitions

Since the transmission and reflection coefficient to the incident wave $Re\ ia_i e^{i(\sigma t - kx)}$ is χ_1 and $\sqrt{1-\chi_1^2}$, respectively, the first wave that transmitted is written as

$$\eta_{t21} = \chi_1\, Re\ ia_i e^{i(\sigma t - kx)} \qquad (13)$$

in the region II and the reflected wave

$$\eta_{r11} = Re\ \sqrt{(1-\chi_1^2)}\ ia_i e^{i(\sigma t - kx)} \qquad (14)$$

in the region I. This transmitted wave propagates along the positive x-direction and reflected by the another barrier of depth d_2. The reflected wave going to negative x-direction is obtained in the same manner as the reflection by the first barrier. The wave can be rewritten as

$$\eta_{t21} = \chi_1\, Re\ ia_i e^{i(\sigma t - k(x-B)) - ikB} \qquad (15)$$

the transmitted of the wave at the second barrier will be

$$\eta_{t31} = Re\ \chi_2 \chi_1\ ia_i\ e^{i(\sigma t - k(x-B)) - ikB} \qquad (16)$$

and reflected wave

$$\eta_{r21}=Re\sqrt{(1-\chi_2^2)}\ \chi_1\ ia_i\ e^{i(\sigma t+k(x-B))-ikB} \quad (17)$$

or

$$\eta_{r21}=Re\sqrt{(1-\chi_2^2)}\ \chi_1\ ia_i\ e^{i(\sigma t+kx)-2ikB} \quad (19)$$

where χ_2 is the transmission coefficient of the second barrier. The reflected wave from the second barrier comes back to the first barrier. The wave again are reflected and transmits. The transmitted wave through the first barrier propagating along the negative x-direction superposes to the reflection wave in the region I, and the reflection wave again propagating along the positive x-direction superposes to the transmitted wave in the region II as

$$\eta_{r12}=Re\sqrt{(1-\chi_2^2)}\ \chi_1^2\ ia_i\ e^{i(\sigma t+kx)-2ikB} \quad (20)$$

and

$$\eta_{i22}=Re\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}\ \chi_1\ ia_i\ e^{i(\sigma t-kx)-2ikB} \quad (21)$$

This wave again reaches the second barrier and transmits and reflects as

$$\eta_{i32}=Re\sqrt{(1-\chi_2^2)}\ \chi_2\sqrt{(1-\chi_1^2)}\ \chi_1\ ia_i\ e^{i(\sigma t-kx)-2ikB} \quad (22)$$

$$\eta_{r22}=Re\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}\ \chi_1\ ia_i\ e^{i(\sigma t+k(x-B))-3ikB} \quad (23)$$

This reflected wave again reaches the first barrier and transmits and reflects as

$$\eta_{r13}=Re\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}\ \chi_1^2\ ia_i\ e^{i(\sigma t+kx)-4ikB} \quad (24)$$

$$\eta_{i23}=Re\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}\sqrt{(1-\chi_1^2)}\ \chi_1\ ia_i\ e^{i(\sigma t-kx)-4ikB} \quad (25)$$

The incident harmonic wave supply the energy continuously. The wave profiles are the sum of the all the waves as

$$\eta_{i2}=\sum_{j=1}^{\infty}\eta_{i2j}=Re\{1+\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}e^{-2ikB}+\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}\sqrt{(1-\chi_1^2)}e^{-4ikB}+\dots\}\ \chi_1\ ia_i\ e^{i(\sigma t-kx)} \quad (26)$$

or

$$\eta_{i2}=Re\{\frac{1}{1-\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}e^{-2ikB}}\ \chi_1\ ia_i\ e^{i(\sigma t-kx)}\} \quad (27)$$

The reflection wave in the region II is obtained in the similar way as

$$\eta_{r2}=\sum_{j=1}^{\infty}\eta_{r2j}=Re\{1+\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}e^{-2ikB}+\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}\sqrt{(1-\chi_1^2)}e^{-4ikB}+\dots\}\ \chi_1\sqrt{(1-\chi_2^2)}\ \ ia_i\ e^{i(\sigma t-kx)}e^{-2ikB} \quad (28)$$

or

$$\eta_{r2}=Re\{\frac{\chi_1\sqrt{(1-\chi_2^2)}\ \ e^{-2ikB}}{1-\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}e^{-2ikB}}\ ia_i\ e^{i(\sigma t+kx)}\} \quad (29)$$

The transmission wave in the region III is then

$$\eta_{i3}\equiv Re\ ia_{i3}\ e^{i(\sigma t-kx)}=Re\{\frac{\chi_1\ \chi_2}{1-\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}e^{-2ikB}}\ ia_i\ e^{i(\sigma t-kx)}\} \quad (30)$$

And the reflection wave at the region I is then

$$\eta_{r1}\equiv Re\ ia_{r1}\ e^{i(\sigma t+kx)}=Re\{\frac{\chi_1^2\sqrt{(1-\chi_2^2)}\ \ e^{-2ikB}}{1-\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}e^{-2ikB}}\ ia_i\ e^{i(\sigma t+kx)}\}+$$

$$Re\{\sqrt{(1-\chi_1^2)}ia_i\ e^{i(\sigma t+kx)}e^{-2ikB}\} \quad (31)$$

The common factor that appears in the refractions has the properties

$$\frac{1}{1-\sqrt{(1-\chi_2^2)}\sqrt{(1-\chi_1^2)}e^{-2ikB}} \quad (32)$$

approaches 0 when $\chi\rightarrow0$, and approaches 1 when $\chi\rightarrow1$

The reflection in the region I and the transmission in the region III has to sat relation $|a_i|^2=|a_{i3}|^2+|a_{r1}|^2$ The result Eqs. (30) and (31), however, do not relationship. A correction factor is introduced as

$$\alpha\equiv\left\{\left|\frac{a_{i3}}{a_i}\right|^2+\left|\frac{a_{r1}}{a_i}\right|^2\right\}^{-1} \quad (32)$$

so that the corrected coefficients are obtained multiplying it to each results, Eq

4. Numerical Results and Comparison

4.1 Single Barrier

Numerical result of the reflection and transmission through the single barrier Weigel's prediction and the 森平's experimental one. The transmission coeffic water depth and barrier depth are shown in Fig. 2. The present computation similar trends to is Weigel's and experimental ones, but higher than them. H in transmission implies that the computed reflection is lower than these other o

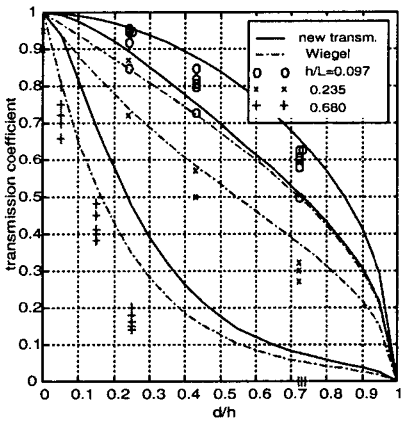

Fig. 2 Wave transmission of the single barrier

4.2 Double Barrier

Wave reflection and transmission through the double barrier is computed and the result of the eigenfunction expansion method and experiment available. Th is obtained for the double barrier depth 0.2m both separated 0.57m submerged in water depth 0.8m. The present computation is performed in the same con result are shown in Fig. 3. While the eigenfunction expansion converges to 1 approaches zero, the present result converges to 0.7, which is close to the expe frequency increases the result shows slow decreasing trend. And the result shows sharp peak at the resonance frequency of the fluid between the barrier. The experimental one does not show that kind of peaks. The calculation shows further sharp peaks which is not apparent in the experiment.

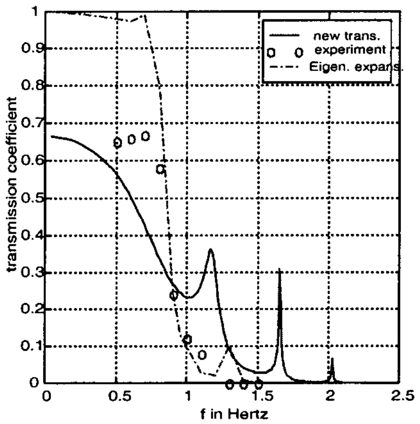

Fig. 3 Wave transmission of the double barrier

5. Conclusion

Two main results are presented in this paper. Wave reflection and transmi single and doble barrier are suggested in explicit form. The computed transmis of the single barrier shows the similar trends but higher than the experiment depth variation.

The computation result of the transmission and reflection of the double barrier asymptotic behavior as the frequency approaches zero closes to the experim decreases as the frequency increases in the middle range, and gives sharp hydrodynamic resonance frequency between the barrier. This computation is explicit formula, which can be easily used in preliminary design.

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