퍼지 집합간의 유사 퍼지 연결성

QUASI FUZZY CONNECTNESS BETWEEN FUZZY SETS

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ABSTRACT

In this paper the concept of fuzzy connectednees between fuzzy sets [8] is generalized to fussy bitopological spaces and some of its properties are studied.

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1. Introduction and preliminaries

Chang [2] used the concept of fuzzy sets to introduce fuzzy topological spaces and several authors continued the investigation of such spaces. Kubiak [7] first introduced the notion of fuzzy bitopological space as a natural generalization of fuzzy topological space(for short, fts), and initiated the the bitopological aspects due to Kelly [6] in the theory of fuzzy topological spaces. Since then several authors [5,9,10,12] have contributed to the subsequent development of various fuzzy bitopological properties. The purpose of this paper is to introduce and study the concept of quasi fuzzy connectedness between fuzzy sets in fuzzy bitopological spaces.

For definitions and results not explained in this paper, we refer to the papers [2,10-12] assuming them be well known. A fuzzy point in X with support $x \in X$ and value $\alpha (0 < \alpha \leq 1)$ is denoted x_α. For a fuzzy set λ of X, $1-\lambda$ will stand for the complement of λ. By 0_X and 1_X we will mean respectively the constant fuzzy sets taking on the values 0 and 1 on X. A fuzzy set λ in X is said to be quasi-coincident [11] with a fuzzy set μ in X, denoted $\lambda q \mu$, if there exists a point $x \in X$ such that $\lambda(x) + \mu(x) > 1$. If λ and μ are two fuzzy sets of X, then $\lambda \leq \mu$ if and only if λ and $1-\mu$ are not quasi-coincident, denoted by $\lambda \not{q} (1-\mu)$.

A fts (X, τ) is said to be fuzzy connected [3] if there is no proper fuzzy set in X which is both fuzzy open and fuzzy closed. A fts (X, τ) is said to be fuzzy connected [8] between its subsets λ and μ if and only if there is no fuzzy closed fuzzy closed fuzzy open set δ in X such that $\lambda \leq \delta$ and $\delta \not{q} \mu$.

A system (X, τ_1, τ_2) consisting of a set X with two topologies τ_1 and τ_2 on X is called a fuzzy bitopological space [7] (for short, fbts). Let (X, τ_1, τ_2) be a fbts and λ be any fuzzy set of X. Then λ is called fuzzy quasi-open (briefly, fqo) [10] if for each fuzzy point $x_\alpha \in \lambda$ there exists either a $\lambda_1 \in \tau_1$ such that $x_\alpha \in \lambda_1 \leq \lambda$ or a $\lambda_2 \in \tau_2$ such that $x_\alpha \in \lambda_2 \leq \lambda$. A fuzzy set λ is fuzzy quasi-closed (briefly, fqc) [10] if the complement $1-\lambda$ is fqo. A fbts (X, τ_1, τ_2) is said to be pairwise fuzzy connected [12] between fuzzy sets λ and μ if there is no fqco set (fqo set and fqc set) δ in X such that $\lambda \leq \delta$ and $\delta \not{q} \mu$.

2. Quasi fuzzy connectedness between fuzzy sets

Definition 2.1. A fbts (X, τ_1, τ_2) is said to be quasi fuzzy connected between fuzzy sets λ and μ if there is no fqco set (fqo set and fqc set) δ in X such that $\lambda \leq \delta$ and $\delta \not{q} \mu$.

Remark 2.1. 1) Quasi fuzzy connectedness between fuzzy sets λ and μ implies pairwise fuzzy connectedness λ and μ.

2) Quasi fuzzy connectedness between λ and μ is fuzzy connectedness of (X, τ_1) and (X, τ_2) between λ and μ.

Example 2.1. Let $X = \{a, b\}$ and let λ, μ, υ_1 and υ_2 be fuzzy sets on X defined as follows:

$$\lambda(a) = 0.2, \quad \lambda(b) = 0.3,$$
$$\mu(a) = 0.5, \quad \mu(b) = 0.4.$$
$$\upsilon_1(a) = 0.3, \quad \upsilon_1(b) = 0.4,$$
$$\upsilon_2(a) = 0.7, \quad \upsilon_2(b) = 0.6,$$

Let $\tau_1 = \{1_X, 0_X, \upsilon_1\}$ and $\tau_2 = \{1_X, 0_X, \upsilon_2\}$ be fuzzy topologies on X. Then (X, τ_1) and (X, τ_2) are fuzzy connected between the fuzzy sets λ and μ but (X, τ_1, τ_2) is neither quasi fuzzy connected not pairwise fuzzy connected between λ and μ.

Example.2.2. Let $X = \{a, b\}$ and let λ, μ, $\upsilon_1, \upsilon_2, \delta_1$ and δ_2 be fuzzy sets on X defined as follows:

$$\upsilon_1(a) = 0.5, \quad \upsilon_1(b) = 0.6,$$
$$\upsilon_2(a) = 0.5, \quad \upsilon_2(b) = 0.7,$$
$$\delta_1(a) = 0.5, \quad \delta_1(b) = 0.4,$$
$$\delta_2(a) = 0.5, \quad \delta_2(b) = 0.3,$$
$$\lambda(a) = 0.5, \quad \lambda(b) = 0.3,$$
$$\mu(a) = 0.5, \quad \mu(b) = 0.2.$$

Let $\tau_1 = \{1_X, 0_X, \upsilon_1, \upsilon_2\}$ and $\tau_2 = \{1_X, 0_X, \delta_1, \delta_2\}$ be fuzzy topologies on X. Then fbts (X, τ_1, τ_2) is pairwise fuzzy connected between λ and μ but not quasi fuzzy connected between λ and μ. Also, neither (X, τ_1) nor (X, τ_2) are fuzzy connected between the fuzzy sets λ and μ.

Theorem 2.1. A fbts (X, τ_1, τ_2) is quasi fuzzy connected between fuzzy sets λ and μ if and only if there is no fqco set δ in X such that $\lambda \leq \delta \leq 1 - \mu$.

Proof. Obvious.

Theorem 2.2. If a fbts (X, τ_1, τ_2) is quasi fuzzy connected between fuzzy sets λ and μ, then λ and μ are non-empty.

Proof. Evident.

Theorem 2.3. If a fbts (X, τ_1, τ_2) is quasi fuzzy connected between fuzzy sets λ and μ and if $\lambda \leq \lambda_1$ and $\mu \leq \mu_1$, then (X, τ_1, τ_2) is quasi fuzzy connected between λ_1 and μ_1.

Proof. Suppose that a fbts (X, τ_1, τ_2) is not quasi fuzzy connected between fuzzy sets λ_1 and μ_1. Then there exists a fqco set δ in X such that $\lambda_1 \leq \delta$ and $\delta \not{q} \mu_1$. Clearly $\lambda \leq \delta$. Now we claim that $\delta \not{q} \mu$. If $\delta \, q \, \mu$, then there exists a point $x \in X$ such that $\delta(x) + \mu(x) > 1$. Thus we have $\delta(x) + \mu_1(x) > \delta(x) + \mu(x) > 1$ and $\delta q \mu_1$, a contradiction. Consequently, a fbts (X, τ_1, τ_2) is not quasi fuzzy connected between λ and μ.

Theorem 2.4. A fbts (X, τ_1, τ_2) is quasi fuzzy connected between λ and μ if and only if it is quasi fuzzy connected between $\mathrm{qcl}(\lambda)$ and $\mathrm{qcl}(\mu)$.

Proof. Necessity follows from Theorem 2.3.

Sufficiency: Suppose that a fbts (X, τ_1, τ_2) is not quasi fuzzy connected between fuzzy sets λ and μ. Then there is a fqco set δ in X such that $\lambda \leq \delta$ and $\delta \not{q} \mu$. Since $\lambda \leq \delta$, $\mathrm{qcl}(\lambda) \leq \mathrm{qcl}(\delta) \leq \delta$ because δ is fqc. Now we have,

$$\begin{aligned} \delta \not{q} \mu &\Leftrightarrow \delta \leq 1 - \mu \\ &\Rightarrow \delta \leq \mathrm{qint}(1 - \mu) \\ &\Rightarrow \delta \leq 1 - \mathrm{qcl}(\mu) \\ &\Rightarrow \delta \not{q} \, \mathrm{qcl}(\mu) \end{aligned}$$

Hence fbts (X, τ_1, τ_2) is not quasi fuzzy connected between $\mathrm{qcl}(\lambda)$ and $\mathrm{qcl}(\mu)$. This is a contradiction.

Theorem 2.5. Let (X, τ_1, τ_2) be a fbts and let λ and μ be two fuzzy sets in X. If $\lambda q \mu$, then (X, τ_1, τ_2) is quasi fuzzy connected between λ and μ.

Proof. If δ is any fqco set in X such that $\lambda \le \delta$, then $\lambda q \mu \Rightarrow \delta q \mu$.

Remark 2.2. The converse of Theorem 2.5 may not be true as is shown by the next example.

Example 2.3. Let $X = \{a, b\}$ and let λ, μ, υ_1 and υ_2 be fuzzy sets on X defined as follows:

$$\lambda(a) = 0.5, \ \lambda(b) = 0.4,$$
$$\mu(a) = 0.3, \ \mu(b) = 0.5.$$
$$\upsilon_1(a) = 0.2, \ \upsilon_1(b) = 0.9,$$
$$\upsilon_2(a) = 0.8, \ \upsilon_2(b) = 0.1.$$

Let $\tau_1 = \{1_X, 0_X, \upsilon_1\}$ and $\tau_2 = \{1_X, 0_X, \upsilon_2\}$ be fuzzy topologies on X. Then (X, τ_1, τ_2) is quasi fuzzy connected between λ and μ but $\lambda \not{q} \mu$.

Theorem 2.6. If a fbts (X, τ_1, τ_2) is quasi fuzzy connected neither between λ and μ_0 nor between λ and μ_1, then (X, τ_1, τ_2) is not quasi fuzzy connected between λ and $\mu_0 \cup \mu_1$.

Proof. Since X is quasi fuzzy connected neither between λ and μ_0 nor between λ and μ_1, there exist fqco sets δ_0 and δ_1 in (X, τ_1, τ_2) such that $\lambda \le \delta_0$, $\delta_0 \not{q} \mu_0$ and $\lambda \le \delta_1$, $\delta_1 \not{q} \mu_1$. Put $\delta = \delta_0 \cap \delta_1$. Then δ is fqco and $\lambda \le \delta$. Now we claim that $\delta \not{q} (\mu_0 \cup \mu_1)$. If $\delta q (\mu_0 \cup \mu_1)$, then there is a point $x \in X$ such that $\delta(x) + (\mu_0 \cup \mu_1)(x) > 1$. This implies that $\delta q \mu_0$ or $\delta q \mu_1$, a contradiction. Hence X is not quasi fuzzy connected between λ and $\mu_0 \cup \mu_1$.

Theorem 2.7. A fbts (X, τ_1, τ_2) is quasi fuzzy connected between every pair of its non-empty fuzzy subsets if and only if there is no non-empty proper fqco set.

Proof. Necessity: Let λ and μ be any pair of non-empty fuzzy subsets of X. Suppose that (X, τ_1, τ_2) is not quasi fuzzy connected between λ and μ. Then there is a fqco set δ in X such that $\lambda \le \delta$ and $\delta \not{q} \mu$. Since λ and μ are non-empty, it follows that δ is a non-empty proper fqco subset of X.

Sufficiency: If there exits a non-empty proper fqco subset δ of (X, τ_1, τ_2), then it is not quasi fuzzy connected between δ and $1-\delta$. This is a contradiction.

Remark 2.3. If a fbts (X, τ_1, τ_2) is quasi fuzzy connected between a pair of its subsets, then it need not necessarily hold that (X, τ_1, τ_2) is quasi fuzzy connected between every pair of its subsets as is shown by the next example.

Example 2.4. Let $X = \{a, b\}$ and let δ_1, δ_2, λ_1, λ_2, μ_1 and μ_2 be defined as follows:

$$\delta_1(a) = 0.4, \ \delta_1(b) = 0.6,$$
$$\delta_2(a) = 0.6, \ \delta_2(b) = 0.4,$$
$$\lambda_1(a) = 0.7, \ \lambda_1(b) = 0.8,$$
$$\lambda_2(a) = 0.3, \ \lambda_2(b) = 0.2,$$
$$\mu_1(a) = 0.8, \ \mu_1(b) = 0.7.$$
$$\mu_2(a) = 0.2, \ \mu_2(b) = 0.3.$$

Let $\tau_1 = \{1_X, 0_X, \delta_1\}$ and $\tau_2 = \{1_X, 0_X, \delta_2\}$ be fuzzy topologies on X. Then (X, τ_1, τ_2) is quasi fuzzy connected between λ_1 and μ_1 but it is not quasi fuzzy connected between λ_2 and μ_2. Also there exists non-empty proper fqco set in (X, τ_1, τ_2).

Theorem 2.8. Let $(Y, (\tau_1)_Y, (\tau_2)_Y)$ be a subspace of a fbts (X, τ_1, τ_2) and let λ, μ be fuzzy sets of Y. If $(Y, (\tau_1)_Y, (\tau_2)_Y)$ is quasi fuzzy connected between λ and μ, then (X, τ_1, τ_2) is also quasi fuzzy connected between λ and μ.

Proof. Evident.

Theorem 2.9. Let $(Y, (\tau_1)_Y, (\tau_2)_Y)$ be a subspace of a a fbts (X, τ_1, τ_2) and λ, μ be fuzzy sets of Y. If (X, τ_1, τ_2) is quasi fuzzy connected between λ and μ and if χ_Y is τ_1-clopen and τ_2-clopen in (X, τ_1, τ_2), then $(Y, (\tau_1)_Y, (\tau_2)_Y)$ is quasi fuzzy connected between λ and μ.

Proof. Suppose that $(Y, (\tau_1)_Y, (\tau_2)_Y)$ is not quasi fuzzy connected between λ and μ. Then there exists a fqco set δ in X such that $\lambda \le \delta$ and $\delta \not q \mu$. Since χ_Y is τ_1-clopen and τ_2-clopen, δ is fqco in (X, τ_1, τ_2). Thus (X, τ_1, τ_2) is not quasi fuzzy connected λ and μ, which is a contradiction.

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