# Estimation of Radar Cross Section for a Swerling 1 Target

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## Abstract

In this paper, we consider the problem of estimation of average radar cross section (RCS) for Swerling 1 fluctuation model, based on the maximum likelihood (ML) estimation method. In a mathematical development, we take into account the event that target strength is lower than detection threshold, or the target is not detected. Our ML estimation for the SNR uses the score function that is the joint probability-pdf of the events and random variables. The solution to the ML estimation reduces to an expression in the form of a contraction mapping. The computational efficiency of the contraction mapping theorem is significant in computing the ML estimation as compared with other root-finding algorithms for most radar tracking conditions.

## I. Introduction

The radar cross section (RCS) of most complex targets depends significantly on the target geometry and the aspect angle of the target relative to the radar[1, 2]. As a target moves, the aspect angle and RCS change. The fluctuation of RCS is described by a stochastic process, that is, the RCS of a target is treated as a random variable with a probability density function (pdf) and a correlation function in time. Often, the RCS fluctuations are characterized as one of four Swerling types depending on the pdf and the correlation function. Moreover, the signal-to-noise ratio (SNR) or received power of a target is proportional to the average RCS of the target.

Recently, the problem of radar resource allocation has been addressed by many authors[8, 9, 10]. van Keuk and Blackman gave a pair of optimal prediction accuracy and optimal SNR to minimize the radar resources required for track maintenance for a phased array radar. Also, Hong and Jung formulated the radar energy- optimal problem into a nonlinear control problem and obtained a pair of optimal sequences of track-update intervals and SNRs for a Swerling 1 target. Blackman, *et al.* presented the manner in which IMM/MHT tracking and data association methods lead to efficient agile beam radar allocation. In literatures [8, 9, 10], SNR is one of the control inputs in optimal resource allocation problem. However, the quantity SNR or average RCS of the target is not a known value practically. Therefore, estimation of SNR using the informations extracted from received signal strength is inevitable so as to control SNR of the target under track.

Numerous data association and target-tracking algorithms have been developed over last three decades[3, 4, 5, 6, 7], and most of them assumed that the target SNR was already known. Accordingly, estimation of SNR is required to ensure best performances of tracking and data association. Techniques for estimating the SNR were discussed by Blair and Brandt-Pearce[11]. They estimated the SNRs of Swerling targets based on the Maximum Likelihood (ML) method for discriminating between different Swerling targets.

In this paper, we derive an expression for estimating SNR for Swerling 1 target, taking into account the event that target signal strength is lower than a pre-specified detection threshold and, thus, the target is not detected. Our ML estimation for the SNR uses the likelihood function that is the joint probability-pdf of the events and random variables[12]. The joint probability-pdf is consisted of the probability of the events that the target is not detected and the pdf of random variables that are the normalized squares of received signal strength. Unfortunately, the solution to the ML estimation cannot be written in an analytic form. It has an expression in the form of a contraction mapping. The efficiency of the contraction mapping theorem is significant in computing the ML estimation for most radar tracking conditions[13]. We discuss about the conditions of convergence in computing the ML estimation via contraction mapping theorem. Also, we show the numerical examples for various nominal SNRs.

## II. Background

The received signals of the radar/sonar system, corrupted by additive noise, are coherently integrated to give the two orthogonal signal components $(A_1+N_1,\ A_2+N_2)$. $A_1$ and $A_2$ represent the instantaneous in-phase and quadrature phase signal amplitudes, respectively, with the signal energy $E[A_i^2]=E_R/2,\ i=1,2$. Specifically we assume that the signal amplitudes $A_1$ and $A_2$ are uncorrelated Gaussian following a Swerling 1 fluctuation model. $N_1$ and $N_2$ represent the receiver noise, respectively, with the noise energy $E[N_i^2]=N_R/2,\ i=1,2$. The quantity SNR is given by $E_R/N_R$. Given that $A_i$, $N_i$ are Gaussian and mutually independent, the sums $A_i+N_i$ again are Gaussian[8, 9, 11]. The signal amplitude $A=(A_1^2+A_2^2)^{1/2}$ for Swerling 1 target is Rayleigh distributed according to

$$f(A) = \frac{A}{A_o^2}\exp\left[-\frac{A^2}{2A_o^2}\right],\quad A \ge 0 \tag{1}$$

where $E[A^2] = 2A_o^2 = E_R$. Because the RCS of the target $\sigma$ is $0.5A^2$, the RCS for Swerling type 1 is exponential distributed, which yields

$$f(\sigma) = \frac{1}{\bar{\sigma}}\exp\left[-\frac{\sigma}{\bar{\sigma}}\right],\quad \sigma \ge 0 \tag{2}$$

where $\bar{\sigma} = E[\sigma] = A_o^2$ is referred to as the average RCS of the target.

Within a range-Doppler cell, the detection of a returned measurement will takes place when the observed SNR (the square of the received signal amplitude) is higher than a specified threshold. That is, we can express the detection process as [8]

$$z = \frac{(A_1+N_1)^2+(A_2+N_2)^2}{N_R} > -\ln P_F \tag{3}$$

in terms of an exponential random variable $z$. The observed SNR or random variable $z$ has distribution

$$f(z) = \frac{1}{1+SNR}\exp\left[-\frac{z}{1+SNR}\right],\quad z \ge 0 \tag{4}$$

where $SNR = 2A_o^2/N_R = 2\bar{\sigma}/N_R$. The target detection (3) defines the relationship between the probability of detection $(P_D)$, the false alarm probability $(P_F)$, and the quantity SNR for the target such that

$$P_D = P_F^{1/(1+SNR)}. \tag{5}$$

Thus, the measurement due to the target has its strength distribution

$$f_T(z) = \frac{1}{P_D}\frac{1}{1+SNR}\exp\left[-\frac{z}{1+SNR}\right],\quad z \ge -\ln P_F. \tag{6}$$

## III. Estimation of SNR for Swerling 1 Target

### *III.1 ML estimation*

A common method for estimation of nonrandom parameters is the maximum likelihood (ML) method. To take into account the event that the target is not detected, we uses the joint probability-pdf as the score function. The joint probability-pdf is consisted of the probability of the events that the target is not detected and the pdf of the observed SNR. If observed SNR is lower than the detection threshold, it can not be usable to estimate the SNR, because it seems to be originated from random clutter or false alarm. Thus, we can write the probability of the event that the target is not detected as following:

$$P\{z \in [0, -\ln P_F]\} = \int_0^{-\ln P_F} f(z)\,dz = 1 - P_F^{1/(1+SNR)}. \tag{7}$$

Suppose that $k$ represents a discrete time index and consider a case that $M$ measurements of $N$ observed SNRs, $z^N = \{z_i,\ i=1,\dots,N\}$, are lower than the detection threshold. Let $I_M$ denote the time index set of $M$ observed SNRs which are lower than the detection threshold. In this case, the joint probability-pdf of events and random variables is defined as

$$\Lambda_N = \prod_{\substack{k=1\\k\notin I_M}}^{N} f_T(z_k) \prod_{k\in I_M} P\{z_k \in [0, -\ln P_F]\}. \quad (8)$$

Substituting (6) and (7) into (8), we can rewrite the score function as follows:

$$\Lambda_N = \prod_{\substack{k=1\\k\notin I_M}}^{N} \frac{1}{P_D}\frac{1}{1+SNR}\exp\left[-\frac{z_k}{1+SNR}\right] \times \prod_{k\in I_M}(1-P_F^{1/(1+SNR)})$$
$$= \left(\frac{1}{P_D}\frac{1}{1+SNR}\right)^{N-M}\exp\left[-\frac{1}{1+SNR}\sum_{\substack{k=1\\k\notin I_M}}^{N} z_k\right] \times (1-P_F^{1/(1+SNR)})^M. \quad (9)$$

The ML estimate of SNR is given by

$$\widehat{SNR} = \arg\max_{SNR} \Lambda_N = \arg\max_{SNR} \ln\Lambda_N. \quad (10)$$

To maximize the log score function, one sets its derivative with respect to SNR to zero. Then $\widehat{SNR}$ satisfies

$$\frac{d}{dSNR}\ln\Lambda_N\Big|_{SNR=\widehat{SNR}} = 0. \quad (11)$$

Thus, the ML estimate of the SNR is given by

$$\widehat{SNR} = -1 + \ln P_F + \frac{1}{(N-M)}\sum_{\substack{k=1\\k\notin I_M}}^{N} z_k + \frac{M}{(N-M)}\frac{P_F^{1/(1+\widehat{SNR})}}{(1-P_F^{1/(1+\widehat{SNR})})}\ln P_F. \quad (12)$$

Note that the ML estimate of the SNR cannot have an expression in an analytic form. We can represent the equation (12) as $\widehat{SNR} = g(\widehat{SNR})$. In order to obtain the ML estimate of the SNR, we must solve the nonlinear equation $\widehat{SNR} = g(\widehat{SNR})$. We solved the nonlinear equation of (12) using the contraction mapping theorem[13]. The computational efficiency of the contraction mapping is significant in computing the ML estimation as compared with other root-finding algorithms. The procedure of solving the nonlinear equation performs the following steps:

*STEP 1*: Set the initial value $\widehat{SNR}$ to a proper value $\widehat{SNR}_0$

*STEP 2*: Set $i = 1$

*STEP 3*: Set $\widehat{SNR}_i = g(\widehat{SNR}_{i-1})$

*STEP 4*: If $|\widehat{SNR}_i - \widehat{SNR}_{i-1}| < \text{TOL}$, stop and return $\widehat{SNR} = \widehat{SNR}_i$

otherwise, set $i = i + 1$ and proceed to step 3

Note that the above iteration procedure is terminated less than 8 iterations in most simulation situations if it converges to global solution.

### *III.2 Convergency of ML estimation solution*

In a previous section, we solved the nonlinear equation using the contraction mapping theorem. As shown in a previous section, $g(\widehat{SNR})$ is a continuous function. Also, $g'(\widehat{SNR})$ exist on $(-\infty, \infty)$ and it is a continuous function. If $|g'(\widehat{SNR})|$ is less than 1, it converges to an unique solution theoretically. That is, it must satisfy the following inequality so as to guarantee the convergence.

$$|g'(\widehat{SNR})| = \frac{M}{(N-M)}\frac{\widehat{P_D}}{(1-\widehat{P_D})^2}(\ln\widehat{P_D})^2 < 1 \quad (13)$$

where $\widehat{P_D} = P_F^{1/(1+\widehat{SNR})}$. Fig. 1 shows the region of the pair $(M/N, \widehat{P_D})$ satisfying the condition (13). The shaded region is for guaranteeing the convergence. From Fig. 1, we know that the sufficient condition for the convergence is $M/N < 0.5$.

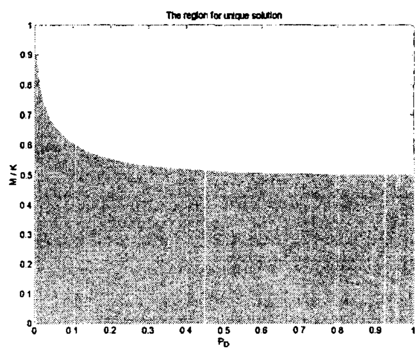



Fig. 1. The region for guaranteeing global convergence.

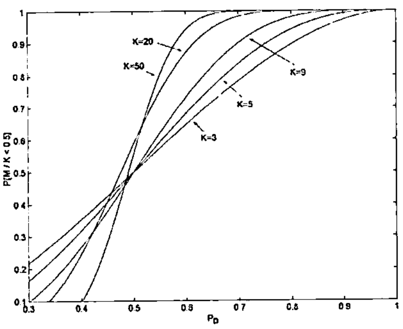



Fig. 2. The probability of convergence.

Here, we can express the probability of convergence $P_{conv}(P_D)$ as a function of $P_D$. First, the probability that $M$ measurements of $N$ observed SNRs are lower than the detection threshold for given $P_D$ and $N$ is given by the binomial probability law:

$$P\{M \textit{ misses} \text{ for } \textit{given } P_D \text{ and } N\} = \binom{N}{M} (1-P_D)^M P_D^{(N-M)} \quad (13)$$

$M/N$ must be less than 0.5 at most in order to guarantee the convergence. Thus, the probability of convergence $P_{conv}(P_D)$ for given $P_D$ and $N$ can be written as follows:

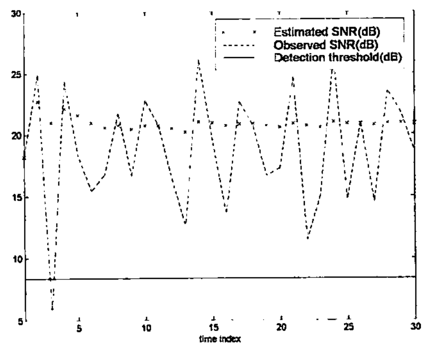
$$P_{conv}(P_D) = P\left\{\frac{M}{N} < 0.5\right\} = \sum_{M, \frac{M}{N}<0.5} \binom{N}{M} (1-P_D)^M P_D^{(N-M)} \quad (14)$$

Fig. 2. shows the probability of convergence $P_{conv}(P_D)$ as a function of $P_D$ for $N=3, 5, 9, 20, 50$. We observed that increasing the probability of detection leads to the enhancement of convergence probability from Fig. 2.

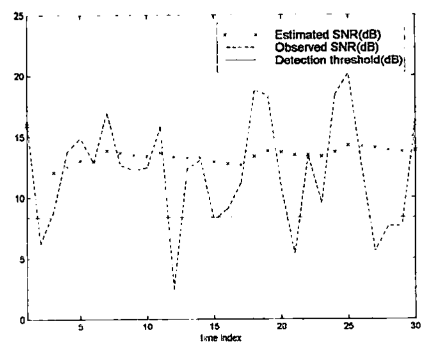
## IV. Numerical Examples

The estimation problem for a Swerling 1 target was solved numerically for three nominal SNRs to examine the performance of the proposed estimation algorithm. We set $P_F$ to $10^{-3}$ and set TOL used in contraction mapping theorem to $10^{-2}$. If it does not satisfy convergence condition after 30 iterations, we stop the contraction mapping and suspend the estimation.

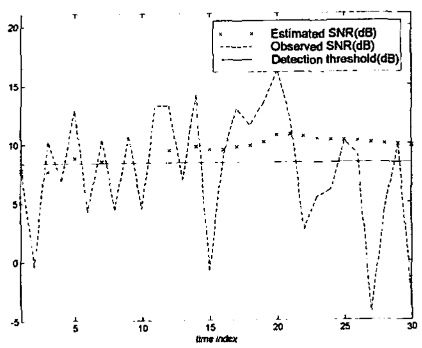
We present results for nominal SNR = 20dB, 14dB, and 10dB are shown in Fig. 3, 4 and 5, respectively. The detection probabilities are 0.934, 0.77 and 0.534, respectively. In these figures, we show the estimated SNRs, observed SNRs and detection threshold. From the figures we see that the performance is as expected: as the detection probability increases, the number of failure of convergence decreases. In a case of 20dB, ML estimates are obtained via contraction mapping at all scans in spite of occurrence of no detection at $k=3$. In a case of 14dB, 7 observed SNRs are lower than the detection threshold and ML estimates are obtained at all scan except for $k=2$. In a case of 10dB, 13 observed SNRs are lower than the detection threshold and ML estimates cannot be obtained eight times.




(a)




(b)




(c)

Fig. 5. The estimated SNR.
(a) nominal SNR = 20dB.
(b) nominal SNR = 14dB.
(c) nominal SNR = 10dB.

## IV. Conclusion

In this paper, we considered the problem of estimation of SNR for Swerling 1 target to control the transmitted power and ensure the best performances of tracking and data association, based on the Maximum Likelihood method. The event that the target is not detected has been taken into account in the mathematical development. The solution to the ML estimation has an expression in the form of a contraction mapping. To obtain the ML estimate of SNR, We solved the nonlinear equation via a contraction mapping theorem. This results can be easily extended for other Swerling targets. The future research topic is the estimation of SNR in case that the returns may be originated from not only the target but also the random clutter.

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