나이테 자료에 대한 비선형 확정론의 검토
Testing for Nonlinear Determinism in Tree-Ring Data

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1. Introduction

Low dimensional strange attractors have been investigated in the past for paleoclimatic data such as marine isotopes and tree ring records (Nicolis and Nicolis, 1984, 1987; Grassberger, 1986, 1987; Jeong and Rao, 1996). Data that are characterized by chaotic dynamics, depend on only a few degrees of freedom and will be deterministic rather than stochastic. Metric approaches based on correlation dimension and Lyapunov exponent have been widely accepted for practical applications. However, for small sample data and for noisy data sets, it may not be clear distinguishing deterministic chaos from stochastic systems based on metric approaches. To overcome the drawbacks of metric methods, a topological technique was recently developed. It is based on the observation that two mechanisms are responsible for the creation of a strange attractor: stretching and folding. We use a topological technique called close returns plot for searching for deterministic chaos in six tree ring data sets of the Sacramento River basin, USA. Close returns plot is applicable to short records, is robust to noise, and is less effected by nonstationarity because of time-ordering analysis. Each tree ring time series data consists of 312 yearly data. Our conclusion from the close returns plot is that tree ring time series exhibit many degrees of freedom which implies that stochastic models rather than deterministic chaos are more suitable to represent such type of data.

2. Close Returns Plot

A key feature of strange attractors is that they are filled with unstable periodic orbits (of which unstable fixed points are a special case). As discussed in Gilmore (1993), when a trajectory passes close enough to one of these orbits, it remains close to it for a short time. This leads to short intervals

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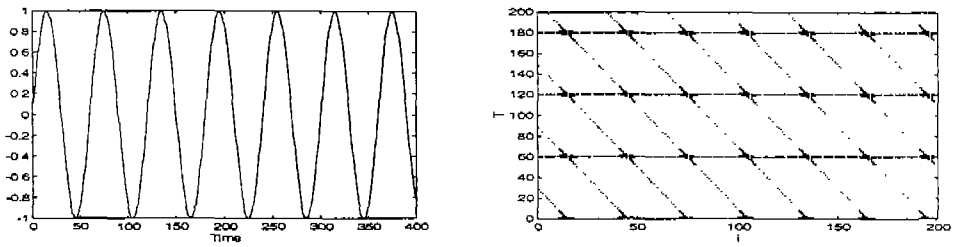
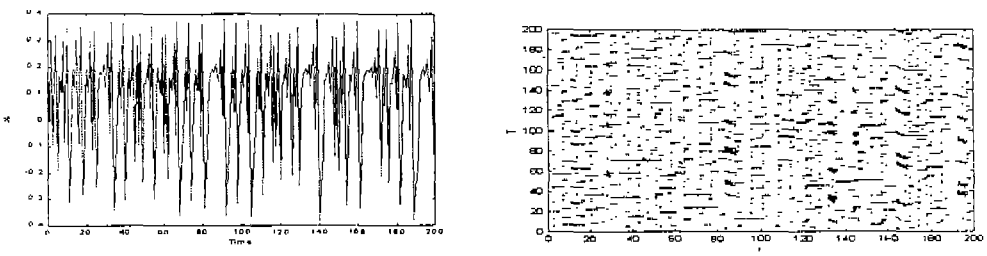
of periodicity in chaotic time series which are not present in stochastic time series.

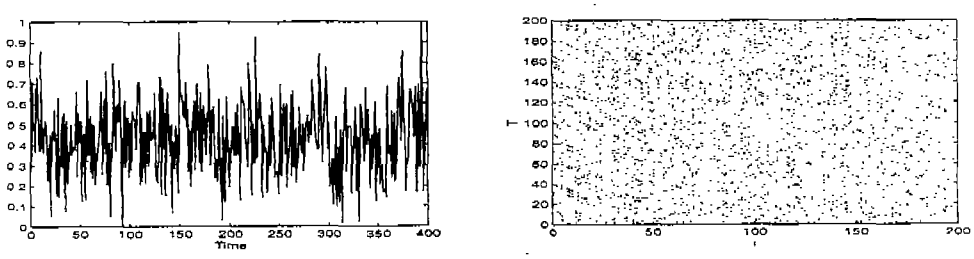
If one of the data points x_i lies near an unstable periodic orbit, then subsequent data points will stay near that orbit for a certain time until the trajectory is repelled away by the exponential divergence. If the trajectory remains near the unstable periodic orbit long enough, it will return to the neighborhood of x_i after some time T (the period of the orbit). In this case, the difference $\delta = | x_i - x_{i+T} |$ will be small for several successive values of i. This may be visualized by plotting i vs. T and indicating by black points those pairs (i, T) for which $\delta \le r$. Intervals of periodicity will appear as horizontal line segments in such a plot, and these segments should be present for a range of r values.



(a) Periodic time series plot and close returns plot



(b) Chaotic (Henon map) time series plot and close returns plot



(c) Stochastic (AR(1)) time series plot and close returns plot

Figure 1. Time series and close returns plots for the systems.

In close-returns plot (CRP), these horizontal line segments are the key to distinguish chaos from noise. In such a plot, periodic time series will generate a series of almost equally spaced horizontal line segments (Fig. 1 (a)). If short horizontal line segments exist, it indicates chaotic behavior in the time series (Fig. 1 (b)). However, if a time series is stochastic, it yields a stochastic pattern without horizontal line segments (Fig. 1 (c)).

3. Application for Tree Ring Data

The climatic characteristics of an area are reflected in the growth of the trees. Consequently, the characteristics of tree rings are studied to determine the long-term climatic behavior of a region.

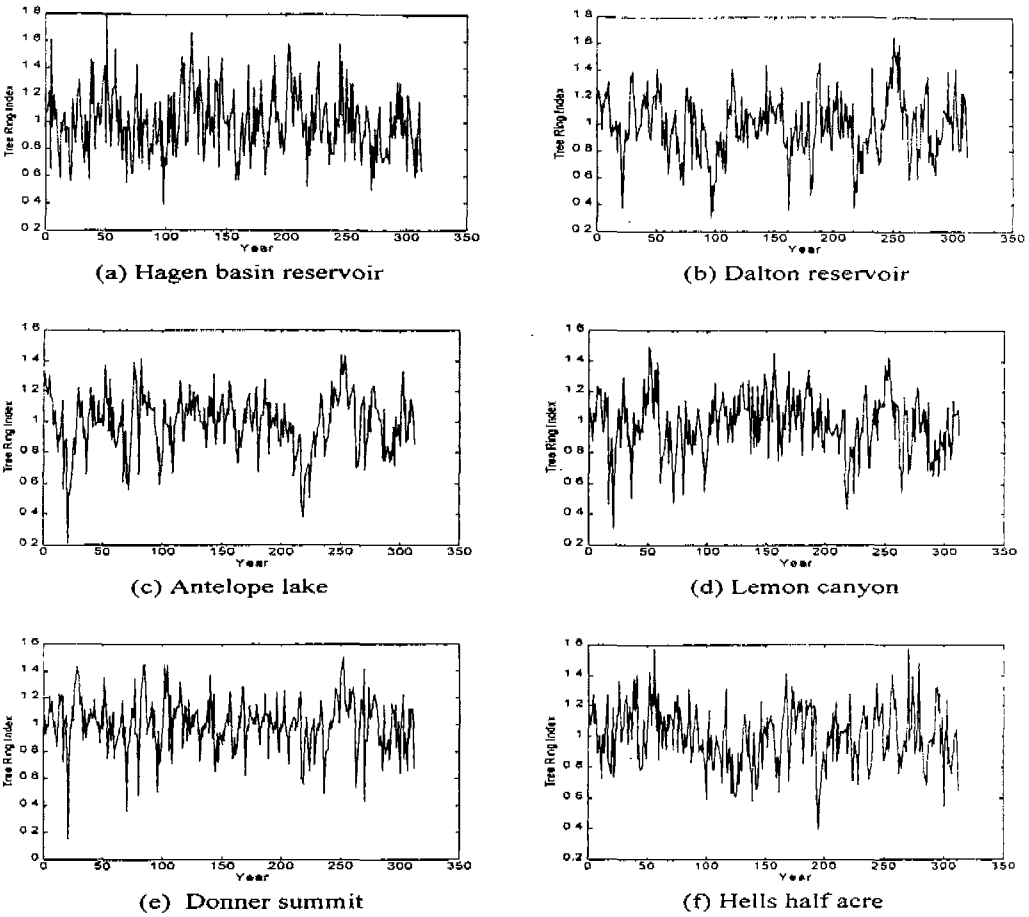


Figure 2. Time series plots of the tree rings.

In the present study, 6-tree ring series from the Sacramento river basin in California, USA are analyzed to determine their nonlinear determinism. The data sets of tree ring index series consist of the period of 1560 to 1871 and the sites are Hagen basin reservoir, Dalton reservoir, Antelope lake. Lemon canyon, Donner summit, and Hells half acre. Tree ring data sets are recorded as local climate properties and annual time scale. Each time series plot is shown in Fig. 2.

The autocorrelation functions of the data series indicate that there is some weak correlations up to a few lags of the series and the power spectrum of the data series shows that there may be some low frequency oscillations, but that they are not consistent (Fig. 3). We show that the autocorrelation function and power spectrum are just for two sites of Antelope lake and Lemon canyon because the series have similar characteristics for other sites.

The close returns plots of all sites are also similar with those of the tree ring series of Antelope lake and Lemon canyon sites (Fig. 4). In each case, the plots do not produce the evidence for the presence of unstable periodic orbits which is a characteristic of nonlinear determinism. Say there is no pattern of horizontal line segments. Therefore, the close returns plot does not provide any support for the positive indications of chaos. Nicolis and Nicolis (1984, 1986) obtained the correlation dimension of 3.1 for the a marine isotopic record but Grassberger (1986, 1987) did not agree with the work Nicolis and Nicolis. Also Jeong and Rao (1996) tried to estimate the correlation dimension for the tree ring series but they could not find the evidence of the chaotic behavior.

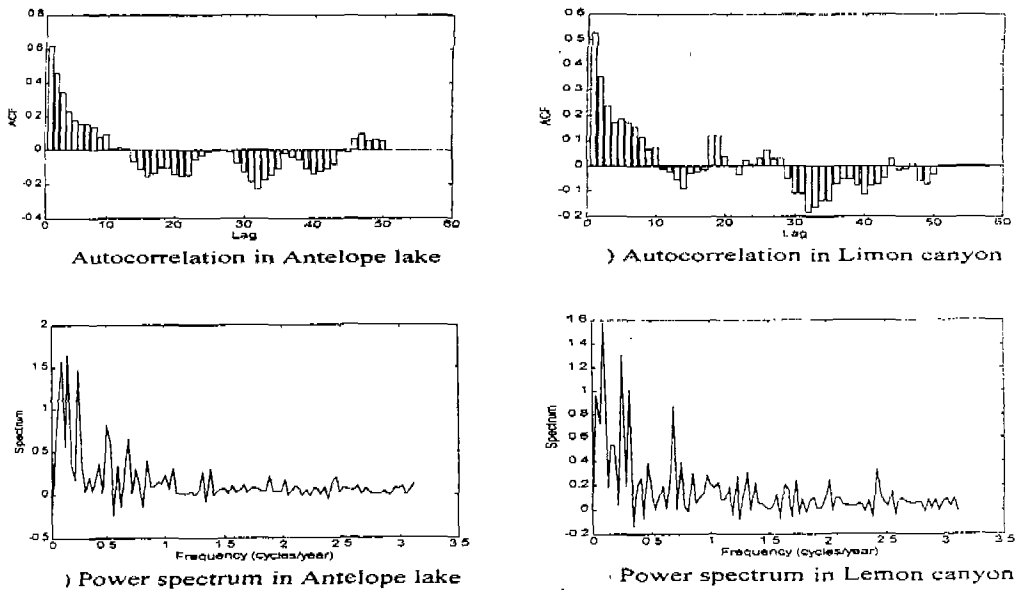


Figure 3. Autocorrelation functions and power spectrums for the tree ring series of Antelope lake and Lemon canyon

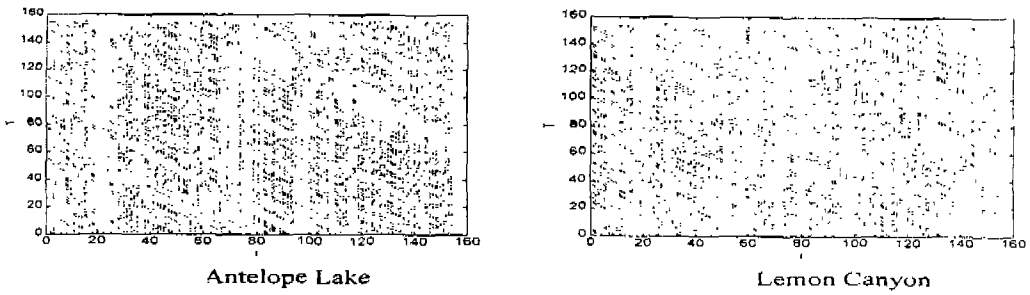


Figure 4. Close returns plots for the tree ring series of Antelope lake and Lemon canyon.

4. Conclusions

Nicolis and Nicolis (1984) investigated the nonlinear determinism for the tree ring series and obtained the correlation dimension of 3.1 with the saturated embedding dimension of 4 for a marine isotopic record. However, Grassberger (1986, 1987) criticized the work of Nicolis and Nicolis for the number of data size and for the technical issues. He goes to state spaces of dimension up to 11 for tree ring series of 7,100 data size and could not obtain the saturated correlation dimension. Jeong and Rao (1996) also tried to obtain the correlation dimension for tree ring data sets but they failed to get the correlation dimension and concluded there is no evidence of chaotic behavior in tree ring series.

Researchers investigating chaotic behavior of tree ring series have used a metric approach of the correlation integral method. However, the method has some limitations such as data size, dynamic correlation, noise and so on. In this study we have used a topological method which has some advantages for compensating for the drawbacks of the correlation integral method and a topological method may be more reasonable than the correlation integral method, especially for small data size.

We have seen that the tree ring series has no evidence of nonlinear determinism and this may give us that the series is stochastic rather than chaos. We also support the previous results which the tree ring series has no chaos characteristics.

5. References

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