The Mathematic Model of "Pressing Complexion" Differential Coefficient Countermeasure Decision for Collision-avoidance

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Abstract: In this article, we have done some analysis about the collision-avoidance specialty of "pressing complexion" and "pressing danger" in the meet process of two boats, and offered a mathematic model of differential coefficient countermeasure decision for collision-avoidance, which adapt to the right complexion. The basal idea is, in the right condition whatever do the coming boat do, and our boat will always adopt dynamic, continuous and the best countermeasure. When both the controlling capabilities of two boats have advantage and inferior position, we can working-out with the qualitative differential coefficient countermeasure.
Key Words: Collision-avoidance of Shipping, Differential Coefficient Countermeasure, Automatic Collision-avoidance

The collision of shipping is one of the most serious maritime accidents. In order to reduce the shipping collision, the IMO which as a spokesman of the international seafaring had adopted many acts, e.g. constituting and actualizing the "International Shipping Collision-avoidance", "International Pact of Sailor's Training. Certification and Keeping watch Standard In 1979 " and some other statutes. advancing "Attitude. Reason and Aim about Sailor's Complications" and some other advices, above all. had acquired finer effect. However, if we want to reduce the shipping collision accidents radically, we must fetch some new technical instruments, the mean idea is equipping shipping with the ACAS.

There are some investigated productions about the collision-avoidance countermeasure of shipping which is the hard-core soft of ACAS. In this article, we advanced a collision-avoidance countermeasure soft, which based on the operational research. It can be generalized as follow: when the collision danger is on distance or in simple condition, we can obtain the countermeasure with the geometrical collision-avoidance arithmetic. In the complicated condition, we can obtain it with the geometrical matrix countermeasure. While it is in the pressing complexion, we need obtain dynamic and continuous optimized countermeasure with the differential coefficient. The ACAS we had studied is based on the "controlling capabilities of coming boat differentiate subsystem" and the "coming boat dynamic differentiate subsystem" specially. We can obtain the T、K index and the turning, shifting dynamic information rapidly by the subsystems above[1][2]. In this article we just introduce the mathematic model of pressing complexion differential coefficient countermeasure decision for collision-avoidance of the ACAS.

1.About the Pressing Complexion

1.1 There are many annotations about the "Pressing Complexion", the one regarded by our nation seafaring is:

——The is a moment or a process.

——"If the distance the two boats are closed must lead to be incapable of passing safely when only one boat adopt action, include the great action", which can be judged having formed the pressing complexion.

——The lower limit of "Pressing Complexion" is the "Pressing Danger", which is "collision is can't be avoided only depend on the action of one boat".

——The "Pressing Complexion" is a moment too, whose lower limit is "collision can't be avoided", the aim of shipping action is "to reduce the losing which the collision can do[3][4]".

1.2 The occasion of the straight moving boat "can adopt action" should before the forming of pressing complexion. From the point of view of the mathematic model, the occasion of end of action should just before the forming of pressing complexion.

1.3 The decision for collision-avoidance of "Pressing Complexion" should adopt the differential coefficient countermeasure.

1.4 When the shipping is in the "Pressing Danger", we can adopt the controlling graph to avoid the collision.

2.The Mathematic Model of Two-ship Coefficient Countermeasure Decision for Collision-avoidance

2.1 Basal Conception

2.1.1 Differential coefficient countermeasure can be defined in this way: When every part takes its own countermeasures, we should use differential equations to describe the phenomenon and the rules. This method can be divided into two categories: determining the nature and fix the quantity. The distinguish of the two approach is that the former is up to find a performance guideline such as the time of avoiding collision, while the latter is up to research whether such a result can be reach, for example whether we can keep off the coming ship in a certain distance.

We think that we can obtain decision-making of collision-avoidance in pressing complexion by the means of the two ways. But here, making the countermeasure by determining the nature seems more close to the target of collision-avoidance safely. Consequently, in the pressing complexion, both the course and the speed are all should be altered, so it is better to use the countermeasure of determining the nature to alter double vehicles' speed and course.

There are 4 probabilities of the result of the countermeasure, they are all shown in the below chart: S—speed, GD--gyration diameter

No.	Own Ship	Coming Ship	Results
1	S(H) GD(H)	S(L) GD(L)	If the coming ship in the own side of boundary, there always have possibility of collision.
2	S(H) GD(L)	S(L) GD(H)	We can always keep away from the coming ship
3	S(L) GD(H)	S(H) GD(L)	The coming ship can always keep away from our own ship.
4	S(L) GD(L)	S(H) GD(H)	If our ship in the coming ship's side of the boundary, there always have possibility of collision.

Correctly speaking, if the collision-avoidance side have the absolute superiority, there is always have the possibility of collision or collision-avoidance. If the both sides are in the same condition, the result belongs to the position in the very beginning. In another words, if the coming ship in the outer side of the boundary, and only we control rationally, there always have possibility of collision-avoidance. If the coming ship is in the inner side of the boundary and always out of line with us, there always have the possibility of collision.

2.1.2 Boundary

The boundary divides the two-side of collision-avoidance which both have advantage and inferior position of the controlling capabilities into two parts: one is the capture area, in this area, once our own ship get in, whatever we do, the collision should never be avoided. The other part is dodge area, in this area, once our own ship get in, whatever the coming ship do, the collision should never befell. When the two-side both have the advantage, and they both do the collision-avoidance out of phase, and they are in the condition of antagonism, the boundary is just the best trajectory of the two ships' action, and they can pass safely along the boundary. It seems like the "danger pie slice" of the geometrical collision-avoidance.

In the theory of geometrical collision-avoidance, we know if the our own speed is lower than which of the coming ship, then a pie slice can be eradiated from our own position, if the coming ship is in this pie slice, the collision can't be avoided. If we adopt right action and let the coming ship out of the pie slice, then the collision would not happen.

The meaning of boundary seems as the critical side, the boundary line is not beeline, but is the curve according to the complicated condition.

2.1.3 Many practices on the sea had indicated that in the pressing complexion or pressing danger, because of the function of circumstance and mentality complication,two ships' avoidance manoeuvrability is not always harmony and even has character of antagonism.This is the reason that in this circumstance we should adopt differential coefficient countermeasure.Even the ship is provided with AIS(Automatic Identification System),when two ships are in the complexion of many ships'encounter and just in the pressing complexion,this decision-making system is also adoptable.If the two sides are not antagonistical,the avoidance decision received by differential coefficient decision-making though is not the best decision,but is optimum decision.It is helpful to the avoidance of collision.

2.2 Mathe Model[6][7]

2.2.1 We can suppose that W standing for our ship and M standing for the target ship can make changed-speed and changed-course movement .The two speed is V_W, V_m,the two course is ϕ_W, ϕ_m, the two acceleration is a_W, a_m,the two turning control is K_W, K_m,the two acceleration is ψ_W, ψ_m.Suppose that W'target volume(viz.ship domain,or a safety area around the ship body according thus complexion) is a circle whose radius is "r",namely

$$D(x,y) = x^2 + y^2 - r^2 = 0 \tag{1}$$

2.2.2. Movement Equation

(1) Absoluteness Coordinate System Movement Equation

Suppose (x_w, y_w), (x_m, y_m) are W、M's coordinate position,then we have :

$$\begin{cases} \dot{x}_w = V_w \operatorname{Sin}\phi_w \\ \dot{y}_w = V_w \operatorname{Cos}\phi_w \\ \dot{x}_m = V_m \operatorname{Sin}\phi_m \\ \dot{y}_m = V_m \operatorname{Cos}\phi_m \\ \dot{\phi}_w = \dfrac{V_w}{R_w} k_w \\ \dot{\phi}_m = \dfrac{V_m}{R_m} k_m \\ \dot{V}_w = a_w \cdot \psi_w \\ \dot{V}_m = a_m \cdot \psi_m \end{cases} \tag{2}$$

Rw、Rm stands for W、M's minimum turning radius .

(2) Relative Coordinate System Movement Equation

By rotating transform

$$\begin{cases} x = (x_m - x_w)\cos\phi_w - (y_m - y_w)\sin\phi_w \\ y = (x_m - x_w)\sin\phi_w + (y_m - y_w)\cos\phi_w \end{cases} \tag{3}$$

The coordinate base on W,W's course is y axis of the coordinate system.Then in relative coordinate system,the movement equation is :

$$\begin{cases} \dot{x}(t) = V_m \mathrm{Sin}\psi - y\dfrac{V_w}{R_w}k_w \\ \dot{y}(t) = V_m \mathrm{Cos}\psi + x\dfrac{V_w}{R_w}k_w - V_w \\ \dot{\psi}(t) = \dfrac{V_m}{R_m}k_m - \dfrac{V_w}{R_w}k_w \\ \dot{V}_w(t) = a_w \cdot \psi_w \\ \dot{V}_m(t) = a_m \cdot \psi_m \end{cases} \tag{4}$$

Here, $\psi = \phi_m - \phi_W$, (4) is state variable (x, y, ψ, Vw, Vm)'s non-linearity differential equation

2.2.3. Seeking For Countermeasure Problem's Solution

(1) Write Out Hamilton Function

$$\begin{aligned} H(x,y,\psi,Vw,Vm,t) &= \lambda^T \cdot (\dot{x},\dot{y},\dot{\psi},\dot{V}w,\dot{V}m) \\ &= \lambda_1\left(Vm\sin\psi - y\frac{Vw}{Rw}Kw\right) + \lambda_2\left(Vm\cos\psi + x\frac{Vw}{Rw}Kw - Vw\right) \\ &+\lambda_3\left(\frac{Vm}{Rm}Km - \frac{Vw}{Rw}Kw\right) + \lambda_4 a_w \psi_w + \lambda_5 a_m \psi_m \end{aligned} \tag{5}$$

(2) According to Determined Nature Double Sdes Extremum Principle :

$$\max\min H(\overline{\lambda},\overline{x},\psi_w,Kw,Km) = \min\max H(\overline{\lambda},\overline{x},\psi_w,Kw,Km) \tag{6}$$

by(4),gained :

$$\begin{aligned} H &= (-\lambda_1 y + \lambda_2 x - \lambda_3)\frac{Vw}{Rw}Kw + \lambda_3\frac{Vm}{Rm}Km + \lambda_4 a_w \psi_w \\ &+\lambda_5 a_m \psi_m + (\lambda_1 Vm\sin\psi + \lambda_2 Vm\cos\psi - \lambda_2 Vw) \end{aligned} \tag{7}$$

the best strategy is

$$\begin{cases} Kw^* = sign(-\lambda_1 y + \lambda_2 x - \lambda_3) \\ Km^* = sign\lambda_3 \\ \psi_w^* = sign\lambda_4 \\ \psi_m^* = -sign\lambda_5 \end{cases} \tag{8}$$

the symbol function—signZ defines as :

$$signZ = \begin{cases} 1 & Z > 0 \\ 0 & Z = 0 \\ -1 & Z < 0 \end{cases} ; \tag{9}$$

the accompanied equation is

$$\begin{cases} \dot{\lambda}_1 = -\dfrac{\partial H}{\partial x} = -\dfrac{Vw}{Rw} Kw \\ \dot{\lambda}_2 = -\dfrac{\partial H}{\partial y} = \dfrac{Vw}{Rw} Kw \lambda_1 \\ \dot{\lambda}_3 = -\dfrac{\partial H}{\partial \psi} = -\lambda_1 Vm \cos\psi + \lambda_2 Vm \sin\psi \\ \dot{\lambda}_4 = -\dfrac{\partial H}{\partial Vw} = -\dfrac{Kw}{Rw}(-\lambda_{1}y - \lambda_2 x - \lambda_3) + \lambda_2 \\ \dot{\lambda}_5 = -\dfrac{\partial H}{\partial Vm} = -\lambda_1 \sin\psi - \lambda_2 \cos\psi - \lambda_3 \dfrac{Km}{Rm} \end{cases} \tag{10}$$

the end value condition is

$$\begin{cases} \lambda_1(t_1) = -\mu \dfrac{\partial D(x,y)}{\partial x} = -2\mu x(t_1) \\ \lambda_2(t_1) = -\mu \dfrac{\partial D(x,y)}{\partial y} = -2\mu y(t_1) \\ \lambda_3(t_1) = -\mu \dfrac{\partial D(x,y)}{\partial \psi} = 0 \\ \lambda_4(t_1) = -\mu \dfrac{\partial D(x,y)}{\partial Vw} = 0 \\ \lambda_5(t_1) = -\mu \dfrac{\partial D(x,y)}{\partial Vm} = 0 \end{cases} \tag{11}$$

2.2.4. Write Out the Inverted Equation

In order to gain the solution,we must know the initial value.Because at this point ,the boundary and the target volume's cut point is likely to be gained exactly.So we must seek the solution backward.In order to determine the target volume's usable boundary, the equation above is need to write out backward equation also.

The backwards trajectory equation is:

$$\begin{cases} \bar{x}(\tau) = -V_m \sin\psi + y \dfrac{V_W}{R_W} K_W & x(0) = x_0 \\ \bar{y}(\tau) = -V_m \cos\psi - x \dfrac{V_W}{R_W} K_W + V_W & y(0) = y_0 \\ \bar{\psi}(\tau) = \dfrac{V_W}{R_W} K_W - \dfrac{V_m}{R_m} K_m & \psi(0) = \psi_0 \\ \bar{V}_W(\tau) = -a_W \psi_W & V_W(0) = V_W^0 \\ \bar{V}_m(\tau) = -a_m \psi_m & V_m(0) = V_m^0 \end{cases} \tag{12}$$

the backwards accompanied equation is :

$$\begin{cases} \dot{\lambda}_1(\tau) = \dfrac{V_W}{R_W} K_W \lambda_2 \\ \dot{\lambda}_2(\tau) = -\dfrac{V_W}{R_W} K_W \lambda_1 \\ \dot{\lambda}_3(\tau) = \lambda_1 V_m \cos\psi - \lambda_2 V_m \sin\psi \\ \dot{\lambda}_4(\tau) = \dfrac{K_W}{R_W}(-\lambda_1 y + \lambda_2 x - \lambda_3) - \lambda_2 \\ \dot{\lambda}_5(\tau) = \lambda_3 \dfrac{K_m}{R_m} + \lambda_1 \sin\psi + \lambda_2 \cos\psi \end{cases} \tag{13}$$

the initial value is

$$\begin{cases} \lambda_1(0) = -2\mu x(0) = \lambda_1^0 \\ \lambda_2(0) = -2\mu y(0) = \lambda_2^0 \\ \lambda_3(0) = \lambda_3^0 \\ \lambda_4(0) = \lambda_4^0 \\ \lambda_5(0) = \lambda_5^0 \end{cases} \tag{14}$$

2.2.5. The formula used to solve the best trajectory and the best control is quite complicated,we can refer to [7].

2.3 the solution procedure

(1)First determine the useable part boundary BUP.

(2)Calculate the initial best control variable $K_W^*(0)$、$K_m^*(0)$、$\psi_W^*(0)$、$\psi_m^*(0)$.

(3)By trajectory formula,calculate next time's $x(\Delta)$, $y(\Delta)$, $\psi(\Delta)$, $V_W(\Delta)$, $V_m(\Delta)$.

(4)Choose the appropriate positive value μ ,then could calculate $\lambda_1(\Delta)$、$\lambda_2(\Delta)$、$\lambda_3(\Delta)$、$\lambda_4(\Delta)$、$\lambda_5(\Delta)$.

(5)By the best control variable seek function,gain next time's $K_W(\Delta)$、$K_m(\Delta)$、$\psi_W(\Delta)$、$\psi_m(\Delta)$.

(6)Circulate calculation,until the end of boundary or beyond the countermeasure space.

3 Manoeuvrability Graphology Method

In the condition of pressing danger,because the ship's inertia is very big ,the decision commonly is one-off,not successional,then we can consider adopting manoeuvrability graph method.

When carrying the method into practise,we can adopt collision avoidance manoeuvrability graph commended by the navigation institute of Britain[5] or the scheme commended by literature [8]

According to a company's disposal of important overseas and domestic ship collision case in recent years at sea and the investigation and testing of the graduate and teacher of navigation academy,the ocean captain,navigating officer,literature [8] draw a conclusion of the pressing danger upper limit prevention decision which is similar to Britain navigation institute manoeuverability graph.

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