Optimum Directivity Synthesis of Ultrasonic Transducers Using Direct Inversion in Combination with Iterative DFP Method

K. R. Cho*, C. D. Kim**, T. Tsuchiya***, Y. Kagawa***

**Dept. of Electronic Communication Engineering, Yosu National University, Korea*
***Dept. of Electronic Engineering, Pukyong National University, Korea*
****Dept. of Electronics and Information Systems Engineering, Akita Prefectural University, Japan*

Abstract - Optimum directivity synthesis of ultrasonic transducers in linear array is considered. To realize the desired directivity, a robust and efficient method is proposed which is the direct inversion combined with the iterative DFP (Davidon-Fletcher-Powell) method. A quasi-ideal beam with the beam width and the steering beam angle specified are chosen for the numerical demonstrations. The demonstration is then extended to the case of multi-beams. The proposed combination method shows quick convergence over the single LMS or DFP method at the expense of the system matrix inversion.

Index Terms - Optimum directivity synthesis, Direct method, DFP method, Ultrasonic transducers

1 INTRODUCTION

Control Systems using the optimization method have been studied extensively and applied to various problems [1-3]. The ultrasonic transducers have been used for medical diagnosis, SONAR and nondestructive testing. Their radiation characteristic is controlled by combining with the element array configuration and the exciting distribution, for which the optimization method plays an important role to achieve the desired directivity.

Numerous papers have been published on the improvement of the performance of directivity synthesis [4-9] using various optimization methods. Because each optimization method has different characteristics in convergence characteristic, it is very important to choose the proper optimization method for the robust and efficient directivity synthesis.

Direct inversion can calculate driving distribution of the linear array to achieve the desired directivity in one operation. However, it lacks flexible control. On the other hand, the iterative method such as the LMS method or the DFP method, sometimes fails to achieve the optimum solution, without the proper initial values chosen. The LMS method also requires the proper step parameter [10].

In this paper, we propose an method which is the direct inversion combined with the iterative DFP method to overcome the disadvantages when the direct inversion or the iterative method is simply used. In the proposed method, firstly, exciting distribution for the linear array is approximately calculated by the direct inversion, secondly, this is chosen as the initial distribution to the DFP method.

To confirm the validity of this method, a quasi-ideal beam [11] with the beam width and the steering beam angle specified are chosen for the numerical demonstrations. The demonstration is then extended to the case of multi-beams.

2 DIRECTIVITY SYNTHESIS BY MEANS OF LINEAR ARRAY

Fig. 1 shows a linear array consisting of point sound sources (point source array). The sound pressure p_m, at the observation point m [12], is given by

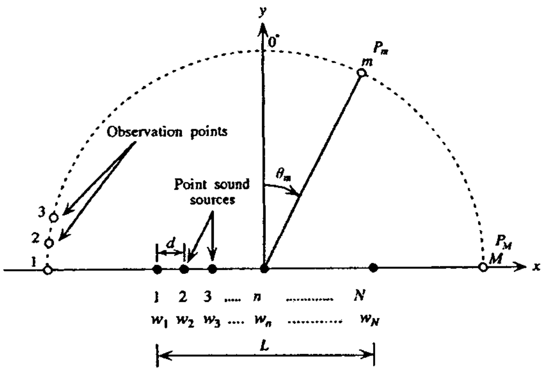


Fig. 1 *Linear array*

$$p_m = \sum_{n=-(N-1)/2}^{(N-1)/2} s_{mn} w_n \quad (m = 1, 2, .., M) \tag{1}$$

where, s_{mn} and w_n are

$$s_{mn} = \exp\left(jkd\frac{2n-N-1}{2}\ \sin\theta_m\right) \tag{2}$$

$$w_n = A_n e^{-j\phi_n} \tag{3}$$

where, A_n and ϕ_n are magnitude and phase of n-th point source. k is the wave number (= ω/c = $2\pi/\lambda$, where c is the sound speed, ω is angular frequency, λ is wavelength), d and L are interval between two point sources and array length, M *and* N are the number of observation point and the number of point sources, and θ_m is directional angle (-90° $\leq \theta_m \leq$90°) to the m-th observation point. The directivity vector is represented in the matrix form as follows

$$\{P\} = [S]\{W\} \tag{4}$$

where

$$\{P\} = \{\ p_1\ p_2 \cdots p_M\ \}^T \tag{5}$$

$$[S] = \begin{bmatrix} s_{11} & s_{12} & \cdot & \cdot & s_{1N} \\ s_{21} & s_{22} & \cdot & \cdot & s_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{M1} & s_{M2} & \cdot & \cdot & s_{MN} \end{bmatrix} \tag{6}$$

$$\{W\} = \{\ w_1\ w_2 \cdots w_N\ \}^T \tag{7}$$

where, { } indicates a vector and [ ] is a matrix. T is the transpose of matrix or vector. $[S]$ is the shape function matrix relating the position of the point source to the direction of the observation point. $\{W\}$ is the weighting vector representing the exciting distribution in the array. In Eqn. (4), directivity vector $\{P\}$ is represented by the product of the shape matrix $[S]$ and the weighting vector $\{W\}$. If the configuration is fixed, the shape function matrix $[S]$ will be constant. Thus, the directivity vector $\{P\}$ is only depends on the weighting vector $\{W\}$, and the desired directivity can be achieved by adjusting the weighting vector $\{W\}$. This is an inverse problem in which the weighting vector $\{W\}$ is to be obtained from the desired directivity.

3 THE PROPOSED ALGORITHM

3.1 Direct inversion

When the directivity vector $\{P\}$ is the desired directivity $\{Z\}=\{z_1\ z_2 \cdots z_M\}^T$, the Eqn. (4) can be rewritten as

$$[S]\{W\}=\{Z\} \tag{8}$$

If the shape function matrix $[S]$ is square, the weighting vector $\{W\}$ can be directly obtained by calculating the inverse matrix $[S]^{-1}$. $[S]$ is not usually square. Therefore, both sides of Eqn. (8) is multiplied by the transpose matrix $[S]^T$ in order to convert the matrix $[S]$ into a square matrix.

$$[S]^T[S]\{W\} = [S]^T\{Z\} \tag{9}$$

Eqn. (9) can be rewritten as

$$[A]\{W\}=\{b\} \tag{10}$$

where $[A]=[S]^T[S]$ and $\{b\}=[S]^T\{Z\}$. The weighting distribution with respect to the desired directivity is solved by direct inversion. But, this does not always guarantees the optimum solutions.

3.2 DFP method [13]

With the DFP method, the optimum weighting distribution is obtained for the linear array in one sense that the mean square error between the desired directivity and the iteratively calculated directivity is minimized (ideally 0). An error vector for the i-th iteration is defined as

$$\{\varepsilon\}_i = \{Z\} - \{\,|P|\,\}_i \tag{11}$$

where $\{\,|P|\,\}_i$ denotes magnitude of the directivity vector. Hence, the mean square error or the objective function E_i^2 is given as

$$\begin{aligned} E_i^2 &= \frac{1}{M}\{\varepsilon\}^T\{\varepsilon\} \\ &= \frac{1}{M}(\{Z\}^T\{Z\} - 2\{|P|\}_i^T\{Z\} \\ &\quad + \{|P|\}_i^T\{|P|\}_i) \end{aligned} \tag{12}$$

When the objective function is minimized, the norm between $\{Z\}$ and $\{P\}$ is minimum. Here, the gradient vector $\{\nabla\}_i$ is introduced with respect to the weighting vector $\{W\}_i$ [14]

$$\{\nabla\}_i = \left\{ \frac{\partial E_i^2}{\partial w_{1i}} \quad \frac{\partial E_i^2}{\partial w_{2i}} \quad \cdots \quad \frac{\partial E_i^2}{\partial w_{Ni}} \right\}^T \tag{13}$$

where

$$\frac{\partial E_i^2}{\partial w_{ni}} = -\frac{2}{M}\sum_{m=1}^{M} \frac{\{\varepsilon\}_i}{\{\,|P|\,\}_i}\{P\}_i[\,\bar{S}\,]_{mn} \tag{14}$$

where, $\bar{\ }$ indicates the complex conjugate.

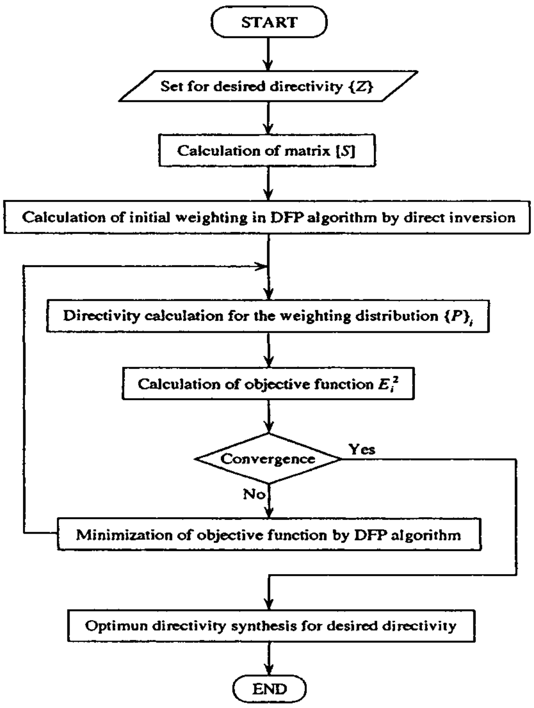


Fig. 2 *Flowchart for optimum directivity synthesis by proposed method*

DFP method uses the line searching to determine the optimum parameter for convergence stability in the iterative calculation process.

Fig. 2 is a flowchart for the procedure of directivity synthesis by the proposed method.

4 NUMERICAL SIMULATION FOR THE DIRECTIVITY SYNTHESIS

The proposed method evaluates the error vector $\{\varepsilon\}_i$ by Eqn. (10). The error is overestimated although the sidelove level (SLL) is satisfied to the given specification. To deal with these situations, the error at the observation point which satisfies the given specification is evaluated by 1/100. The given specification would be

1. The ripple in the main lobe must be within -3dB.
2. The sidelobe level (SLL) must be below an assigned value (for example below -30dB).

To confirm the validity of the error evaluation, the quasi-ideal beam with the beam width and the steering beam angle specified are chosen for the numerical demonstrations. The demonstration is then extended to the case of multi-beams. An ideal beam is defined as the sensitivity is unity within the main lobe and there is no sensitivity in one rest at all. However, it is impossible to realize physically. Therefore, if the SLL satisfies the given specification, the quasi-ideal beam is assumed to be realized approximately.

For all calculations to follow, the linear array consists of N = 20 point sources, the array length is L = 5.0 λ and the number of the observation points is M = 181 (the interval is chosen to be θ_m = 1°). In the iterative method, the magnitude is set to 1 at the 10th and the 11th point source for the initial values and the other magnitudes zero. Phases are all set to be 0. In the LMS method, the step parameter is properly chosen in consideration of the stability and the speedy convergence. However, in the proposed method, the initial values are obtained from the solution of direct inversion.

In the numerical demonstrations, the direct inversion, the DFP method, the LMS method and the proposed combination method are examined by comparing the capability of the directivity synthesis.

The simulation is performed on a PC (Pentium 400 ㎒, RAM 64 MByte). The language used Microsoft Fortran 77.

4.1 Directivity synthesis of a quasi-ideal beam

To confirm the validity of the proposed method, the directivity synthesis for the quasi-ideal beam with the beam widths of 10° or 30°, and with the SLL -30dB is considered. The step parameter μ is chosen to be 0.1 in the LMS method.

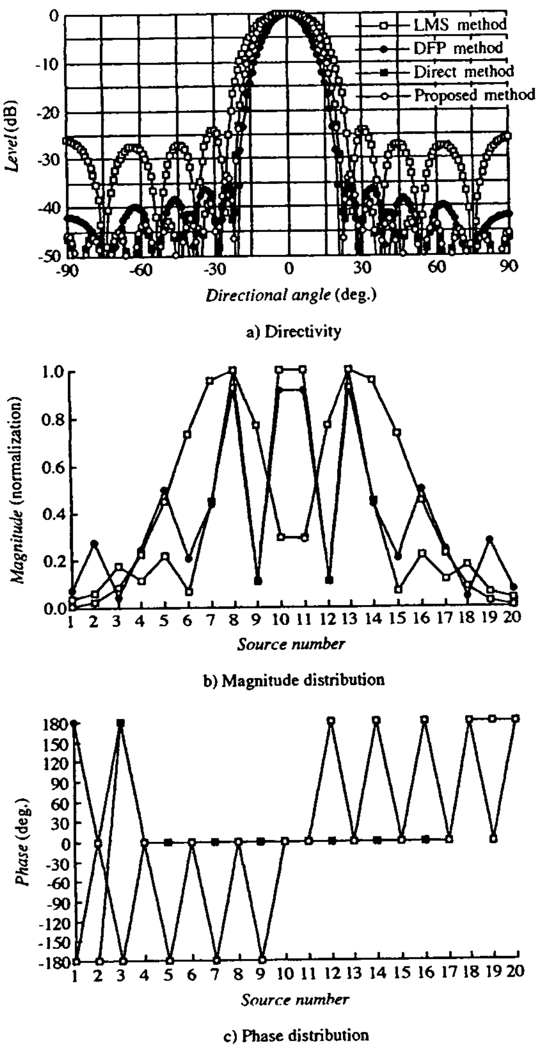


Fig. 3 *Synthesized directivity pattern and weighting distribution (quasi-ideal beam with the beam width* 10°. L = 5.0λ, N = 20, *and* M = 181. μ = 0.1 *in the case of the LMS method)*

The calculated directivity for the quasi-ideal beam and the corresponding weighting distribution are shown in Fig. 3, in which the magnitude is normalized with respect to the maximum value. The directivity represents the results at 5th and 100th iteration, in the DFP method, the LMS method and the proposed method, respectively. The result of direct inversion is obtained directly. The methods all give the resonable results, satisfying the given specification. However, the proposed method and the direct method is shown to provide better SLL suppression capability than the LMS method or the DFP method.

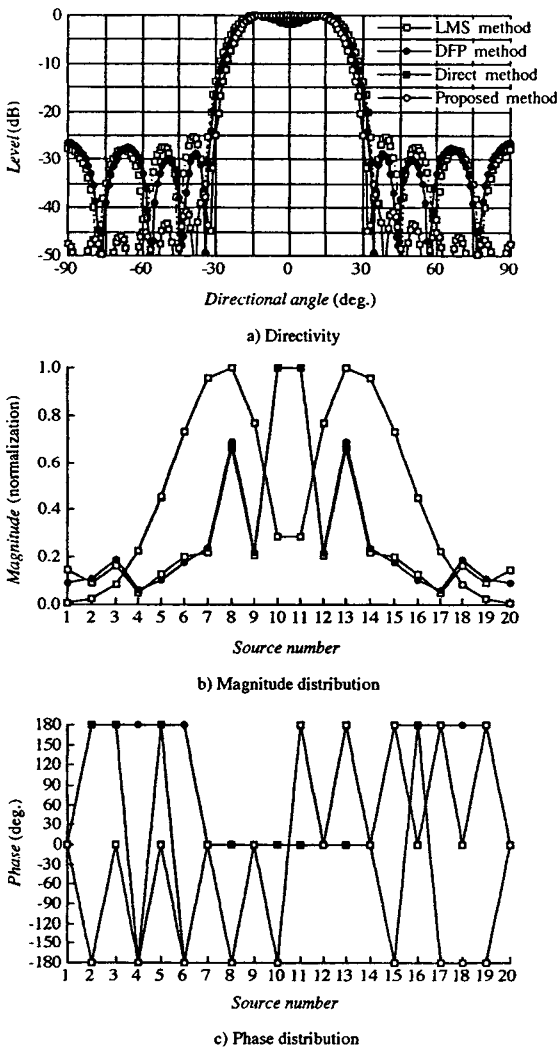


Fig. 4 *Synthesized directivity pattern and weighting distribution (quasi-ideal beam with the*

beam width 30°. L = 5.0 λ, N = 20, *and* M = 181. μ = 0.1 *in the case of the LMS method)*

Fig. 4 shows the case with the beam width of 30°. The directivity represents the results at 9th and 100th iteration in the DFP and the LMS method respectively.

The result of the proposed method is obtained directly without iterative calculation process, so the weighting distribution of the proposed method is same with the direct inversion. In the proposed method and the direct inversion, the sensitivity is flat in the main lobe although the ripples of 1.22dB and 1.78dB are observed in the LMS method and the DFP method.

The convergence characteristics of the objective function against the iterations are presented in Figs. 5 a) and 5 b). When the objective function became less than -100dB, the objective function is assumed to be converged and the iterative calculation process is then terminated.

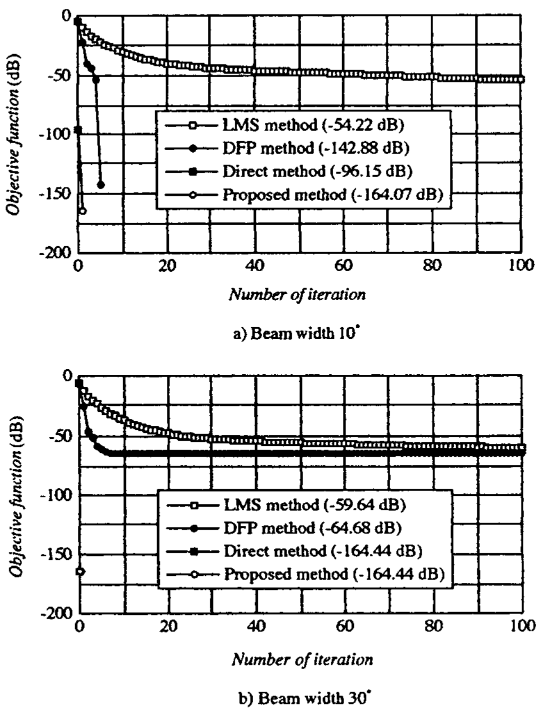


Fig. 5 *Convergence characteristics of objective function against number of the iteration*

For the case with the beam width of 10°, the objective function indicates -54.22dB at 100th iteration in the LMS method, -142.88dB at 5th in the DFP method, and -164.07dB at 1st in the proposed method.

In the case of the beam width 30°, the proposed method have more rapid convergence capability than the LMS method or the DFP method is simply used.

In the above demonstration, the desired directivity can be realized only by the direct inversion. But, if the source conditions are changed, there is the case that the synthesized directivity can not satisfy the given specifications in the direct inversion.

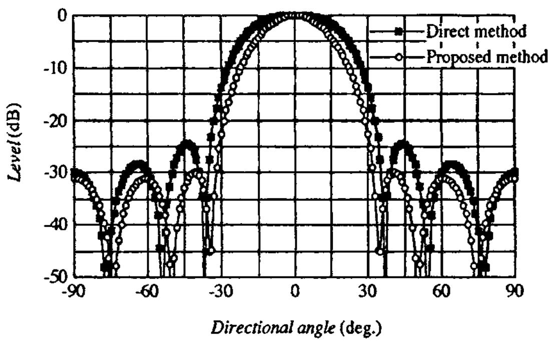


Fig. 6 *Comparison of synthesized directivity (main directional angle* θ_m = 0°, *Beam width* 30°, L = 3.0 λ, N = 10, *and* M = 181)

Fig. 6 shows the result when the number of point sources and array length are to be 10 and 3.0 λ, respectively. The direct inversion can not achieve the desired directivity, while the proposed method satisfies the given specifications. Thus the proposed method is more flexible.

4.2 Beam steering

Steering of the beam angle without the mechanical rotation of the transducers axis is sometimes required. This can be achieved by the adjusting the weighting distribution of the array transducer electrically. The given specifications must be satisfied regardless the steered direction.

The synthesized directivity when the beam width and the main directional angle are specified to be 10° and -40°, respectively are shown in Fig. 7. The step parameter μ is chosen to be 0.15 in the LMS method. Each directivity represents the result at 100th, 61st and 1st iteration of the LMS method, the DFP method and the proposed method respectively. The methods all give the reasonable results, satisfying the specified directivity characteristics. However, the direct inversion and the proposed method provide better SLL suppression capability than the LMS method or the DFP method.

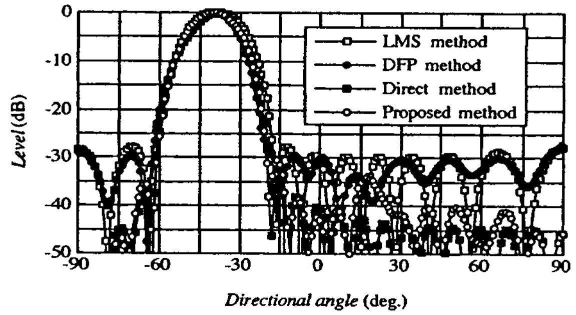
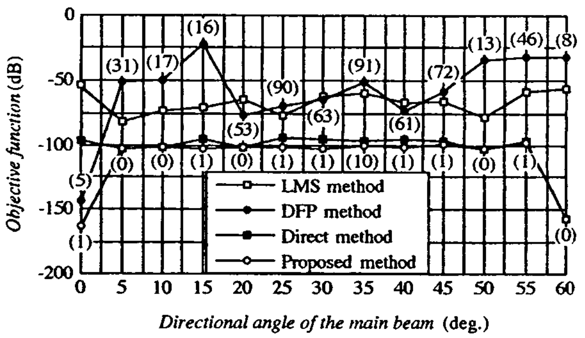


Fig. 7 *Directivity synthesis for the steered beam (beam width* 10°, *main directional angle* θ_m = -40°, L = 5.0λ, N = 20, M = 181, *and* μ = 0.15 *in case of the LMS method)*

Fig. 8 *Change of objective function against the main directional angle*

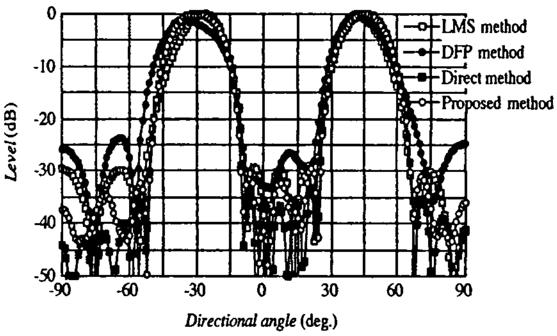


The change of the objective function against the angle of the main beam (from 0° to 60°), as the beam with a beam width of 10°, is examined. Fig. 8 shows the achieved level of the objective function. The number in the bracket is the iterated number. The proposed method shows the faster convergence than the single use of the LMS method or the DFP method. 0 iteration means that the solution is obtained only by the direct inversion.

4.3 Multi-beam

For the simultaneous communication with 2 or more stations, the directivity characteristics of multi-beam is required. In this case, the two main beam angles, the two main beam widths and the SLL should be specified. Fig. 9 shows the synthesized results. The two beam angles are -30° and 45° with the beam width of 10°. The step parameter μ is chosen to be 0.06 in case of the LMS method.

Fig. 9 *Synthesized directivity with two multi-*



beam (main directional angles θ_m = -30° and 45°, *beam width* 10°, *respectively.* L = 5.0λ, N = 20, *and* M = 181. μ = 0.06 *in case of the LMS method)*

In the proposed method and the direct inversion, the two beam angles are properly realized and the SLL is maintained under -30dB. For the main beam directions with the single use of the LMS or the DFP method, the errors are as much as 2° or 3°. Especially, the DFP method can not satisfy the specified SLL -30dB.

5 CONCLUSIONS

In this paper, we propose a method which is the direct inversion combined with the iterative DFP method.

The validity of the proposed method was confirmed through the numerical simulations of several directivity synthesis examples. The derived conclusions are as follows;

1. The proposed method provides faster and more efficient convergence than that the single use of the LMS method or the DFP method.

2. The direct inversion is not flexible for the directivity, while the proposed method can meet the error control about the given specifications.

REFERENCES

1. B. Widrow and S. D. Steans, "Adaptive Signal Processing," Prentice-Hall, Inc., New Jersey (1985).
2. K. Kido, "The technologies for active noise control," J. Acoust. Soc. Jpn. (E) 12, 6, 245/253 (1991).
3. A. Kataoka and Y. Ichinose, "A microphone-array configuration for AMNOR (Adaptive microphone-array system for noise reduction)," J. Acoust. Soc. Jpn. (E) 11, 6, 317/325 (1990).
4. K. R. Cho, J. N. Lee, C. S. Yi and M. S. Lee, "Directivity synthesis simulation of ultrasonic transducer using Gauss Elimination Method," Electromagnetic Eng. Soc. Kor, (K) 4, 3, 20/27 (1995)
5. T. Tsuchiya, H. Khono and Y. Kagawa, "Optimal directivity synthesis for ultrasonic transducers," J. Simulation. Jpn, (J) 12, 1. 61/69 (1995).
6. K. R. Cho, "Simulation of optimal directivity synthesis by ultrasonic radiation point source array," J. Commu. Soc. Kor, (K) 20, 7, 1834/1840 (1995).
7. K. R. Cho, "Adaptive directivity synthesis simulation of point source array using algorithm combined directive and recursive method (LMS method)," J. Commu. Soc. Kor, (K) 6, 11, 1453/1462 (1996).
8. K. R. Cho, T. Tsuchiya and Y. Kagawa, "Optimal directivity synthesis of ultrasonic transducers using a combined algorithm," J. Acoust. Soc. Kor, (K) 19, 1, 25/31 (2000).
9. Francisco J. Ares-Pena, Juan A. Rodriguez-Gonzalez, Emilio Villanueva-Lopez, S. R. Rengarajan, "Genetic algorithms in the design and optimization of antenna array patterns," IEEE Trans., AP, (E) 47, 3, 506/510 (1999).
10. H. Hamada, "Fundamentals of adaptive digital filter (Part 2)," J. Acoust. Soc. Jpn. (J) 45, 8, 317/325 (1989).
11. T. Sasakura, "Optimal design of ultrasonic transducers," Computrol, No. 39, Corona Co., 106/112 (1992).
12. K. Kido, "Acoustical Engineering," Corona Co., (1991).
13. T. Watanabe, M. Natori and T. Oguni, "Numerical Calculation Software," Maruzen. Co., (1989).
14. S. Takahashi and K. Onishi, "Mathematics for optimization method," Computrol, No. 39, Corona Co., 113/117 (1992).