FINITE ELEMENT ANALYSIS FOR DISCONTINUOUS MAPPED HEXA MESH MODEL WITH IMPROVED MOVING LEAST SQUARES SCHEME

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ABSTRACT

There is a big issue to generate 3D hexahedral finite element (FE) model, since a process to divide the whole domain into several simple-shaped sub-domains is required before generating a continuous mesh with mapped mesh generators. In general, it is nearly impossible to set up proper division numbers interactively to keep mesh connectivity between sub-domains on a complicated arbitrary-shaped domain. If mesh continuity between sub-domains is not required in an analysis, this complicated process can be omitted. Element-free Galerkin method (EFGM) can accept discontinuous meshes, which only requires nodal information. However it is difficult to choose a reasonable influenced domain in moving least squares scheme with non-uniformly distributed nodes in discontinuous FE models. A new FE scheme for discontinuous mesh is proposed in this paper by applying improved EFGM with some modification to derive FE approximated function in discontinuous parts. Its validity is evaluated on linear elastic problems.

1 INTRODUCTION

In recent years, the finite element method (FEM) is used in various fields of research and design. Introducing CAE such as FEM drastically shortens the term in design process. Although the hexahedral element is commonly used in three-dimensional case, full-automatic mesh generators for hexahedral element have not been developed yet [1], which is a bottleneck in CAE. Most of conventional mesh generators with hexahedral element are based on an interactive mapped mesh approach, which accepts only simple-shaped domain. Therefore dividing the whole domain into several simple-shaped sub-domains, considering the mesh connectivity between them, is definitely a prerequisite for hexahedral mesh generation. With this reason, the pre-process of FEM, mesh generation, requires much more manpower and time than those in the main-process, the three dimensional FEA itself. If discontinuous mapped mesh (DCMM), generated individually in each sub-domain, can be treated in an analysis, this time consuming process can be omitted, approximately by half. In industrial aspects, we have following three requirements. First, the method should be easily applicable to conventional FEM software. Secondly, it should have acceptable computational efficiency. Finally, reasonable accuracy should be guaranteed. In this paper, FE related method for discontinuous mesh with these three requirements is considered. Some of already proposed methods related with the treatment of discontinuous mesh are summarized below.

Nodal Linear Constraint Method (NLCM) was used for discontinuous nodes in h-adaptive refinement. In this method, the displacements of discontinuous nodes are constrained by linear function with those of related continuous nodes. If there are many discontinuous constrained nodes, the degrees of freedom become lower and accuracy is poor in NLCM.

Domain composition method [2], proposed by W. W. Charlesworth, can handle a complicated shape by setting sub-domains crossed. However the solution might be changed according to the cross section's ratio against the whole domain. If sub-domains have touched and do not cross, this method is ascribed to NLCM.

Overlaying mesh method [3] proposed by J.Fish can analyze the global-local problem by overlapping local model with fine mesh for partly high resolution on global model with coarse mesh. However local model should be contained in global model, this method cannot be used for discontinuous mesh.

Since meshless method doesn't require mesh, it is applicable to discontinuous mesh. Element-Free Galerkin Method (EFGM) [4] proposed by T. Belytschko is one of the most useful methods in meshless approaches. Since a moving least squares method is chosen to generate an approximate function, the function can be derived only with local information of nodal locations. Because the time complexity in EFGM is lower than that in FEM, the method of coupling FEM and EFGM is also proposed by T.Belytschko [(5)(6)]. It is reported that accuracy of EFGM is affected by the selection of nodes in MLSM [(7)-(9)].

In this paper, EFGM based approach for discontinuous mapped mesh model is discussed. EFGM is summarized in section 2. The problems on the application of EFGM to DCMM are discussed in section 3. Then a new feasible FEM for DCMM is proposed by applying improved EFGM[(10)] with some modification to derive FE approximated function in discontinuous parts of mapped mesh in section 4. Finally, its validity is evaluated on linear elastic problems, including an optimal sizing design in section 5.

2 ELEMENT FREE GALERKIN METHOD (EFGM)

The approximated function of EFGM, governed by Galerkin method, is based on Moving Least Squares Method (MLSM). The approximation function in MLSM can be derived only with the information of nodal coordinates around an evaluation point. (See Figure1). MLSM is summarized as follows.

If the approximated function u^h on physical values at an evaluated point x is represented by Eq. (1), EFG approximated function u^h is obtained by minimizing objective function J with respect to coefficient vector a_J.

$$u^h(\mathbf{x}) = \sum_{J=1}^{m} p_J(\mathbf{x}) a_J(\mathbf{x}) \equiv \sum_{I=1}^{n} \phi_I(\mathbf{x}) u_I \qquad (1)$$

$$J = \sum_{I}^{n} w_I(\mathbf{x}-\mathbf{x}_I) \left[\sum_{J=1}^{m} p_J(\mathbf{x}) a_J(\mathbf{x}) - u_I \right]^2 \qquad (2)$$

Where m , n, P and a_J are the number of bases, the number of points in the domain of influence, the bases of approximated functions, coefficients in the approximated function, while x,Φ ,u and w are coordinates, EFG approximated function, physics values, weight function, at interpolation node I, respectively.

$$u^h(\mathbf{x}) = \sum_{J=1}^{m} p_J(\mathbf{x}) a_J(\mathbf{x}) \equiv P^T(\mathbf{x}) a(\mathbf{x}) \qquad (3)$$

Here, considering the combination of EFGM and FEM, the number of bases P is chosen as the same number as that of FE approximated function in 8-node hexahedral element.

$$P^T(\mathbf{x}) = [1, x, y, z, xy, yz, zx, xyz] \quad (m=8) \qquad (4)$$

Minimization of J on coefficient a_J is ascribed to the following matrix equation.

$$A(\mathbf{x})\, a(\mathbf{x}) = B(\mathbf{x})\, u \qquad (5)$$

Equation (1) can be re-written with matrices above.

$$u^h(\mathbf{x}) = \sum_{I}^{n} \sum_{J}^{m} p_J(\mathbf{x}) (A^{-1}(\mathbf{x}) B(\mathbf{x}))_{JI} u_I \qquad (6)$$

Therefore, the basis function in EFGM is given by Eq. (7). Finite element is used as a background cell for gauss integration.

$$\Phi_I(\mathbf{x}) = \sum_{J}^{m} p_J(\mathbf{x}) (A^{-1}(\mathbf{x}) B(\mathbf{x}))_{JI} \qquad (7)$$

There are several weight functions available. In weight functions, as node approaches an evaluated point, the value of a weight function increases, which has zero value on the boundary of the influenced domain. In this paper, exponential weight function, which is numerically stable, is chosen in the numerical EFG calculations at section 5, as shown in Eq. (8), where d_I is a distance between an evaluation point and a node I in the domain of influence, d_{mI} is the radius of influenced domain and c is a representative size of the influenced domain.

$$w_I(d_I) = \begin{cases} \dfrac{e^{-(d_I/c)^2} - e^{-(d_{mI}/c)^2}}{\left(1 - e^{-(d_{mI}/c)^2}\right)} & d_I \le d_{mI} \\ 0 & d_I > d_{mI} \end{cases} \qquad (8)$$

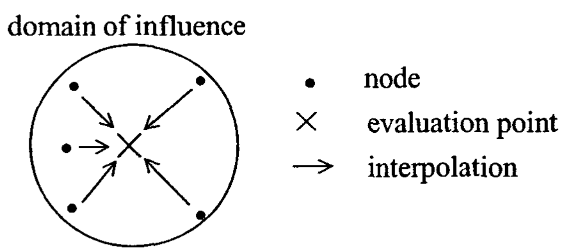


Figure 1: Moving least squares scheme

The radius of the influenced domain is multiplied by 1.01, to include the nodes on the edge of the influenced domain. If FEM and EFGM are applied together, a certain adjustment is needed on common boundaries, because the nodes on FE-EFG boundary have two values of the shape function in FEM and the basis function in EFGM. To hold those continuities, the method proposed by T. Belytschko should be applied [(5)(6)].

It should be noted that the raw value u_I at node I, calculated in EFGM, doesn't represent the physical value u_I at node I. To set up boundary conditions, this nature should be considered.

3 CHARATERICTICS OF EFGM

It is reported that the accuracy of EFGM depends on the selection of nodes in MLSM [(7)-(9)]. Since the nodes are non-uniformly distributed in discontinuous mesh, the selection of nodes for interpolation is very difficult. The following characteristics are investigated for an EFG approximated function in this research.

(1) At least, some numbers of nodes are necessary to construct a MLS function. For example, 8 nodes are needed for tri-linear basis in the three-dimensional case.
(2) The resolution of the MLS function is affected by the value of the support's radius. If there are too many nodes involved in the support, the MLS function becomes too smooth and hence cannot get an accurate EFG solution, due to the nature of least squares.
(3) If only one support radius is commonly used over the domain, the number of the involved nodes in MLS varies at every integration point with non-uniform nodal distribution. This means that the resolution is uncontrolled without any physical meanings.
(4) MLS is the function of the support's radius, not of the directional distributions of nodes. In other words, MLS compresses three-dimensional information on xyz-coordinates to one-dimensional information on support's radius in its derivation. This is a reason that EFGM is week at non-uniformly distributed nodes.
(5) If the nodes involved in the support domain are located on a straight line or on a plane in the three dimensional case, away from an integration point, a MLS function cannot be defined at the integration point. In this case, MLS is failed and EFG approximation function cannot be obtained at that integration point. (See Figure 2)

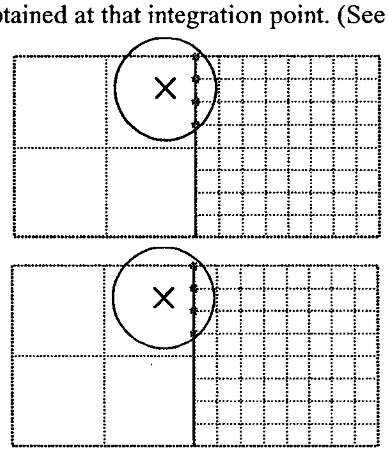


Figure 2: Typical nodal distribution on discontinuous mesh in 2D

By summarizing these features, the condition to obtain decent approximated functions in MLSM is that an evaluation point should be surrounded by minimum necessary nodes with a good balance, which might be difficult in the case of non-uniformly distributed nodes. In DCMM, independently generated subdomains with different mesh sizes are attached to get a discontinuous mesh. Since mesh sizes are different between faced subdomains, the nodes are non-uniformly distributed on the discontinuous parts. Therefore, if a conventional EFGM is applied to a discontinuous mesh, only a poor accuracy is obtained.

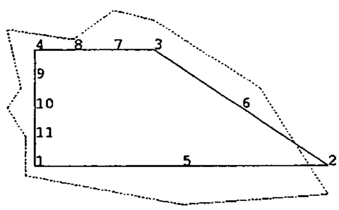
4 DISCONTINUOUS MAPPED MESH FEM (DCMM-FEM)

Improved EFGM satisfies the condition of Kronecker's delta in MLS, by introducing singular weight function shown in Eq. (9), where d_I is a distance between an evaluation point and a node I in the domain of influence, and d_{mI} is the radius of influenced domain. Due to this nature, the approximated function is not as smooth, even if too many nodes are in the support. Since treatment of influence domain is the same as in EFGM, it is still poor at non-uniformly distributed nodes.

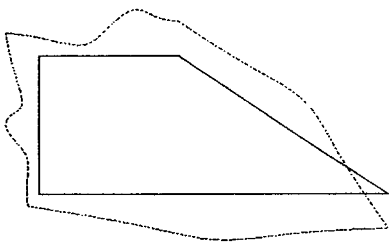
$$w_I(d_I)=\begin{cases}\left(\dfrac{d_{mI}}{d_I}\right)^2-2\left(\dfrac{d_{mI}}{d_I}\right)+1 & d_I \le d_{mI} \\ 0 & d_I > d_{mI}\end{cases} \quad (9)$$

According to the discussion in previous section, the necessary condition in MLS with good accuracy is that the selected nodes should surround an evaluation point by minimum necessary nodes with a good balance. If MLS is applied in an element, this condition is almost satisfied. However this is not still sufficient, if the shape of an applied element is distorted. Additionally if MLS is applied to an element, C0 continuity problem occurs in usual EFGM. Because of Kronecker delta's condition, improved EFGM meets C0 continuity at nodes, but not on element boundaries.

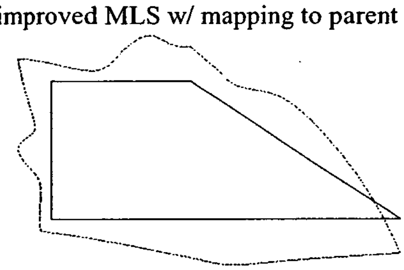
If improved MLS is applied to an element in natural coordinates, previous two problems might be solved. First, the nodes related with an element, including discontinuous nodes, are mapped onto natural coordinates, as in usual FE shape function. Then, improve EFGM is applied in a discontinuous element in parent domain. Finally, approximated function is mapped back to a general coordinate. With this simple setting, we have always well-balanced nodes even in distorted mesh, and C0 continuity is approximately satisfied on element boundaries (See Figure 3)



Imposed displacements at irregular QUAD11

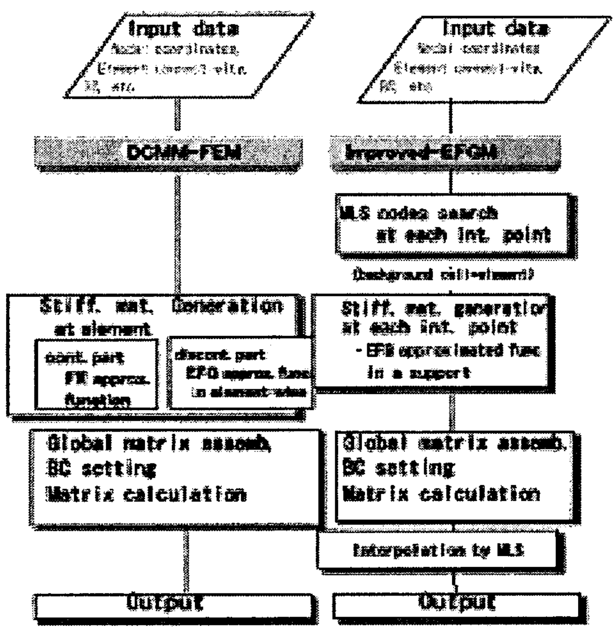


By improved MLS w/ mapping to parent domain



By improved MLS w/o mapping to parent domain

Figure 3: C0 continuity on distorted element boundary



DCMM-FEM Improved-EFGM

Figure 4: Algorithms of DCMM-FEM and improved EFGM

On backgrounds above, the discontinuous mapped mesh (DCMM) FEM can be proposed to overcome the problem on the selection of nodes due to non-uniform distribution of nodes in MLSM, by applying improved MLS element-wise in natural coordinates. The algorithm is provided in Fig. 4, accompanied with that of improved EFGM for a reference. Compared with improved EFGM, the proposed DCMM-FEM is a sort of FEM and it is easily applicable to usual FEM code just by exchanging the subroutine of a shape function. Since improved MLS is only applied in discontinuous elements, the time complexity is almost equal to that in FEM, if there are not so many discontinuous elements in DCMM.

It should be noted that DCMM-FEM is only valid for HEXA8 based mesh with tri-linear basis in improved MLS because of linear based C0 compatibility between discontinuous elements. Because of this limitation, if there exist too many discontinuous nodes related with an element, improved MLS has some problems due to insufficient number for such an element.

5 NUMERICAL EXAMPLES

5.1 Cantilever with T-cross section

The proposed method is evaluated on an elastic cantilever with T-cross section, with equally distributed loads imposed on its free end, as shown in Fig. 5. The model is made up of independently meshed two sub-domains. Regular mapped mesh and distorted mapped mesh are prepared. Continuous mesh is for FEM, as a comparison, while discontinuous mesh is for NLCM, FE-EFGM, and the proposed method, DCMM-FEM. EFGM is applied only on discontinuous nodes, coupling with FEM in the rest of the domain. As shown in Fig. 6, there are no differences among these four methods with regular mesh. With distorted mesh in Fig. 7, DCMM-FEM shows almost equivalent deformation to that in FEM, although FE-EFGM and NLCM achieve poor accuracy with rather rigid deformations. As for FE-EFGM, because of the problems mentioned before, no decent support's radius is found even with parameter study. In NLCM, since the constraints are almost spread over the discontinuous plane, only the linearly constrained deformation is obtained. The distorted mesh case is rather an exaggerated case, but the similar cases might be expected in DCMM. With this example, it can be concluded that NLCM and FE-EFGM are not good for DCMM.

One more thing should be added on EFGM. Previous results on FE-EFGM are even the best solutions after parameter tunings by trial and error. The graphs in Fig. 8 show how EFGM depends on the choice of MLS nodes in a support. Because of characteristics on EFGM, discussed in section 3, it is not a clever idea to apply EFGM in non-uniformly distributed nodes.

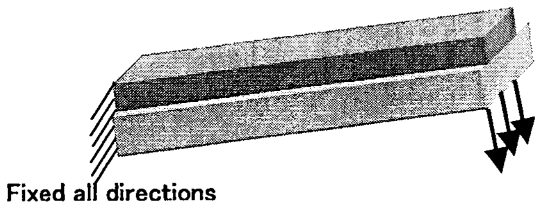
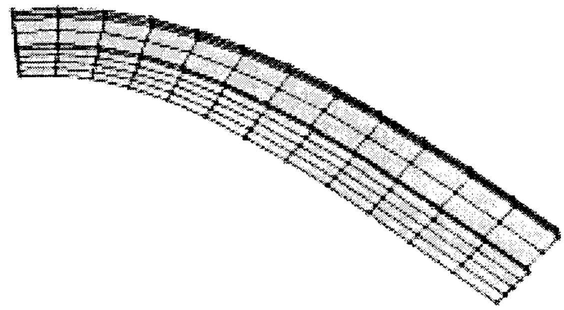
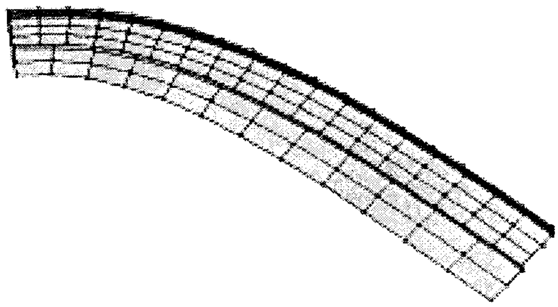


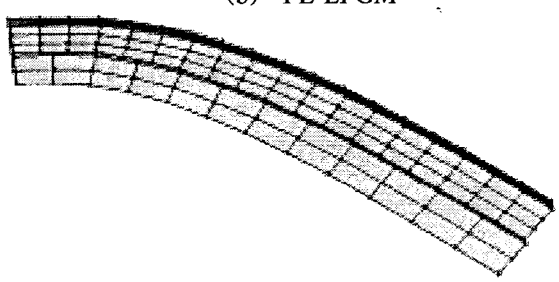
Figure 5: Cantilever beam with T's cross-section



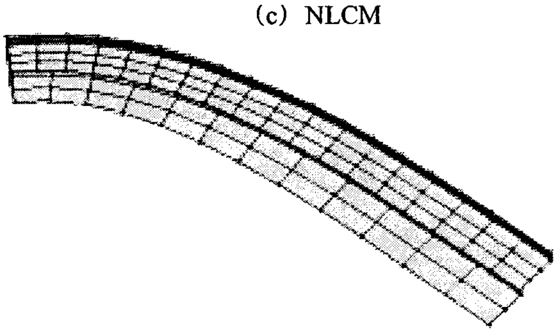
(a) FEM



(b) FE-EFGM

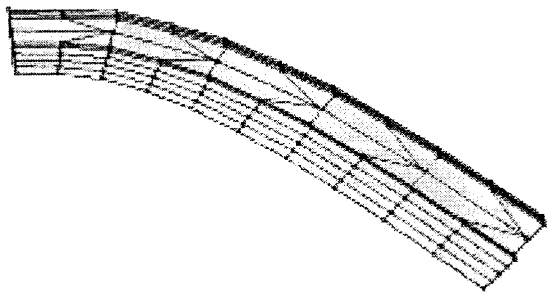


(c) NLCM

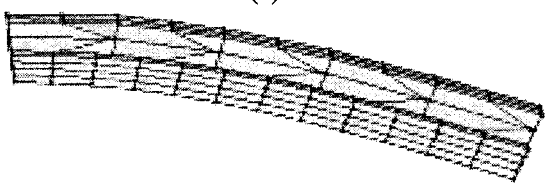


(d) DCMM-FEM

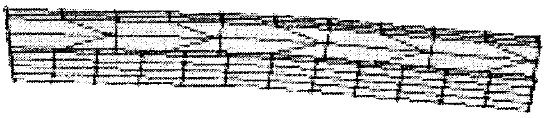
Figure 6: Deformations with regular mesh (scaling 10,000,000)



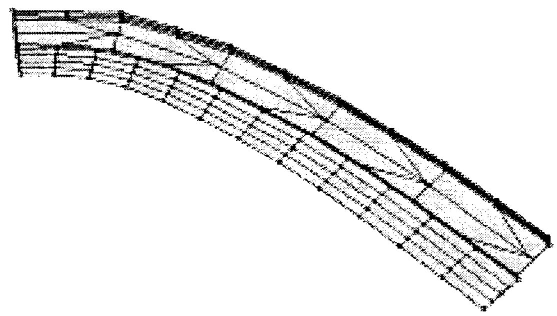
(a) FEM



(b) FE-EFGM

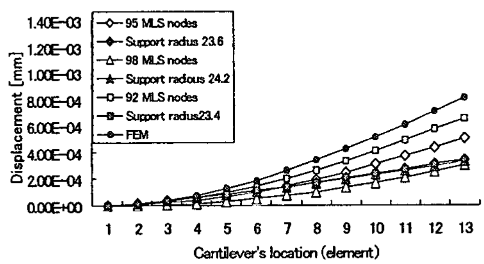


(c) NLCM



(d) DCMM-FEM

Figure 7: Deformations with distorted mesh (scaling 10,000,000)



Parameter study in EFGM (Cantilever with distorted mesh)

Figure 8: Parameter study on the choice of MLS nodes

5.2 Pulled plate with a pipe

Young's modulus 2.1×10^5 Mpa, Poisson's ratio 0.3

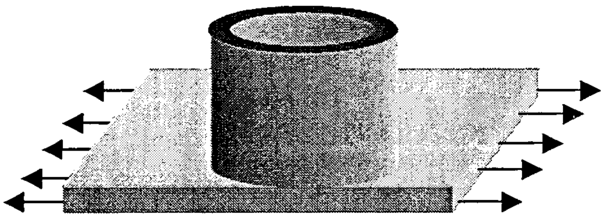
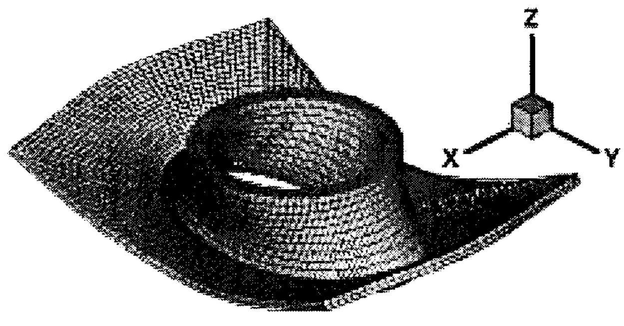
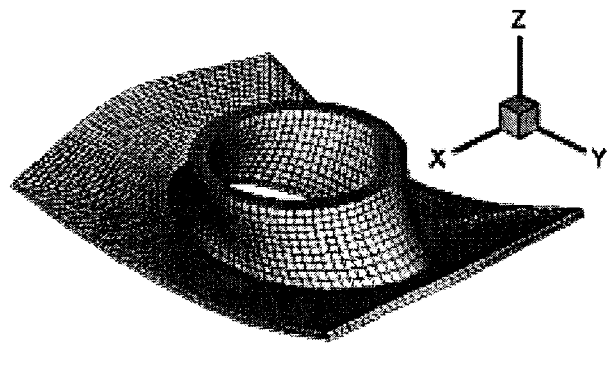


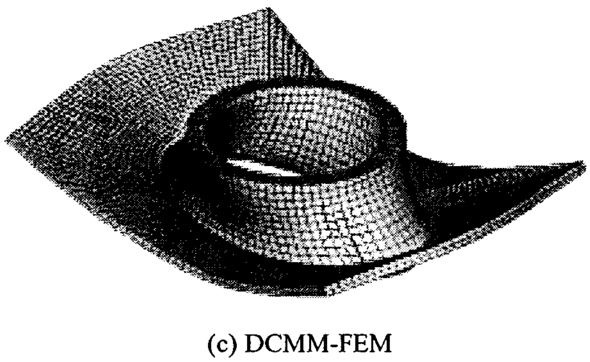
Figure 9: Meshed models of pulled plate combined with a pipe



(a) FEM



(b)NLCM



(c) DCMM-FEM

Figure 10: Deformations of pulled plate with a pipe

As shown in Fig. 9, the elastic model of a pulled plate combined with a pipe is analyzed, where the top of the pipe is fixed in all directions and the plate is pulled by uniformly distributed load in y-direction. Although the discontinuous mesh part, the cross section of the pipe attached with the plate, seems like a smooth circle, it is composed of not curves but straight lines in reality. For this reason, in the junction of the pipe and the plate, there are almost no coincided nodes and an irregular discontinuous mesh is formed. An extra number of nodes are constrained in NLCM, which is the reason that unacceptable rigid deformations are represented as shown in Fig. 10. Since some nodes are very close together although not coincided, which leads that the node distribution is extremely biased in this model, it is impossible to determine effective influenced domains in FE-EFGM. On the other hand, the numerical result in DCMM-FEM is very close to that in FEM (with continuous mesh). As described previously, the proposed method cannot guarantee perfect C0 compatibility between elements, however it is within an acceptable limit in an engineering sense.

With the examples above, the superiority of the proposed approach is confirmed as compared with the result in NLCM.

6 CONCLUSIONS

The enormous time and manpower are required to generate hexahedral element mesh by an interactive manner with a mapped mesh approach, which is one of the causes that design process couldn't be shortened yet. The division of domain into several simple sub-domains, considering mesh connectivity between them, is definitely required as a prerequisite for mapped mesh generation, which is the most time consuming part. If discontinuous mesh (DCMM), with individually generated sub-domains, can be treated in an analysis, this problem can be half omitted. In this research, the new scheme which can support discontinuous mesh is proposed to omit the treatment of mesh continuity between partial sub-domains.

First we showed NLCM (nodal linear constraint method) and EFGM (element-free galerkin method) were not appropriate for DCMM. NLCM represented only rigid deformations in some discontinuous mesh models. EFGM, whose function is derived only with information on the coordinates of surrounding nodes, can be applied to discontinuous mesh. However, the selection of nodes with fine accuracy in EFGM is very difficult, if nodes are non-uniformly distributed, which is a common case in a discontinuous mesh.

We proposed DCMM (discontinuous mapped mesh)-FEM which can treat discontinuous mesh with good time complexity and accuracy. The proposed method introduces improved MLSM with an element-wise treatment, combined with FEM to prevent depressing time complexity. After evaluations on a few kinds of discontinuous mesh models in linear elastic problems, the superiority of our proposed method was indicated.

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