MODELING AND SIMULATION FOR GAS PIPELINE SYSTEMS

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ABSTRACT

City gas is one of the most important necessities of daily city life and social infrastructures. City gas is delivered to every user through a pipeline network. The gas pressure in the pipeline is regulated by gas regulator. In the pressure control system, characteristics of gas pipeline is as important as characteristics of regulator. There are many reports about the transfer function model of the fluid pipeline. But suitable model about the gas transmission pipeline is not known. In this paper, as the transfer function model of the gas pipeline, new model considering the heat transfer between pipe wall and gas and temperature change of gas is proposed. To evaluate this model, frequency response tests are used. As the result, the proposed model shows a better agreement when compared with the experimental result than conventional models. The results show the effectiveness of the model.

1. INTRODUCTION

City gas is one of the most important necessities of daily city life and social infrastructures. City gas is delivered to every user through a pipeline network. The gas pressure in the pipeline network is regulated by gas regulators. The gas regulators characteristics are influenced by the pipeline. Therefore, the study of the flow characteristics of the city gas through the pipeline is very important.

There are many reports about the transfer function model of the fluid pipeline. In 1962, the model proposed by F. T. Brown gives good simulation results for laminar flow area [1]. However when Reynolds number is large, as in general pipeline systems where Reynolds number is in the order of 10^4 or in high pressure pipeline systems where the Reynolds number is larger than 10^6, results using the Brown model have large errors. Thus the model proposed by Brown is not appropriate as the pipeline model. In 1984, J. Kralik proposed the transfer function model for the gas pipeline with turbulent flow subjects to the assumption of isothermal state change [2]. But this assumption becomes a source of error when calculating the resonance frequency.

In this paper, we will propose a new model considering the heat transfer between pipe wall and gas and temperature change of gas. Then we will carry out the frequency response tests to validate this model, and compare with simulation results.

NOMENCLATURE

A	:	cross sectional area of pipe	[m^2]
e	:	internal energy of gas	[J]
D	:	inner diameter of pipe	[m]
G	:	mass flow rate	[kg/s]
h	:	heat transfer coefficient	[W/(m^2.K)]
L	:	pipe length	[m]
P	:	pressure	[Pa]
q'	:	heat energy that transfers to outside from system per unit time per unit mass	[J/(s.kg)]
s	:	Laplace operator	[-]
t	:	time	[s]
u	:	velocity of gas flow in pipe	[m/s]
x	:	displacement along pipe axis	[m]
κ	:	thermal coefficient ratio	[-]
λ	:	pipe friction coefficient	[-]
ρ	:	density	[kg/m^3]
θ	:	temperature	[K]

subscribe

0 : steady state value

superscript

^ : value after Laplace transfer

other

Δ : very small variation from steady state value

2. TRANSFER FUNCTION MODEL

2.1 Basic Equations

When one dimensional flow is assumed, next basic equations are held.

[Equation of continuity]

$$\frac{\partial \rho}{\partial t}+\rho\frac{\partial u}{\partial x}+u\frac{\partial \rho}{\partial x}=0 \quad (1)$$

[Equation of motion]

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{2}\frac{\lambda}{D}u^2=0 \quad (2)$$

[Equation of state]

$$P=\rho R\theta \quad (3)$$

[Equation of energy]

$$\frac{\partial}{\partial t}\left[\rho A\left(e+\frac{u^2}{2}\right)\right]+\frac{\partial}{\partial x}\left[\rho A\left(e+\frac{u^2}{2}+\frac{P}{\rho}\right)\right]+\rho q' A=0 \quad (4)$$

2.2 Assumptions and Linearization

To develop the transfer function model, we assumed:

1. The pipeline is evenness, its cross sectional area is constant and the pipe wall is rigid.
2. The velocity of flow in the pipe must be under 30[m/s]
3. When it is steady state, gas temperature is equal to the outside temperature.
4. When it is steady state, state values are not distributed in the pipe.
5. Convection term in the basic equations can be neglected

It is decided that the velocity of gas flow in the pipeline must be under 30[m/s] by the design basis for the gas supply system. Thus, above assumptions is stable for modeling of the gas pipeline. Under these assumptions, we will linearize the basic equations, and introduce the very small variation from steady state value.

$$P=P_0+\Delta P \quad (5)$$

$$\rho=\rho_0+\Delta\rho \quad (6)$$

$$\theta=\theta_0+\Delta\theta \quad (7)$$

$$u=u_0+\Delta u \quad (8)$$

In the steady state, we assume that flow temperature in the pipeline is equal to the temperature of pipe wall. And using the heat transfer coefficient, energy equation become:

$$\frac{\partial}{\partial t}\left[\rho A\left(e+\frac{u^2}{2}\right)\right]+\frac{\partial}{\partial x}\left[\rho A\left(e+\frac{u^2}{2}+\frac{P}{\rho}\right)\right]+h\pi D\Delta\theta=0 \quad (9)$$

Using these assumptions, convection terms are neglected. Then we substitute eq.(5)-(8) for eq.(9), expand it, and neglect the infinitesimal term, we can get following equations. Their coefficients are summarized as Table 1.

[Linearized equation of motion]

$$m_1\frac{\partial \Delta G}{\partial t}+m_2\frac{\partial \Delta P}{\partial x}+m_3\Delta G=0 \quad (10)$$

[Linearized equation of continuity]

$$c_1\frac{\partial \Delta\rho}{\partial t}+c_2\frac{\partial \Delta G}{\partial x}=0 \quad (11)$$

[Linearized equation of state]

$$\frac{\Delta P}{P_0}=\frac{\Delta\rho}{\rho_0}+\frac{\Delta\theta}{\theta_0} \quad (12)$$

[Linearized equation of energy]

$$e_1\Delta G+e_2\Delta\rho-e_3\Delta P-e_4\frac{\partial \Delta P}{\partial t}+e_5\frac{\partial \Delta\rho}{\partial t}=0 \quad (13)$$

Table 1 Coefficients of Linearized Equations

m_1	1	e_1	$(\kappa-1)\frac{3}{2}\frac{\lambda}{D}\frac{R^2\theta_0^2G_0^2}{P_0A^3}$
m_2	A	e_2	$(\kappa-1)\left(\frac{\lambda R^3\theta_0^3G_0^3}{2DP_0^2A^3}+\frac{4hR\theta_0^2}{D}\right)$
m_3	$\frac{\lambda R\theta_0G_0}{DP_0A}$	e_3	$(\kappa-1)\frac{4h\theta_0}{D}$
c_1	1	e_4	P_0
c_2	$\frac{1}{A}$	e_5	$\kappa R\theta_0P_0$

2.3 Transfer Function Matrix

After the Laplace transformation, and eliminating the $\Delta\hat{\rho}$, we get:

$$A_1\frac{\partial^2\Delta\hat{P}}{\partial x^2}-A_2\frac{\partial\Delta\hat{P}}{\partial x}-A_3\Delta\hat{P}=0 \quad (14)$$

$$A_1\frac{\partial^2\Delta\hat{G}}{\partial x^2}-A_2\frac{\partial\Delta\hat{G}}{\partial x}-A_3\Delta\hat{G}=0 \quad (15)$$

Solving the these equations along the pipe direction, following transfer matrix model can be obtained.

$$\begin{bmatrix}\Delta\hat{P}(0)\\ \Delta\hat{G}(0)\end{bmatrix}=\begin{bmatrix}\hat{F}_{11} & \hat{F}_{12}\\ \hat{F}_{21} & \hat{F}_{22}\end{bmatrix}\begin{bmatrix}\Delta\hat{P}(L)\\ \Delta\hat{G}(L)\end{bmatrix} \quad (16)$$

Where

$$\begin{bmatrix}\hat{F}_{11} & \hat{F}_{12}\\ \hat{F}_{21} & \hat{F}_{22}\end{bmatrix}=\frac{e^{-\gamma_1 L}}{\gamma_2}\begin{bmatrix}\gamma_1\sinh(\gamma_2 L)+\gamma_2\cosh(\gamma_2 L) & Z\sinh(\gamma_2 L)\\ -\frac{\gamma_1^2-\gamma_2^2}{Z}\sinh(\gamma_2 L) & -\gamma_1\sinh(\gamma_2 L)+\gamma_2\cosh(\gamma_2 L)\end{bmatrix} \quad (17)$$

As the boundary condition at the end of pipe, using the constant value K that holds $\Delta\hat{G}(L)=K\cdot\Delta\hat{P}(L)$, the transfer function from pressure at the inlet of the pipe to pressure at outlet of pipe became:

$$\frac{\Delta P(L)}{\Delta P(0)}=\frac{\gamma_2 e^{\gamma_1}}{\gamma_1\sinh(\gamma_2 L)+\gamma_2\cosh(\gamma_2 L)+KZ\sinh(\gamma_2 L)} \quad (18)$$

Their coefficients are summarized as Table 2.

Table 2 Coefficients of Transfer Function Model

A_1	$\frac{c_2}{c_1 s}$	Z	$\frac{m_1 s+m_3}{m_2}$
A_2	$-\frac{e_1}{e_5 s+e_2}$	γ_1	$-\frac{A_2}{2A_1}$
A_3	$-\left(\frac{m_1 s+m_3}{m_2}\right)\left(\frac{e_4 s+e_3}{e_5 s+e_2}\right)$	λ_2	$\frac{\sqrt{A_2^2-4A_1A_3}}{2A_1}$

3. EFFECTS OF HEAT TRANSFER ON FREQUENCY RESPONSE

Kralik model is expressed by following equations [2]:

$$\begin{bmatrix}\Delta\hat{P}(0)\\ \Delta\hat{G}(0)\end{bmatrix}=\begin{bmatrix}\hat{F}_{11} & \hat{F}_{12}\\ \hat{F}_{21} & \hat{F}_{22}\end{bmatrix}\begin{bmatrix}\Delta\hat{P}(L)\\ \Delta\hat{G}(L)\end{bmatrix} \quad (19)$$

where

$$\begin{bmatrix}\hat{F}_{11} & \hat{F}_{12}\\ \hat{F}_{21} & \hat{F}_{22}\end{bmatrix}=\frac{e^{-a_{11}/2}}{b}\begin{bmatrix}\frac{a_{11}}{2}\sinh(b)+b\cosh(b) & a_{21}\sinh(b)\\ a_{12}\sinh(b) & -\frac{a_{11}}{2}\sinh(b)+b\cosh(b)\end{bmatrix} \quad (20)$$

and their coefficients are calculated by

$$a_{11}=\frac{\lambda}{2DA^2}\frac{|G_0|G_0R\theta_0L}{P_0^2}+\frac{2G_0Ls}{AP_0} \quad (21)$$

$$a_{12}=-\frac{\lambda}{DA^2}\frac{G_0R\theta_0L}{P_0^2}-\frac{Ls}{A} \quad (22)$$

$$a_{21}=-\frac{AL}{R\theta_0}s \quad (23)$$

$$a_{22}=0 \quad (24)$$

$$b=\frac{1}{2}\sqrt{a_{11}^2+4a_{12}a_{21}} \quad (25)$$

To compare with Kralik model, simulation was carried out under the same condition showed in Fig.4. The dashed line in Fig.1 shows the simulation results by Kralik model,

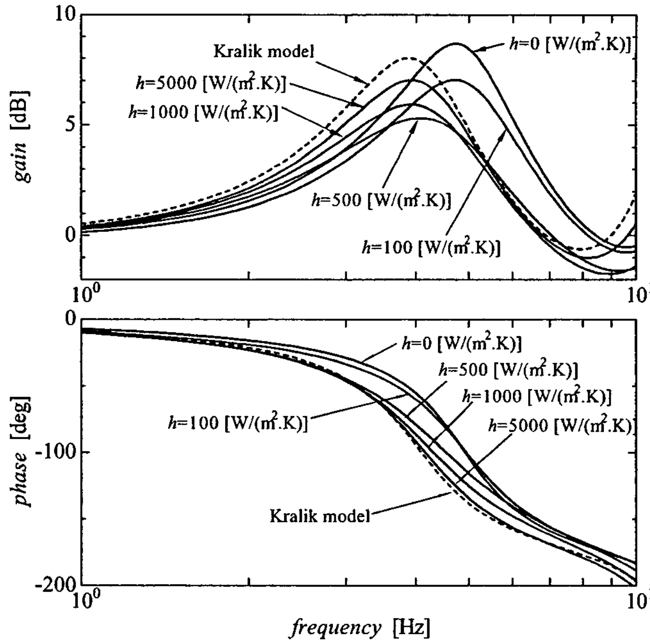

Fig.1 Kralik model and New model (condition A)

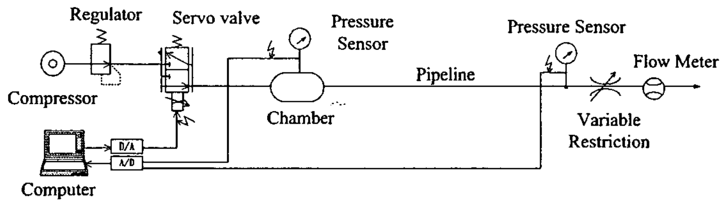

Fig.2 Experimental Apparatus

and solid lines show the simulation results by new model. As shown in this figure, we found that the heat transfer coefficient become large, these gains decreases from adiabatic, phase shift to left then gain increases to Kralik model or isothermal. This is the same phenomena in a pneumatic nozzle flapper cleared by Kagawa [3].

4. EXPERIMENT

Fig.2 shows the experimental apparatus for measuring the frequency response of the pipe. The pipe was made of polyethylene, inner diameter was 13.0[mm] and length was 18[m]. We could neglect the change of cross sectional area with change of pressure in the pipe. The signal from the pressure sensor at the inlet of pipe was sent to the computer through the A/D converter. Using this signal, PI control was carried out. The pressure in the chamber was changed as a sine wave by using the servo valve. The pressure at the inlet of pipe and the pressure at the outlet pipe was measured. The pressure in the chamber on the steady state was 395.53[kPa], sinusoidal amplifier was 0.98[kPa]. And flow rate was regulated by variable restriction at the outlet of pipe, and measured by flow meter. Experiments were carried out in the four cases of steady flow. Then steady flow rates and Reynolds numbers are summarized as Table 3.

Table 3 Experimental conditions

Pressure [kPa]	Flow rate [kg/s]	Velocity [m/s]	Reynolds number [-]	condition
393.71	4.07×10^{-3}	6.43	2.2×10^{4}	A
391.81	6.10×10^{-3}	9.70	3.3×10^{4}	B
389.34	8.14×10^{-3}	13.0	4.4×10^{4}	C
387.26	9.20×10^{-3}	17.0	5.5×10^{4}	D

5. RESULTS AND DISCUSSION

From the Fig.3 to Fig.6, results of frequency responses test are shown.

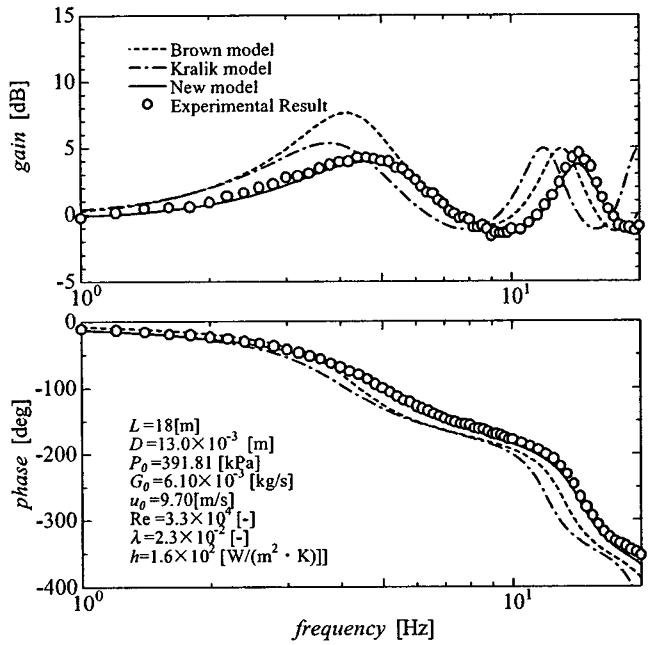

Fig.4 Frequency Characteristics of Pipeline (condition B)

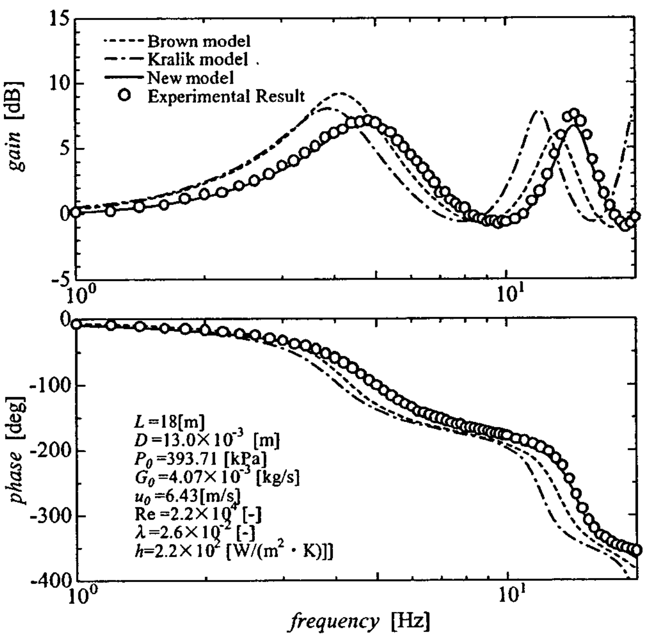

Fig.3 Frequency Characteristics of Pipeline (condition A)

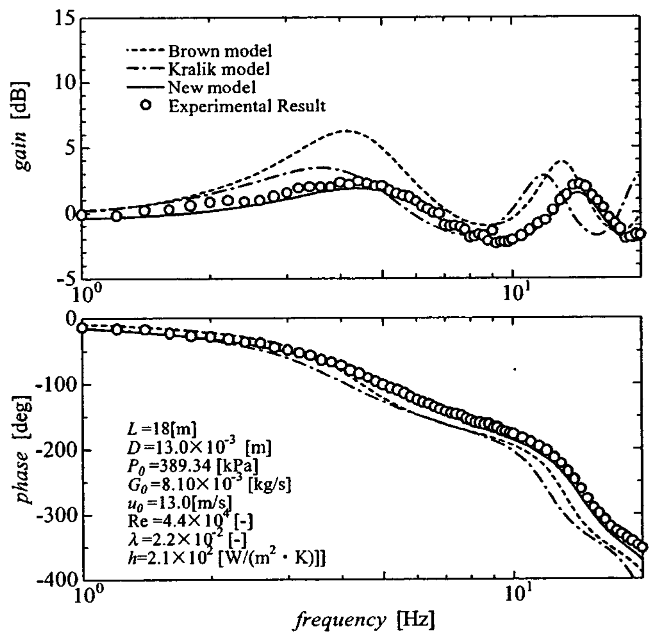

Fig.5 Frequency Characteristics of Pipeline (condition C)

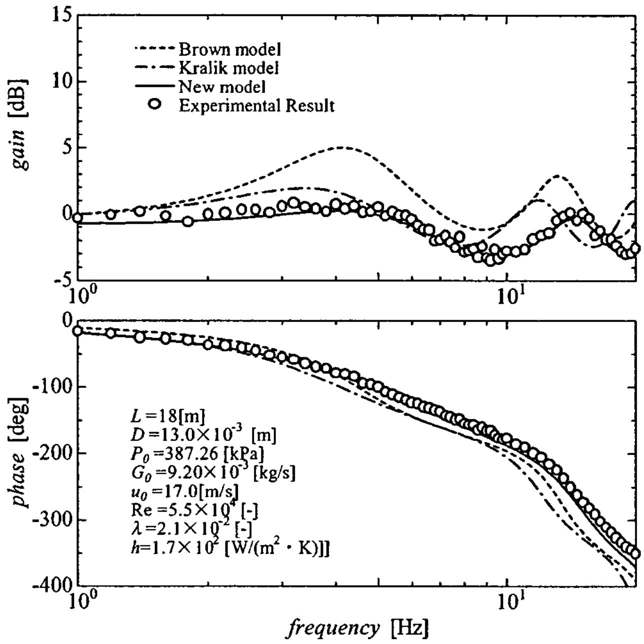

Fig.6 Frequency Characteristics of Pipeline (condition D)

In these simulations, heat transfer coefficients are calculated by Nusselt number using the Dittus-Boelter formula:

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4} \qquad (26)$$

Pipe friction coefficients are calculated by Blasius formula:

$$\lambda = 0.3164 \cdot \mathrm{Re}^{-0.25} \qquad (27)$$

In above figures, solid lines show the simulation results using by new model and dashed lines show the simulation results calculated by Kralik model and dashed-solid lines show the Brown model. From these figure, Reynolds number became large, gain is decreased by pressure loss. Reynolds number become larger than larger, calculated results by Browm model had large error. And calculated results by Kralik model, it is found that phase is shift to left. New model shows a good agreement. The results show the effectiveness of the model.

6. CONCLUSION

In this paper, we have proposed a new model considering the heat transfer between pipe wall and gas and temperature change of gas. Then we will carry out the frequency response tests to validate this model, and compare with simulation results. As the result, the proposed model shows a better agreement when compared with the experimental result than conventional models. The results show the effectiveness of the model.

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