A NEW APPROACH OF FAULT DETECTION BASED ON WAVEARX NEURAL NETWORK OBSERVER

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ABSTRACT

A novel approach based on WaveARX neural network observer is proposed for the fault detect of a class of nonlinear systems which consist of known linear part and unknown nonlinear part. A linear observer is first designed, then a nonlinear compensation term in the nonlinear observer is estimated by using a deconvolution method. The WaveARX network is used to model the obtained compensation term. At last, the residual for fault detection is generated based on the analysis of the upper bound approximate error. Simulation results have shown the feasibility and effectiveness of the method.

1 INTRODUCTION

Over the past two decades, much attention has been paid to the problem of fault detection in dynamic systems. This is due to both a higher demand for reliability and safety of industrial processes, and to economic and environmental constraint. When a fault occurs, it may affect the efficiency of the process, and if it is not located at an early stage it could lead to a catastrophic scenario including injuries to personnel. Many methods have therefore been developed, the observer-based approaches have been proven to be capable of successfully detecting certain types of system faults.

It is well known that the core element of model-based fault detection in control systems is the generation of residual signals which act as indicators of faults. The residual signals are generated using estimates of and a comparison with real measured quantities. For the design of residual generators, various approaches have been discussed in the literature.Various mathematical-model-based algorithms have been achieved for linear systems. However, there are few fruitful results for nonlinear dynamic systems[1]. The first reason is that it is very difficult to develop an exact mathematical-model for a nonlinear dynamic system in practice. The second is that there are few methods to construct a state estimator (filter) for a nonlinear dynamic system, so the residuals for fault detection can not be calculated.

Extended Kalman Filter (EKF) which is used widely for state estimation relies on linearized state and output equations to estimate the states. Therefore it can lead to divergence when modeling error exists in underlying nonlinear dynamic systems[2]. Wen Chen[3] presented a variable structure adaptive observer approach based on known bound of the nonlinear part, which is not always achieved for real plant.

Neural network has proven to be an universal approximator. It can successfully approximate nonlinear function. Only few papers have applied neural network approach to the state estimation of nonlinear systems.

In this paper, a new method of fault dection is proposed for a class of nonlinear system which consists of known linear part and unknown nonlinear part. A linear observer is first designed, then a nonlinear compensation term in the nonlinear observer is estimated by using a deconvolution method. And this obtained compensation term is modeled by a WaveARX neural network. At last, the residuals is generated for fault detection based on the analysis of the upper bound approximated error. A simulation example is given to show the effectiveness of this proposed approach.

2 WAVEARX NEURAL NETWORK OBSERVER

Consider a class of nonlinear systems described by:

$$x(k+1) = Ax(k) + f(x(k)) \qquad (1)$$

$$y(k) = Cx(k) \qquad (2)$$

where: A is the linear part of this system and it is supposed known.. $f(x)$ is the unknown nonlinear function. (A,C) is observable.

A method for fault detection based on WaveARX neural network observer is proposed following.

2.1 Nonlinear Observer Design

Firstly, a linear observer is first designed based on the known linear part of system (1) and (2).

$$\hat{x}_l(k+1) = A_0\hat{x}_l(k) + Ky(k) \tag{3}$$

$$A_0 = A - KC \tag{4}$$

where K is the gain of observer and it may be designed by some methods[4,5] which are already exist. $\hat{x}_l$ is the state of the linear observer.

Nonlinear observer is defined as following:

$$\hat{x}(k+1) = A_0\hat{x}(k) + Ky(k) + S(k) \tag{5}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{6}$$

where $S(k)$ is the compensation term according to the nonlinear part $f(x)$. It is relative to the initialization condition of the observer.

2.2 Nonlinear Compensation Term Estimation

In this step, we estimate the unknown nonlinear compensation term $S(k)$ using a deconvolution procedure. Because there is a convolution relation between $S(k)$ and the $\hat{y}$ which is the output of the nonlinear observer, if make $\hat{y}$ equal to the $y(k)$, then $S(k)$ will be achieved. From equation (5) and (6), following equation can be gotten.

$$\begin{aligned}\hat{y}(k) &= C\hat{x}(k) \\ &= C[A_0\hat{x}(k-1) + Ky(k-1) + S(k-1)] = \cdots = \\ & CA_0^k\hat{x}(0) + \sum_{i=1}^{k} CA_0^{i-1}Ky(k-i) + \\ & \sum_{i=1}^{k} CA_0^{i-1}S(k-i)\end{aligned} \tag{7}$$

$$\begin{aligned}\tilde{y}(k) &= y(k) - \hat{y}(k) \\ &= y^*(k) - \sum_{i=1}^{k} CA_0^{i-1}S(k-i)\end{aligned} \tag{8}$$

where

$$y^*(k) = y(k) - CA_0^k\hat{x}(0) - \sum_{i=1}^{k} CA_0^{i-1}Ky(k-i) \tag{9}$$

and $\hat{x}(0)$ can be selected arbitrarily, but the equation $C\hat{x}(0) = \hat{y}(0) = y(0)$ should be satisfied, thus $y^*(k)$ is known.

Let $\tilde{y}(k) = 0$ and $C_i = CA_0^{i-1}$, $k = 1, 2, \cdots, M+1$, from equation (8), we can get

$$\begin{cases} C_1S(0) = y^*(1), \\ C_1S(1) + C_2S(0) = y^*(2), \\ \vdots \\ C_1S(M) + \cdots + C_{M+1}S(0) = y^*(M+1) \end{cases} \tag{10}$$

When $rank(C) = n$ and $p \geq n$, where p is the dimension of output, n is the dimension of state vector. Thus $S(k)$ can be calculated as following:

$$\begin{cases} S(0) = C^+y^*(1) \\ S(k) = C^+[y^*(k+1) - \sum_{i=0}^{k-1} C_{k+1-i}S(i)] \end{cases} \tag{11}$$

$C^+ = (C'C)^{-1}C'$, $k = 1, \cdots, M$, C' is the transposed matrix of C.

Thus if the initialization condition $\hat{x}(0)$ is certain, then $S(k)$ can be calculated.

2.3 Modeling nonlinear compensation term using WaveARX neural network

In this paper, we will adopt WaveARX neural network to model the nonlinear compensation term. Zhao Zhong[6] presented that WaveARX neural network has the ability of approximation. Here $S(k)$ is supposed to be a scalar function (while $S(k)$ is a vector function, its components can be treated similarly). A WaveARX neural network is used to model the compensation term obtained in step 2.

For arbitrary function $f(x): R^n \to R$, it is can be approximated by a multi-input single output (MISO) WaveARX model. The architecture of the WaveARX neural network can be written as,

$$f(X) \approx \sum_{l,m}^{N} C_{l,m}(f)g\,(\|a^{-1}x - bme\|) + c^Tx + c_0 \tag{12}$$

where $C_{l,m}(f)$ are the network coefficients, when choose dilating scale a, translating scale b properly, g can form an affine wavelet frame, $e \in R^n$ which all elements are one and $c \in R^n$, c and c_0 are the coefficients of linear ARX model and can be determined using linear modeling techniques, N is the total number of wavelet functions selected.

Let the input of the nonlinear part be the input and obtained compensation term be the output of the WaveARX neural network. Here the input variable of network is supposed to $\hat{x}(k)$ and the output variable is $\hat{f}(\hat{x}(k))$. Thus, the neural network observer can be written as

$$\hat{x}(k+1) = A_0\hat{x}(k) + Ky(k) + \hat{f}(\hat{x}(k)) \quad (13)$$

$$y(k) = C\hat{x}(k) \quad (14)$$

3 FAULT DETECTION BASED ON WAVEARX NEURAL NETWORKS OBSERVER

After the nonlinear observer is constructed, it can be used for fault detection. At this time, it is not used as an observer, but a normal model for the system with no faults. The fault detection scheme is shown in Figure 1.

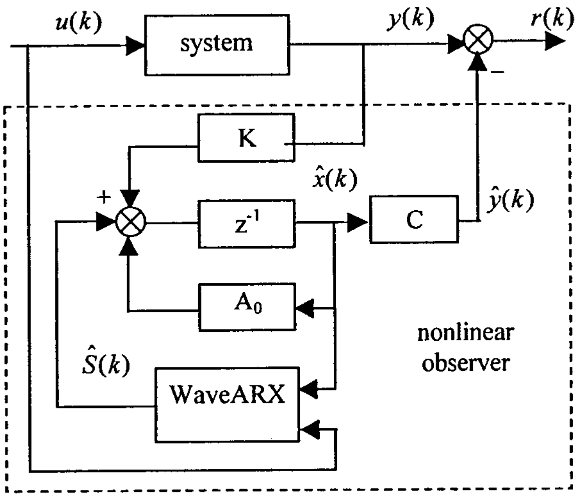

Figure 1: Fault detection scheme

The key problem of model-based fault detection is how to generate the residuals. The residuals are referred to the errors of the functions formed by the observed data and their estimation values. When there is no fault, the residuals are affected by state estimation errors and noises, they are usually small while the state estimation errors are small enough. When there is a fault, the residuals appear large biases. So the fault can be detected by observing the changes of residuals. It means that this model can must accurately predict the output of the system. The convergence of the proposed nonlinear observer is demonstrated following.

Define the error of system output estimation as

$$r(k+1) \equiv y(k+1) - \hat{y}(k+1) = Ce(k) \quad (15)$$

and the maximum error and the Lipschitz continuity of the approximated function as

$$\begin{cases} \| f(x(k)) - \hat{f}(x(k)) \|_\infty \le e_f \\ \| f'(x(k)) - f'(x(k)+e(k)) \|_\infty \le \alpha \| e(k) \|_\infty \end{cases} \quad (16)$$

where $e(k) = x(k) - \hat{x}(k)$

$$\hat{f}(x(k)) = f'(x(k)) + C^T x(k) + c_0$$

$$f'(x(k)) = \sum_{l,m}^{N} C_{l,m}(f) g\ (\|a^{-1}x(k) - bme\|)$$

When the process and measurement noises are small, the estimation of e_f and α can be obtained from the training data. Then from equation (1) and (13), we can get

$$\begin{aligned} e(k+1) &= x(k+1) - \hat{x}(k+1) \\ &= A_0 e(k) + [f(x) - \hat{f}(\hat{x})] \\ &= A_0 e(k) + \Delta\hat{f}(k) + \varepsilon(k) \end{aligned} \quad (17)$$

where $\Delta\hat{f}(k) = \hat{f}[x(k)] - \hat{f}[x(k) - e(k)]$,

$$\varepsilon(k) = \hat{f}[x(k)] - \hat{f}[\hat{x}(k)]$$

A_0 is a Hurwitz matrix, so for an any given positive definite real symmetric matrix Q, there is a positive definite real symmetric matrix P, which satisfies the equation

$$A_0' P A_0 - P = -Q \quad (18)$$

Consider the Lyapunov function $V(k) = e'(k)Pe(k)$, and at first suppose there is no error of the approximation network, that is to say $\varepsilon(k) = 0$. Then from equation (17), following equations can be gotten.

$$\begin{aligned} V(k+1) &= e'(k+1)Pe(k+1) = e'(k)A_0' P A_0 e(k) + \\ &\quad 2\Delta\hat{f}'(k) P A_0 e(k) + \Delta\hat{f}'(k) P \Delta\hat{f}(k) \end{aligned} \quad (19)$$

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) = -e'(k)Qe(k) + \\ &\quad 2\Delta\hat{f}(k) P A_0 e(k) + \Delta\hat{f}'(k) P \Delta\hat{f}(k) \end{aligned} \quad (20)$$

According to the definitions in equation (16), we can obtain following equation

$$\Delta V(k) \le (-\lambda_{q\min} + 2\alpha\lambda_{p\max}\sigma_{\max} + \alpha^2\lambda_{p\max}) \| e(k) \| \quad (21)$$

where $\lambda_{q\min}$ is the minimum eigenvalue of Q, $\lambda_{p\max}$ is the maximum eigenvalue of P, $\sigma_{\max}$ is the maximum singular value of A_0. If $-\lambda_{q\min} + 2\alpha\lambda_{p\max}\sigma_{\max} + \alpha^2\lambda_{p\max} < 0$, then $\Delta V(k) < 0$, which means $e(k) \to 0$, as $t \to \infty$.

In practice, there always exists error of network, that is, for all k, $\| \varepsilon(k) \|$ is not equal to zero all the time. Thus we can analyze the upper bound approximate error. From equation (17), we have

$$\| e(k+1) \| \le \| A_0 e(k) \| + \| \Delta\hat{f}(k) \| + \| \varepsilon(k) \|$$

$$\leq (\sigma_{\max} + \alpha) \| e(k) \| + \| \varepsilon(k) \| \tag{22}$$

If $(\sigma_{\max} + \alpha) < 1$,equation (21) will be stable. So when k is enough large, we have

$$e(k+1) \approx e(k) \tag{23}$$

Substituting equation (22) into (21), we get:

$$\| e(k) \| \leq \frac{\| \varepsilon(k) \|}{1 - (\sigma_{\max} + \alpha)} \tag{24}$$

Then the residuals for fault detection can be designed as

$$\| r(k) \| \leq \| C \| \frac{e_f}{1 - (\sigma_{\max} + \alpha)} \equiv \beta \tag{25}$$

When a fault is exist, equation (25) cann't be satisfied. Therefore, β is the threshold of residuals and the change of $r(k)$ can be used for fault detection.

4 ILLUSTRATIVE EXAMPLE

In this section , the proposed approach is applied to an example of nonlinear system to show its effect validity. Consider the motion equation of a single-link robot constrained in vertical plane[8]. This motion can be expressed in joint space as

$$\begin{aligned} &M\ddot{q} + 0.5mgl \sin q = u \\ &y = q \end{aligned} \tag{26}$$

where q represents the generalized coordinate (joint position), u is the applied joint torques, M is the moment of inertia, g is the constant of gravity, m and l represent the quality and the length of the arm respectively. Here the value of these parameters are given in table 1 and all of them are SI units .

Table1: Robot parameters

m	l	M	g
1	1	0.5	9.8

Suppose $x_1 = q, x_2 = \dot{q}$, $u = \sin 20t + \cos 20t$, then equation (26) can be expressed as

$$\begin{aligned} &\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\sin 20t + \cos 20t - o.5mgl \sin x_1) / M \\ &y = x_1 \end{aligned} \tag{27}$$

First , we design the linear observer which has the same form as

$$\begin{aligned} &\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} y \\ &\hat{y} = \hat{x}_1 \end{aligned} \tag{28}$$

then discretize it as a sampling interval 0.05 seconds, and add a nonlinear compensation term $S(k)$ to form a nonlinear observer. The designed discrete nonlinear observer has the following form

$$\begin{aligned} \begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0.9512 & 0 \\ -0.0476 & 0.9512 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} \\ &+ \begin{bmatrix} 0.0488 \\ 0 \end{bmatrix} y(k) + \begin{bmatrix} 0 \\ S(k) \end{bmatrix} \end{aligned} \tag{29}$$

where the nonlinear compensation term $S(k)$ is determined using decovolution method , modeled by a WaveARX network. Based on the knowledge of system's nonlinearity the input of network is taken as $\hat{x}_1$, i.e., $S(k)$ is approximated by network output $\hat{S}(\hat{x}_1(k))$ which is a function of $\hat{x}_1(k)$. After the network is successfully trained, the nonlinear observer is design. the maximum error and the Lipschitz constant are estimated as $e_f = 1.1709$, $\alpha = 0.1202$. Figure 2 shows the comparison of the real motion with no fault and the output of the observer designed above.

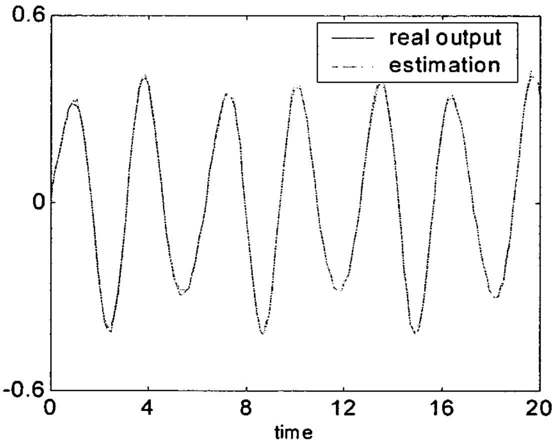

Figure 2: Comparison of motion estimation and actual output

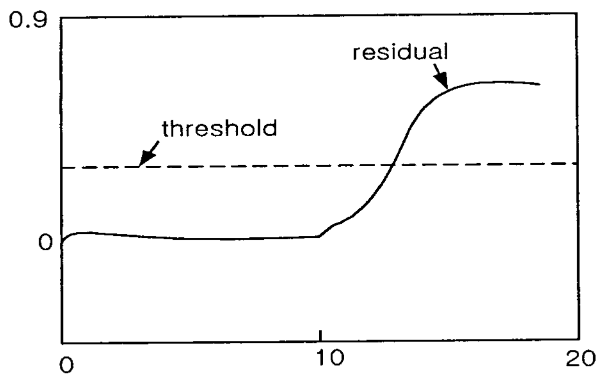

Figure 3: Residual generated in a fault

Figure 3 shows the residual when a fault is present. It can be seen that the residual has changed a lot after the fault. Thus, the fault can be detected easily by using appropriate threshold.

5 CONCLUTION

This paper reported on the use of WaveARX Neural network to build a nonlinear observer that is used to detect faults in nonlinear systems. The proposed approach can be applied to a class of nonlinear systems described in (1) and (2). The linear part of the system is assumed known however the nonlinear part can be unknown and there is no restriction on its type. The analysis based on Lyapunov function in part 3 of this paper guarantees the convergence of this method. Simulation results showed that the proposed system can effectively detect faults.

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