주성분 분석을 이용한 블라인드 신호 분리

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Blind Source Separation by PCA

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abstract

Various methods for blind source separation (BSS) are based on independent component analysis (ICA) which can be viewed as a nonlinear extension of principal component analysis (PCA). Most existing ICA methods require certain nonlinear functions, the shapes of which depend on the probability distributions of sources (which is not known in advance), whereas PCA is a linear learning method based on only second-order statistics. In this paper we show how BSS can be achieved by PCA, provided that sources are spatially uncorrelated but temporally correlated.

1. Introduction

Blind source separation (BSS) is a statistical method which aims at recovering unknown sources from their linear instantaneous mixtures without any prior knowledge of the mixing matrix nor sources. Most existing methods for BSS are based on ICA. These methods exploit the higher-order statistical structure of the data which is statistically independent[1] either implicitly or explicitly. Alternatively the task of BSS can be achieved by the nonlinear PCA[2] where certain nonlinear functions employed in the standard PCA according to the probability distributions of sources which are unknown in advance. These methods are also based on higher-order statistics.

In this paper we show that the standard PCA can be applied to the task of BSS, provided that sources are spatially uncorrelated but temporally correlated. The resulting method is based on only second-order statistics, so it avoids nonlinear function. Moreover, the method is also able to separate the mixtures of several colored Gaussian sources, whereas the ICA method can not.

2. Problem Formulation

In the simpliest form of BSS, it is assumed that the n-dimensional observation vector $\boldsymbol{x}(t) = [\, x_1(t) \cdots x_n(t)]^T$ is generated by

$$\boldsymbol{x}(t) = A\, \boldsymbol{s}(t), \qquad (1)$$

where $A \in R^{n \times n}$ is called the *mixing matrix*, $\boldsymbol{s}(t)$ is the n-dimensional vector whose elements are called *sources*.

The task of BSS is to recover source vector $\boldsymbol{s}(t)$ up to its possibly scaled and re-ordered version. That is, the estimate of source vector, $\hat{s}(t)$ satisfy $\hat{s}(t) = P\Lambda\, s(t)$ where P is some permutation matrix and Λ is some nonsingular diagonal matrix. The transformation by generalized permutation matrix, $P\Lambda$ is referred to as a *transparent transformation*. In other words, the task is to build a linear transformation W (demixing matrix) such that $WA = P\Lambda$ is satisfied.

Throughout this paper the following assumptions are made:

AS1 The mixing matrix A is of full rank.

AS2 Sources are spatially uncorrelated but temporally correlated stochastic signals with zero mean and unit variance, i.e.,

$$E\{\, s_i(t)\ \ s_j(t-\tau)\} = r_i(\tau)\ \delta_{ij}, \quad \forall \tau, \quad i, j = 1, \ldots, n, \qquad (2)$$

where δ_{ij} is the Kronecker delta equal to 1 for $i=j$, otherwise it is zero, $r_i(\tau)$ is the arbitrary variable, and $E\{\cdot\}$ denotes the statistical average operator.

3. Proposed Methods

This section describes our method of BSS based on PCA with brief review of PCA and whitening.

3.1 Principal Component Analysis(PCA)

The PCA is a classical multivariate data analysis method that is useful in linear feature extraction and data compression. These methods find a linear transformation $\boldsymbol{y} = H\boldsymbol{x}$, where the row vectors of H

correspond to the normalized orthogonal eigenvectors of the data covariance matrix.

One simple approach to PCA is to use singular value decomposition(SVD). Let us denote the data covariance matrix by $R_x(0)=E\{x(t)\ x^T(t)\}$. Then the SVD of it gives

$$R_x(0)=UD\,U^T, \quad (3)$$

where U is the eigenvector matrix(i.e., modal matrix) and D is the diagonal matrix whose diagonal elements correspond to the eigenvalues of $R_x(0)$. Then the linear transformation H in PCA is given by

$$H=U^T. \quad (4)$$

Adaptive algorithms for PCA are also proposed such as Oja's subspace rule[3], GHA[4], and APEX[5].

3.2 Whitening

The task of whitening is to find a linear transformation $y=Vx$ such that $E\{y(t)\ y^T(t)\}=I$ where $I\in R^{n\times n}$ is the identity matrix. In other words, the whitening aims at eliminating cross-correlations as well as normalizing the variance to be unity. The transformation $V=D^{-\frac{1}{2}}U^T=R_x^{-\frac{1}{2}}(0)$ follows from the decomposition in (3).

It can also be performed in adaptive fashion such as the global decorrelation rule[6].

3.3 BSS Based PCA

The data vector $x(t)$ is first whitened by a linear transformation $V=D^{-\frac{1}{2}}U^T$. Denote the whitened vector by $z(t)=Vx(t)$. Then the whitened data vector $z(t)$ has the form

$$z(t)\ =\ Bs(t), \quad (5)$$

where $B=VA$ is an orthogonal mixing matrix since $E\{z(t)\ z^T(t)\}=I$.

Let us consider the signal $y(t)$ generated by

$$y(t)=z(t)+z(t-1). \quad (6)$$

The correlation matrix of $y(t)$ is

$$R_y(0)=E\{[z(t)+z(t-1)]\ [z(t)+z(t-1)]^T\}$$
$$=2I+R_z(1)+R_z^T(1), \quad (7)$$

where $R_z(1)$ is the time-delayed correlation matrix defined by $R_z(1)=E\{z(t)\,z^T(t-1)\}$.

Since the correlation matrix $R_y(0)$ is symmetric, it has the following eigen-decomposition

$$R_y(0)=U_y\,D_y\,U_y^T. \quad (8)$$

Note that from (5) and (6) we have

$$R_y(0)=B\{2I+R_s(1)+R_s^T(1)\}\,B^T, \quad (9)$$

where $2I+R_s(1)+R_s^T(1)$ is a diagonal matrix from the assumptions *(AS2)* and B is an orthogonal matrix. Thus, it follows from (8) and (9) that the orthogonal mixing matrix B is equal to the eigenvector matrix U_y up to a permutation and sign of eigenvectors, provided that the diagonal elements of $2I+R_s(1)+R_s^T(1)$ are distinct. This leads to the estimate of the mixing matrix, $\hat{A}=V^{-1}U_y$.

The method is summarized below.

Algorithm Outline: BSSPCA-1

- Whiten the data $x(t)$ by $V=D^{-\frac{1}{2}}U^T$, i.e., $z(t)=Vx(t)$.
- Compute $y(t)=z(t)+z(t-1)$.
- Apply the PCA to $y(t)$ to obtain U_y where $R_y(0)=U_y\,D_y\,U_y^T$.
- Obtain the demixing matrix $W=U_y^TV$.

We define $v(t)=Wx(t)=U_y^Tz(t)$. Then one can easily see that both $R_v(0)$ and $R_v(1)+R_v^T(1)$ are diagonalized by the transformation W. The following theorem explains why the simultaneous diagonalization of these two matrices viewing the method BSSPCA-1 gives the solution to the problem of BSS.

Theorem 1: Let $\Lambda_1, \Lambda_2, D_1, D_2\in R^{n\times n}$ be diagonal matrices with nonzero diagonal entries. Suppose that $G\in R^{n\times n}$ satisfies the following decompositions:

$$D_1=G\,\Lambda_1\,G^T \quad (10)$$
$$D_2=G\,\Lambda_2\,G^T \quad (11)$$

Then the matrix G is the generalized permutation matrix, i.e., $G=P\Lambda$ if $D_1^{-1}D_2$ and $\Lambda_1^{-1}\Lambda_2$ have distinct diagonal entries.

The method is described below.

Algorithm Outline: BSSPCA-2

- Whiten the data $x(t)$ by $V=D^{-\frac{1}{2}}U^T$, i.e., $z(t)=Vx(t)$.
- Compute the time-delayed correlation matrix,
  $$M_z(1)=\frac{1}{2}\{R_z(1)+R_z^T(1)\}. \quad (12)$$
- Compute the SVD of $M_z(1)$, $M_z(1)=U_z\,D_z\,U_z^T$ to obtain U_z.
- Compute the demixing matrix $W=U_z^TV$.

In BSSPCA-1, instead of (6), we can consider

$$y(t)=z(t)+z(t-\tau),\quad \tau\in T, \quad (13)$$

as long as $R_s(\tau)$ is invertible diagonal matrix with distinct diagonal elements. Therefore, in BSSPCA-2, instead of (12), we can use

algorithm	performance index(PI)
BSSPCA-1	0.0008
BSSPCA-2	0.0014
JADE[7]	0.0127
fastICA[8]	0.0586
Extended Infomax[9]	0.0472

Table 1.Performance Index of various methods

$$M_z(\tau)=\frac{1}{2}\{ R_z(\tau)+R_z^T(\tau)\}. \qquad (14)$$

Moreover we can also consider a linear sum

$$y(t)=\sum_{i=0}^{L} \alpha_i z(t-i), \qquad (15)$$

and a linear combination $\sum_{i=1}^{L} \beta_i M_z(i)$.

4. Numerical Experiment

We demonstrate the useful behavior of our methods (BSSPCA-1 and BSSPCA-2) through numerical experiment. We have chosen two speech and one music signals that were sampled at 8 kHz, and three independent colored-Gaussian sources. These six sources constituted the 6-dimensional source vector $s(t)$. The 6-dimensional observation $x(t)$ were generated by linear transformation. Total number of samples were 10000.

In order to evaluate the performance of methods, we calculated the performance index(PI) defined by

$$PI=\frac{1}{2(n-1)}\sum_{i=1}^{n}\left\{\left(\sum_{k=1}^{n}\frac{|g_{ik}|}{\max_j|g_{ij}|}-1\right)+\left(\sum_{k=1}^{n}\frac{|g_{ki}|}{\max_j|g_{ji}|}-1\right)\right\} \qquad (16)$$

where g_{ij} is the (i,j)-element of the global system matrix $G=WA$ and $\max_j|g_{ij}|$ represents the maximum value among the elements in the i-th row vector of G, $\max_j|g_{ji}|$ does the maximum value among the elements in the i-th column vector of G. The performance index defined in (16) tells us how far the global system matrix G is from a generalized permutation matrix (transparent transformation). The zero performance index means perfect separation, however in practice very small number (say, .001) is acceptable performance.

In Table 1, we compared the performance index of our methods to that of popular BSS algorithms such as JADE[7], fastICA[8], and extended infomax[9] are based on higher-order statistics, one expect that they have difficulty in separating out several Gaussian sources.

5. Conclusions

In this paper, we have shown that the standard PCA could be applied to the task of BSS, provided that sources are spatially uncorrelated but temporally correlated. We have presented two methods (BSSPCA-1, BSSPCA-2) that are useful for BSS. The adaptive implementation of BSSPCA-1 and BSSPCA-2 is straightforward since any existing adaptive PCA algorithms can be employed. The main advantage of proposed methods is to exploit only second-order statistics, so the methods avoid any nonlinear function, in contrast to the conventional ICA methods. We are currently working on a way of finding a linear combination (15) that guarantees that the corresponding time-delayed correlation matrix of source vector has distinct diagonal elements. We also working on the extension of the proposed methods to handle the noisy observation vector that was not considered in this paper.

6. Acknowledgment

This work was supported by Korea Ministry of Science and Technology under Brain Science and Engineering Research Program and by Ministry of Information and Communication under Advanced backbone IT technology development project and by POSTECH BK 21.

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