비대칭 벽식구조의 변위에 근거한 내진설계

Displacement Based Seismic Design of Asymmetric-Plan Wall Structures

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국문요약

본 논문은 직접 변위 설계법의 기본개념을 이용하여 비대칭 평면을 갖는 벽식구조의 변위에 근거한 내진설계방법을 제안한다. 제안된 설계방법은 구조시스템의 각 벽체의 강도비와 강성비 및 목적 설계 변위를 결정하고, 직접 변위 설계법에 따라 설계하중은 구하는 과정으로 이루어진다. 탄성 영역에서는 강성 편심을, 비탄성 영역에서는 강도 편심을 주요한 설계변수로 사용하였다. 성능에 기초한 내진설계의 개념에 따라 비대칭 평면을 갖는 구조물이 요구되는 성능수준을 효과적으로 만족할 수 있도록 본 논문은 시스템의 비틀림 미케니즘과 각 벽체의 변형능력을 고려하였다. 제안된 설계방법을 이용하여 중진과 강진 지역에 대해 예제 구조물의 설계하중을 구하고, 최적의 설계방법을 제안하였다.

1. Introduction

It is well known that asymmetric-plan buildings are vulnerable during earthquakes. To reduce vulnerability, current seismic code provisions restrict excessive ductility demand of members due to torsion. These code provisions are mainly based on elastic behavior and enable us to proportion strength of walls with assumption that stiffness depends only on wall length (Fig. 1(a)). However, according to the displacement based design proposed by Priestley and Kowalsky [4,5], the yield curvature of a cantilever wall is dependent on wall length (Fig. 1(b)) and the assumption of constant stiffness for walls of equal length leads to significant errors. Contrary to the current code provisions based on the constant stiffness assumption for cantilever walls, Paulay [2,3] identified torsional plastic mechanisms based on the constant curvature assumption and determined system ductility capacity of asymmetric-plan buildings by classifying asymmetric building systems into torsionally restrained and unrestrained systems.

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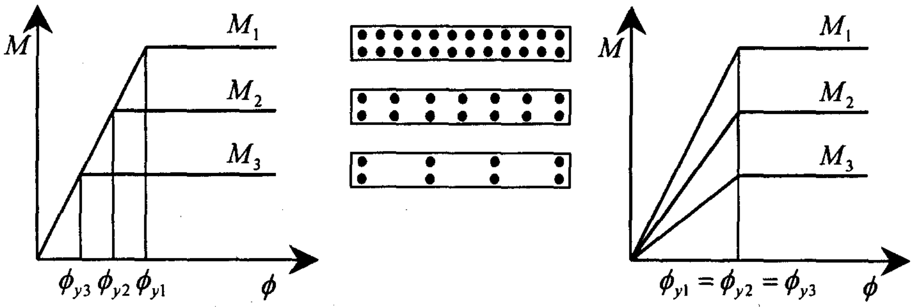
This paper proposes displacement based seismic design method for asymmetric-plan wall buildings. The design method is composed of following three steps. In the initial step, strengths of walls are proportioned. Once the strength ratio among the walls is proportioned, the eccentricity of stiffness and strength, the torsional and lateral stiffness are determined based on the constant yield curvature assumption. The center of stiffness or strength can be used as main design parameters depending on elastic or inelastic behavior. In the next step, the target displacement of system is determined. A Target displacement is limited by the member whose displacement capacity is reached first. In the final step, the total base shear of system is calculated by the direct displacement method and then the base shear of each cantilever wall is distributed.



(a) Constant stiffness **(b) Constant yield curvature**

Fig. 1 Stiffness-strength relationship for cantilever walls of equal length

2. Strength Proportioning

As an initial step to the displacement based design for asymmetric-plan buildings, an appropriate strength ratio among walls is selected. For given geometric properties such as center of mass, dimensions of system, lengths and locations of walls, design parameters involving the eccentricities of strength and stiffness, the lateral and torsional stiffness are found.

Table 1 Strength proportioning methods

Method	Torsional provisions of codes	Method I	Method II	Method III
Design strategy	Locate C.V. between C.S. and C.M.	Locate C.S. at target location	Locate C.V. at target location	Locate C.V. by target rotation
Behavior	Elastic behavior	Elastic behavior	Inelastic behavior	
Application	Force based design	Displacement based design		
Assumption	Constant stiffness	Constant yield curvature		

Based on the constant yield curvature assumption, the determination of strength ratio leads to the determination of other design parameters. According to the design strategy for locating C.V. or C.S., three methods are proposed and detailed procedures of each method for the model in Fig. 2 are described.

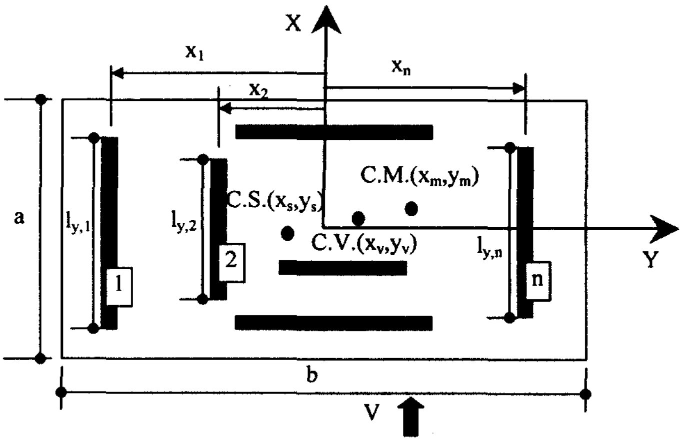


Fig. 2 Model of asymmetric-plan building

2.1 Method I – Locating center of stiffness at target location

If zero eccentricity of stiffness in the asymmetric-plan building (Fig.2) is intended, the following procedures to coincide the center of stiffness (C.S.) with the center of mass (C.M.) can be used.

Step 1 Select initial strength ratio of each wall

Strengths of walls in y-direction (loading direction) are distributed proportionally to the square of each wall length. The stiffness of each wall is calculated by dividing the strength by its yield displacement that is inversely proportional to the wall length. For convenience, the total sum of strength and stiffness of walls in y-direction are assumed to be a unit.

$$V_{y,i} = \frac{\alpha l_{y,i}^2}{\sum V'_{y,i}} = \frac{\alpha l_{y,i}^2}{\sum \alpha l_{y,i}^2} = \frac{l_{y,i}^2}{\sum l_{y,i}^2} \tag{1}$$

$$K_{y,i} = \frac{V_{y,i}/(\beta / l_{y,i})}{\sum K'_{y,i}} = \frac{V_{y,i} l_{y,i} / \beta}{\sum V_{y,i} l_{y,i} / \beta} = \frac{V_{y,i} l_{y,i}}{\sum V_{y,i} l_{y,i}} \tag{2}$$

where $\beta / l_{y,i} =$ the yield displacement and $\alpha, \beta =$ constants. Note that V and K denote the ratios of strength and stiffness and V' and K' the absolute values of strength and stiffness. The stiffness ratio of walls in x-direction (transverse direction) is obtained by assuming that base shears in x and y directions are same.

$$K_{x,i} = \frac{l_{x,i}^{3} \sum l_{y,i}^{2}}{\sum l_{y,i}^{3} \sum l_{x,i}^{2}} \tag{3}$$

Step 2 Find the centers of stiffness and strength

$$(x_s, y_s) = \left(\frac{\sum x_i K_{y,i}}{\sum K_{y,i}}, \frac{\sum y_i K_{x,i}}{\sum K_{x,i}} \right) \tag{4}$$

$$(x_v, y_v) = \left(\frac{\sum x_i V_{y,i}}{\sum V_{y,i}}, \frac{\sum y_i V_{x,i}}{\sum V_{x,i}} \right) \tag{5}$$

Step 3 Calculate normalized eccentricity of stiffness

$$\xi = (x_s - x_m) / b \tag{6}$$

Step 4 Calculate torsional stiffness of system and required additional strength ratio

$$K_T = \sum K_{y,i}(x_i - x_s)^2 + \sum K_{x,i}(y_i - y_s)^2 \tag{7}$$

$$V_{y,i}^{\ T} = \frac{K_{y,i}(x_i - x_s)\xi b}{K_T} = \frac{K_{y,i}(x_i - x_s)\xi}{\rho^2 b} \tag{8}$$

where the normalized radius of gyration of stiffness is calculated by eq. (9)

$$\rho = \frac{1}{b}\sqrt{K_T / \sum K_{y,i}} = \frac{\sqrt{K_T}}{b} \tag{9}$$

Step 5 Update strength and stiffness ratios

New strength ratio is obtained by adding required strength ratio ($V_{y,i}^{\ T}$) to resist torsion to the assumed strength in the k-th step.

$$V_{y,i}^{\ k+1} = V_{y,i}^{\ k} + V_{y,i}^{\ T} \tag{10}$$

New stiffness ratio is also updated by eq. (2)

Step 7 Repeat Step 2 to Step 6 until the eccentricity of stiffness reaches zero.

Non-zero eccentricity of stiffness can allow us an appropriate reinforcement ratio distribution among the walls when there is an excessive eccentricity of stiffness after the first iteration. To achieve this proportionality, the center of mass is assumed to be at the target center of stiffness.

2.2 Method II – Locating center of strength at target location

If zero eccentricity of strength in the asymmetric-plan building (Fig.2) is intended, the following procedures to coincide the center of strength (C.V.) with the center of mass (C.M.) can be used.

p 1 Try an initial strength ratio among walls

Distribute the strength ratio of walls in y-direction (loading direction) by eq.(1).

Step 2 Determine complementary strength ratio for zero eccentricity of strength

A complementary strength ratio is necessary to render the eccentricity of strength zero. They are assumed by eq. (11).

$$V^T_{y,i} = pl_i^2 \text{ (when } x_i < x_v\text{)} \tag{11-a}$$

$$V^T_{y,i} = ql_i^2 \text{ (when } x_i > x_v\text{)} \tag{11-b}$$

In order to find constants p and q, the following two conditions are necessary. The additional strength $V^T_{y,i}$ should satisfy eq. (12) to keep the sum of strengths unchanged and the center of modified strength must coincide with C.M. by eq. (13)

$$\sum pl_i^2 + \sum ql_i^2 = 0 \tag{12}$$

$$\sum x_i V_{y,i}^{\ k+1} = \sum x_i (V_{y,i}^{\ k} + V^T_{y,i}) = x_m \tag{13}$$

Step 3 Modify the strength ratio

The eccentricity of strength can be shifted by modification of the initial strength ratio.

$$V_{y,i}^{\ k+1} = V_{y,i}^{\ k} + V^T_{y,i} \tag{14}$$

2.3 Method III – Locating center of strength by pre-determined target rotation

This method is applicable to tortionally restrained systems [8] where target rotation is restrained by transverse walls which remain elastic. This method determines the target location of C.V. that is calculated from the torsional stiffness and the target rotation angle.

Step 1 Choose target rotation angle by graphical method

In the first step, the target rotation angle θ can be selected between θ_1 and θ_2 that are limited by displacement capacity of walls. The target rotation angle resulting in the smallest base shear is chosen as an optimum rotation angle. The system damping value by eq. (17) is used as a criterion for magnitude of base shear.

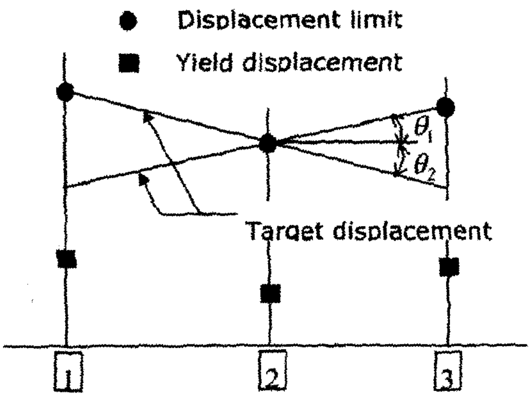


Fig. 3 Target rotation angle

Step 2 Calculate target eccentricity of strength

Torsional stiffness is calculated from the elastic stiffness of transverse walls.

$$K_T = \sum \frac{V_{x,i}}{\Delta_{x,i}} (y_i - y_s)^2 \tag{15}$$

where $\Delta_{x,i}$ is the yield displacement of wall in the transverse direction. The target eccentricity of strength is calculated by equilibrium condition.

$$e_v = \frac{K_T \theta}{\sum V_{y,i}} \tag{16}$$

Step 3 Proportion strength ratio among the walls by Method II

Assume C.M. at the target C.V. and distribute strength by Method II.

3. Target Displacement

After the strength ratio is allocated, the base shear is determined by the direct displacement based design procedure. The selection of target displacement is the first step to the displacement based design. The target displacement of an isolated cantilever wall is determined by the capacity of plastic hinge rotation or code-specified drift limits [1]. When a system consists of a group of walls, C.M. is considered as a reference point of the target displacement. The target displacement at C.M. is determined by a step-by-step procedure. As displacement demand increases, the walls yield and the system properties change. The lateral stiffness, the torsional stiffness and the eccentricity of stiffness are revised at each step and the target displacement is determined when one of walls reaches the displacement limit.

4. Design Base Shear

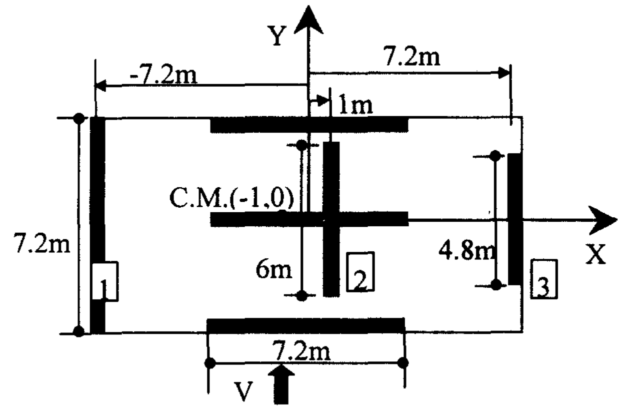
The base shear corresponding to the target displacement is calculated by the design displacement spectrum. The system damping in an equivalent S.D.O.F system is derived from the effective damping of each wall, where a weighted mean average is appropriate, given by eq. (17).

$$\xi_e = \sum_i^n V_{y,i} \xi_{y,i} \tag{17}$$

The displacement spectrum is modified by the system damping and the effective period corresponding to the target displacement is determined from the spectrum. Finally, the design base shear is calculated from the effective period and the stiffness. The base shear is distributed to each wall according to the determined strength ratio.

5. Design Example

Design base shear forces of an example wall building shown in Fig. 4 are calculated by the proposed design methods and Table 2 shows the results. Base shear forces for one example asymmetric building vary to the extent of 48% according to the strength proportioning methods. For implementation of method I, two cases are demonstrated depending on different target location of C.S. In Case 1, C.S. is located at X=-1 and the ratio of amounts of uniformly distributed reinforcements of each wall ($A_{s,i}$) to the sum of them of all walls ($\sum A_{s,i}$) is calculated as (0.23, 0.352, 0.418). In this case the reinforcement ratio of wall 3 is twice larger than that of wall 1. When 10% eccentricity of stiffness is allowed, C.S. moves to the location at X=-1.72 by Case 2, which results in $A_{s,i}$ to $\sum A_{s,i}$ ratio as (0.28, 0.344, 0.376).



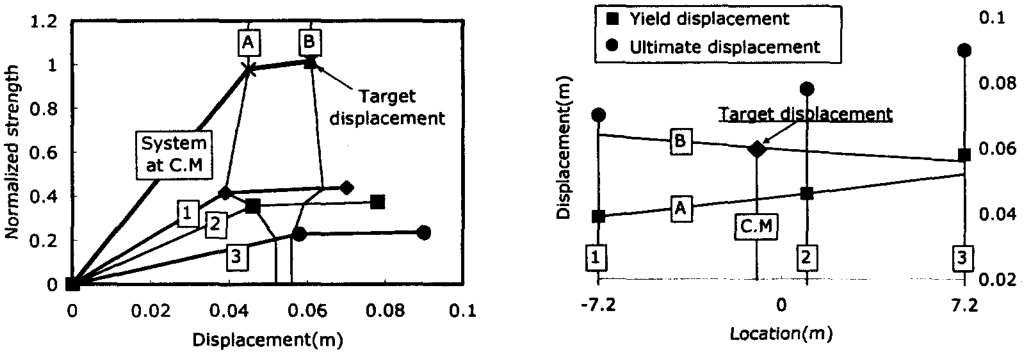
Height: 25m
Story: 10
Story weight: 660 kN
Design zone: Korea zone I

Fig. 4 Design Example

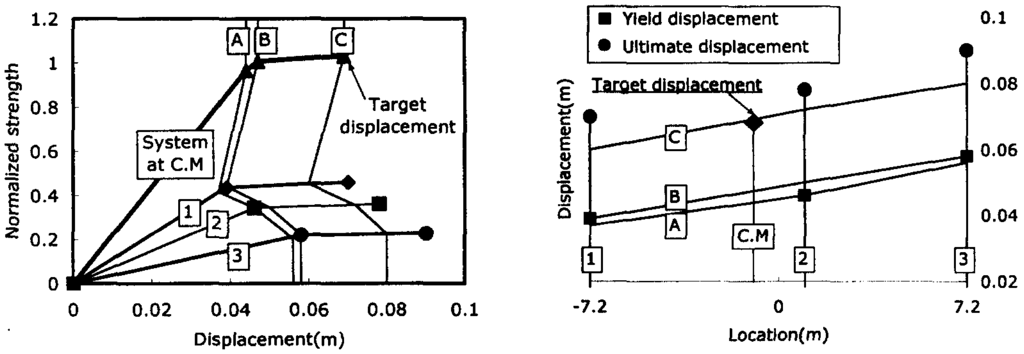
Table 2 Design results

Method	Method I (Case 1)	Method I (Case 2)	Method II	Method III
Design strategy	Locate C.S. at C.M.	Locate C.S. at X=-1.72	Locate C.V. at C.M.	Locate C.V. at X=-1.217
Center of Stiffness	X=-1	X=-1.72	X=-1.857	X=-2.072
Center of Strength	X=-0.113	X=-0.842	X=-1	X=-1.217
Strength ratio	0.348,0.371,0.281	0.409,0.348,0.243	0.416,0.356,0.228	0.437,0.344,0.22
$A_{s,i}$ to $\sum A_{s,i}$ ratio	0.23,0.352,0.418	0.28,0.344,0.376	0.289,0.356,0.356	0.306,0.35,0.347
Target displacements (C.M./members)	0.053 / 0.064,0.049,0.038	0.059 / 0.064,0.057,0.053	0.061 / 0.064,0.059,0.056	0.069 / 0.06,0.072,0.08
Base Shear(Korea/UBC)	642.7kN / 3549kN	587.1kN/ 3393kN	576.8kN / 3362kN	433kN / 2718kN

Displacement profiles of system and members by Method II and III are shown in Fig. 5. Numbers 1, 2 and 3 denote the identification number of walls and A, B and C indicate each step when any walls yield. Line B in Method II, line C in Method III respectively indicate the target displacement and the rotation angle. In Method II, wall 3 doses not yield at the target displacement, meanwhile in Method III, the displacement capacity of all walls contributes to the displacement capacity of system. As a result, the target displacement by Method III is largest and base shear is smallest. Therefore, the strength distribution by Method III is considered as an optimal method.



(a) Method II



(b) Method III

Fig. 5 Displacement profile by Method II and III

6. Conclusion

Based on the constant yield curvature assumption, displacement based design method of asymmetric-plan buildings is proposed. This method determined strength ratio by locating C.S. and C.M. according to the design strategy. Method I focus on elastic behavior and Method II and III focus on inelastic behavior. By the displacement based design method, target displacement and base shear are determined. Base shear forces for the example asymmetric building vary considerably according to the strength proportioning methods. The difference results from extent of utilizing ductility capacity of each wall. Judging from the design results, Method III can utilize ductility capacities of walls most effectively and is considered as an optimal method for seismic design of asymmetric building in inelastic range. The proposed design procedure that considers torsional mechanism and ductility capacity of each wall is appropriate for performance based design of asymmetric-plan buildings.

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