A Note on the Defuzzification Method and Distance Metric of Fuzzy Color Model

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퍼지 컬러 모델의 비퍼지화 방법과 거리 척도의 제안

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Abstract

Most people have to deal with color and color problems occasionally. There are many strange things about color and color vision that most people do not notice. Even though color seems intuitive and simple it is not. In this paper, we modeled the color using fuzzy set theory. The proposed fuzzy color model is based on the Munsell color space. We defined several fuzzy color terminologies, and proposed a extended center of gravity defuzzification method for fuzzy color set. Finally, three distance measures between fuzzy colors were also formulated.

1. Introduction

Color plays an important role in our daily lives. Most people have to deal with color and color problems occasionally, and people who work professionally with color must deal with them everyday. There are many strange things about color and color vision that most people do not notice. Even though color seems intuitive and simple it is not [1][2][3][4].

Two major models in the research field of computer science are RGB model and HSI model. RGB color model is simple and easy to understand and manipulate. HSI color model represents the more similar characteristics of color that our humans perceive. But both RGB and HSI models do not reflect the uncertainty of the color component, which is due to the color quantization and human perception. So in order to overcome these shortcomings, we must design a new color model based on an exact color space [5].

In this paper, we studied a new color model using the fuzzy set theory. The proposed fuzzy color model is based on the Munsell color space. We transformed the Munsell space to fuzzy set domain. And by using three factors, hue, value, and chroma, we designed the fuzzy color model. The shape of fuzzy color set is similar to triangular fuzzy number. In order to defuzzify the fuzzy color, a new defuzzification method called ECOG (extended COG) method was developed. By using the defuzzified singleton value, we formulated three color distance measures between fuzzy color families and fuzzy colors.

The remainder of this paper is organized as follows. Section 2 gives the description of the previous color models. We covers the proposed fuzzy color model, the defuzzification method, and the distance metric in section 3. Finally, we conclude our remarks in section 4.

2. Previous Color Models

As already mentioned in introduction, color seems simple and intuitive but it is not. It's rather complicated. To handle color problem in the computer, people usually adopt either RGB or HSI model.

RGB color model is the most famous color representation model. With the RGB coordinates one obtains the color cube whose vertices are the eight representative colors. The major problem that RGB model suffers is a strong degree of correlation among the three components. The three values change dependently and are highly sensitive to the variation of lightness [5].

HSI color model can represent the subjective color suitable for human observes, and the result does not have correlation among hue, saturation, and intensity. HSI model is one of simple approximation of the Munsell color system. But because HSI model is a symmetric approximation of original Munsell system, the loss of information in the original color representation was occurred due to the simplified symmetric shape. Besides these two color models, CIELAB and CIELUV models are also popular in color industry.

Munsell color space is a systematic way of ordering and describing color [4][6]. The color space has been the most intuitive and the most useful to the artists and designers until now. Figure 1 shows a 3-dimensional representation. The system consists of three components, hue, chroma, and value. *Hue* is a quality which distinguishes one color family from another, as red from yellow, or green from blue. In the Munsell system there are five major hue families: Red,

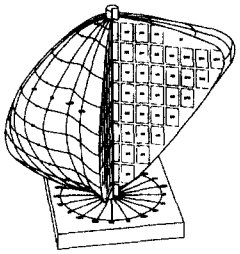

그림 1 Munsell color space

Yellow, Green, Blue, and Purple. *Value* is that quality by which a light color is distinguished from a dark one. The Munsell gray scale is divided into ten value steps. *Chroma* is the strength or vividness of color. It can be also defined as the degree of departure of color from a gray of the same lightness. If hue carries most of the emotional content and value carries the information, then chroma is the attention-getting quality.

3. Fuzzy Color Model

3.1 Fuzzy color set

Original Munsell color system has a asymmetric value-chroma plane. This is an important point to distinguish each color characteristics and calculate the more exact distances between colors. But until now a few research have been done on this matter, and there's no models by which this problem is easy to be represented. We approached this issue by taking a fuzzy set theory into color modeling. A proposed fuzzy color model can easily represent the original Munsell system, and more, can handle the uncertainties resided in natural color [7].

The first thing to acquire fuzzy color model is to cut the Munsell color system vertically. Then we can obtain an value-chroma plane at specific hue point. Next job is to spread the whole value-chroma planes which were cut at every hue points. The unfold shapes seem to look like a mountain range. The key point is that we can approximate each value-chroma planes as the triangular fuzzy numbers. A slice of value-chroma plane corresponds to one fuzzy set. We selected ten pre-defined triangular fuzzy numbers: five major hue colors (red, yellow, green, blue, purple) and five minor hue colors (yellow red, green yellow, blue green, purple blue, red purple). As defined earlier, a fuzzy color set has two axes, value as a domain and chroma as a membership degree for color family. Given a color specification, the membership degree can be easily obtained by computing the value and chroma positions.

3.2 Defuzzification of the fuzzy color set

In order to calculate the distance between fuzzy colors, we adopt the geometrical computation based on the defuzzified values of each fuzzy color sets. In this section we describe the two defuzzification methods: center of gravity method and extended center of gravity method.

3.2.1 Center of gravity (COG)

Probably the best-known defuzzification operator is the *center of gravity (COG)* defuzzification method. It is a basic general defuzzification method that computes the center of gravity of the area under the membership function. We modeled the basic color concept by introducing the fuzzy set theory. Hence we can easily apply the COG method to obtain the defuzzified singleton value (ρ) of the fuzzy color set. The following equation shows the formulae of COG defuzzification in the fuzzy color set.

$$\rho(F_i)=\frac{\sum_{k=\alpha}^{\gamma}\mu_{F_i}(v_i^k)\cdot v_i^k}{\sum_{k=\alpha}^{\gamma}\mu_{F_i}(v_i^k)}$$

In the above equation, In the above equation, F_i denotes the i-th fuzzy color family set, and *ρ-value* represents the defuzzified singleton. The value of $\mu(v_i)$ of F_i in v_i in *Chroma Set* is called the degree of membership of v_i in *Chroma Set*.

3.2.2 Extended center of gravity (ECOG)

COG method shows a good performance in most cases. But what if a user wants to apply a weight or preference value on the domain axis? For example, according to color theory, human perception of lightness is not uniform. A gray reflecting fifty percent of the light falling on the sample doesn't look halfway between black and white. This is because people are much more sensitive to value differences between dark colors than they are to value differences between light colors. So in computing the defuzzified singleton we must consider this fact, namely, the defuzzification method must be sensitive to dark values. This problem can be roughly solved by applying the weight (sensitivity) on the left side of the value axis. Besides the value dimension, a similar color property is found in the chroma dimension. The lower section of chroma axis should have more sensitive weight.

The original COG can't handle this kind of situation. So we need to extend the COG *(Extended COG, ECOG)*, which can generate a better result and reflect the user's preference and weight. To accomplish this, we proposed two new concepts, domain preference on domain axis (*value* dimension) and membership degree preference on the membership axis (*chroma* dimension). By introducing these concepts, user can give a weight to the specific part which they think more important.

By using the value-weight function ($f_v(v)$) and chroma-weight function ($f_c(\mu(v))$), the formulae of ECOG method is as follows.

$$\rho(F_i)=\frac{\sum_{k=\alpha}^{\gamma}f_v(\mu_{F_i}(v_i^k))\cdot f_c(v_i^k)}{\sum_{k=\alpha}^{\gamma}f_v(\mu_{F_i}(v_i^k))}$$

3.3 Distance metric of the fuzzy color set

It is important task to calculate the distance between colors, and several distance measures have been proposed. In this section we describe the distance measure between the fuzzy colors.

3.3.1 Distance between the fuzzy color families

The fuzzy color families include all possible colors at given hue position. For example, we want to calculate the distance between 'red' family and 'blue' family. In order to compute the distance between two color families, people may want to calculate only the hue difference between two families. As already mentioned, hues have a cyclic property. So the angular distance between two hue values are over π, anti-direction short cut angle is selected as the hue distance. The below definition gives the distance concept between hue values. If the hue position is normalized to scalar value (*0.0 - 1.0*), we get the *MAXHUE* value as *0.5*.

Definition 1. *The distance (H_{dist}) between two hue values H_i and H_j is*

$$H_{dist}(i,j)=\begin{cases}|H_i-H_j| & \text{if } |H_i-H_j|\le MAXHUE\\ MAXHUE-|H_i-H_j| & \text{otherwise}\end{cases}$$

But this simple approach is not appropriate because the shapes of the fuzzy color families are different each other. So we must consider the hue difference as well as the shape of the color family in computing the color family distance. The following definition gives the proposed distance measure between the fuzzy color families.

Definition 2. *Let the two color families be F_i and F_j. Then the distance(χ) between color family F_i and F_j is*

$$\chi(F_i,F_j)=\sqrt{H_{dist}^2(i,j)+\rho_{dist}^2(i,j)}$$
$$\rho_{dist}(i,j)=|\rho(F_i)-\rho(F_j)|$$

As we mentioned earlier, each color family(F) is expressed by a representative value (*ρ-value*), in this case the value is computed by the extended center of gravity defuzzification method. So the distance between two families can be considered as a geometrical distance between two centers of gravities. By taking a center of gravity, we can accomplish the objectives we want to obtain.

3.3.2 Distance between the fuzzy color and fuzzy color family

In addition to the distance mentioned in the previous section, there's a need to calculate the distance between the fuzzy color family and fuzzy color. Let's suppose a situation that an arbitrary fuzzy color is given and we should calculate the distance between the color and color family 'red'. What's the distance? To cover this matter, we suggest the following definition.

Definition 3. *Let the fuzzy color family and fuzzy color be F_i and c_j respectively, and F_j is the color family to which c_j belongs. Then the distance(ξ) between F_i and c_j is*

$$\xi(F_i,c_j)=\chi(F_i,F_j)\bullet\mu_{F_j}(c_j)$$

Because a direct calculation between two objects is not easy, we adopted the distance measure between color families and the membership function. It seems simple and intuitive.

3.3.3 Distance between the fuzzy colors

Finally, we can imagine the distance between two fuzzy colors. Given two arbitrary fuzzy colors from a picture, how to compute the color difference? Similar to the measure in the previous section, the definition 4 can give a solution.

Definition 4. *Let two fuzzy colors be c_i and c_j. Then the distance(ζ) between c_i and c_j is*

$$\zeta(c_i,c_j)=min(\xi(F_i,c_j),\xi(c_i,F_j))$$

The distance measure between two fuzzy colors exploits the definition 3 and minimum operator.

4. Conclusion

Color science and color modeling is an interesting research area, and according to their applications, it is necessary to select an appropriate color model. Conventionally, the RGB color model is the most popular one, but it suffers from the correlation of its color coordinates, and more it can't afford to reflect the degree of human perception about color.

In this paper, we proposed a new color model, which is based on the Munsell color space and fuzzy set theory. We introduced the concept of the fuzzy color family and fuzzy color. The fuzzy color model reflects the whole aspects of the Munsell color space without any loss of information. In order to provide color-specific properties, we developed a new defuzzification method which extends the original center of gravity method. Finally, three distance measures were formulated to compute the distance between the fuzzy color families and the fuzzy colors. In near future, we'll plan to show the usefulness of the proposed fuzzy color model by applying it to digital color image problems.

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