# The Optimal Bispectral Feature Vectors and the Fuzzy Classifier for 2D Shape Classification

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**Abstract**

*In this paper, a method for selection of the optimal feature vectors is proposed for the classification of closed 2D shapes using the bispectrum of a contour sequence. The bispectrum based on third order cumulants is applied to the contour sequences of the images to extract feature vectors for each planar image. These bispectral feature vectors, which are invariant to shape translation, rotation and scale transformation, can be used to represent two-dimensional planar images, but there is no certain criterion on the selection of the feature vectors for optimal classification of closed 2D images. In this paper, a new method for selecting the optimal bispectral feature vectors based on the variances of the feature vectors. The experimental results are presented using eight different shapes of aircraft images, the feature vectors of the bispectrum from five to fifteen and an weighted mean fuzzy classifier.*

***Keywords:***

Bispectrum; Pattern Classification; Optimal Feature Selection; Weighted Fuzzy Mean

## 1. Introduction

The accuracy on pattern classification problems, while keeping simplicity of the overall system, depends on some important factors. One is to extract feature vectors representing a 2D object image. The feature vectors should have the small dimensionality for real-time process, the similarity between intraclass. In this paper, the boundary of a closed planar shape is characterized by an ordered sequence that represents the Euclidean distance between the centroid and all boundary pixels since the overall shape information is contained in the boundary of the shape. Then, the contour sequence is normalized with respect to the size of image. This normalization includes the amplitude and the duration of the contour sequence. Next the bispectrum based on third order cumulants is applied to this normalized contour sequence as a means of feature selection. Higher order spectra (bispectrum, trispectrum) play an important role in digital signal processing due to their ability of preserving non-minimum phase information, as well as information due to deviations from Gaussianity and degrees of non-linearities in time series[1]. In the previous works for classification systems[2][3], the spectrum feature vectors were extracted from power spectrum density of contour sequence. However, the power spectrum of contour sequence is corrupted by white Gaussian noise power $E[n^2]=\delta_n^2$ in the all frequency components where the bispectrum is not. The reason for that will be shown in next section and Han's work presents that the bispectral feature vector has a better noisy tolerant characteristics[4]. Therefore, in this investigation of 2D object classification, the bispectral components of the normalized contour sequence of an object image are utilized as feature vectors. These bispectral feature vectors have enough shape information to represent each 2D object, a property to be invariant in size, shift, and rotation, and are used as the input of fuzzy classifier.

Another factor is to select an appropriate classifier architecture for this particular classification task. In a recent year, the neural network algorithms[2]-[4] and the fuzzy memberships functions[5] are widely used. However, the hybrid neural structure with back propagation and counter propagation in [2] and with two fuzzy ART modules in [3] is relatively complicated. Moreover it is hard to select an optimal matching of specific neural network architecture for this kind of classification system among many different neural models. Thus, a triangular fuzzy membership function and an weighted fuzzy mean method are utilized as a classifier. This fuzzy classifier has a simple structure and it can easily improve the classification results by an weighted fuzzy mean extracted from analyzing the bispectral feature vectors.

Another important factor for accurate classification is to select an appropriate feature vectors based on characteristics of the feature values. In this paper, a method for selection of the optimal feature vectors based on the variances of the feature vectors, is proposed. In the experimental procedure, the proposed feature selection method is tested with eight different shapes of aircraft images. The bispectral feature values of eight different shapes of aircraft images and a weighted fuzzy mean classifier are used for the experiment. The results are presented by varying the selection number of sets of feature values from 15 to 5.

## 2. Shape Information and Bispectral Feature Extraction

In this portion of the study, the boundary of a closed planar shape is characterized by an ordered sequence that represents the Euclidean distance between the centroid and all contour pixels of the digitized shape. Clearly, this ordered sequence carries the essential shape information of a closed planar image. The bispectral feature extraction from a closed planar image is done as follows. First, the boundary pixels are extracted by using contour following algorithm and the centroid is derived [6][7]. The second step is to obtain an ordered sequence in a clockwise direction, b(i), that represents the Euclidean distance between the centroid and all boundary pixels. Since only closed contours are considered, the resulting sequential representation is circular as equation (1).

$$b(i) = \sqrt{(x_i - x_c)^2 + (y_i + y_c)^2} \quad (1)$$

and $\quad b(i + PN) = b(i) \quad i = 1,2,3,\ldots\ldots. PN$

where $(x_c, y_c)$ : the centroid of an image, $(x_i, y_i)$ : the contour pixel, and *PN*(period): the total number of boundary pixels.

This Euclidean distance remains unchanged to a shift in the position of original image. Thus the sequence b(i) is invariant to translation. The next step is to normalize the contour sequence with respect to the size of image. Scaling a shape results in the scaling of the samples and duration of the contour sequence. Thus scale normalization involves both amplitude and duration normalization. The normalized duration of the sequence, 256 points fixed, is obtained by resampling operation and function approximation. This is shown in equation (2).

$$c(k) = b(k * N / 256) \qquad k = 1,2,3,\ldots\ldots\ldots,256 \quad (2)$$

where *c(k)*: the duration normalized sequence.

After duration normalization, amplitude is divided by sum of contour sequence and removed the mean. It is shown in equation (3) and (4).

$$d(k) = c(k)/s \qquad k = 1,2,3,\ldots\ldots\ldots,256 \quad (3)$$

$$d(k) = d(k) - mean(d(k)) \quad (4)$$

where $s = c(1) + c(2) + c(3) + \ldots\ldots + c(256)$.

The sequence *d(k)* is invariant to translation and scaling. In a fourth, bispectral feature measurement is taken into the contour sequence. The spectral density of the sequence *d(k)* is derived by using a third-order moment, called a bispectrum. In general, the higher-order spectra can address noise suppression, and preserve non-minimum phase information as well as the information due to degrees of non-linearities[8]. In this study to 2D shape classification, the bispectrum of contour sequence *d(k)*, instead of power spectrum, is investigated for feature vectors because of its better noisy-tolerant characteristic [4][8]. The nth order cumulants spectrum of contour sequence *d(k)* is defined as

$$\begin{aligned} H_n &= (\omega_1, \cdots, \omega_{n-1}) \\ &= \frac{1}{(2\pi)^{n-1}} \sum_{\tau_1=-\infty}^{+\infty} \cdots \sum_{\tau_{n-1}=-\infty}^{+\infty} C_d(\tau_1, \cdots, \tau_{n-1}) \\ &= F(\omega_1) \cdots F(\omega_{n-1}) F^*(\omega_1 + \cdots + \omega_{n-1}) \end{aligned} \quad (5)$$

where $C_d$, F are cumulants and Fourier transform of the sequence *d(k)*, respectively. For the special cases where n=2(power spectrum) and n=3 (bispectrum) :

$$\begin{aligned} H_2(\omega) &= \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} C_d(\tau) e^{-j\omega\tau} \\ &= F(\omega) F^*(\omega) \end{aligned} \quad (6)$$

where $C_d(\tau) = E[d(k)d(k+\tau)]$: Expectation of $d(k)d(k+\tau)$.

$$\begin{aligned} &H_3(\omega_1, \omega_2) \\ &= \frac{1}{(2\pi)^2} \sum_{\tau_1=-\infty}^{+\infty} \sum_{\tau_2=-\infty}^{+\infty} C_d(\tau_1, \tau_2) e^{-j(\omega_1\tau_1 + \omega_2\tau_2)} \\ &= F(\omega_1) F(\omega_2) F^*(\omega_1 + \omega_2) \end{aligned} \quad (7)$$

where $C_d(\tau_1, \tau_2) = E[d(k)d(k+\tau_1)d(k+\tau_2)]$ and $|\omega_1| \le \pi, |\omega_2| \le \pi, |\omega_1 + \omega_2| \le \pi$.

If the observed contour sequence *d(k)=s(k)+n(k)* where *s(k)*: the zero mean contour sequence without noise, *n(k)*:

the zero mean white Gaussian noise sequence and they are independent, equations (6) and (7) becomes

$$H_2(\omega) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{+\infty} C_s(\tau)e^{-j\omega\tau} + \frac{1}{2\pi}\sum_{\tau=-\infty}^{+\infty} C_n(\tau)e^{-j\omega\tau} \quad (8)$$

$$= H_s(\omega) + H_n(\omega) = H_s(\omega) + \frac{1}{2\pi}\left(\frac{N_0}{2}\right) \quad (9)$$

where $C_n(\tau) = E[n(k)n(k+\tau)] = \frac{N_0}{2}\delta(\tau)$ and $C_s(\tau) = E[s(k)s(k+\tau)]$.

$$H_3(\omega_1,\omega_2) = \frac{1}{(2\pi)^2}\sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty} C_s(\tau_1,\tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)} + \frac{1}{(2\pi)^2}\sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty} C_n(\tau_1,\tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)} \quad (10)$$

$$= H_s(\omega_1,\omega_2) + H_n(\omega_1,\omega_2) = H_s(\omega_1,\omega_2) + \gamma_n = H_s(\omega_1,\omega_2) + E[n^3(k)] \quad (11)$$

where $C_s(\tau_1,\tau_2) = E[s(k)s(k+\tau_1)s(k+\tau_2)]$ and $C_n(\tau_1,\tau_2) = E[n(k)n(k+\tau_1)n(k+\tau_2)] = \gamma_n\delta(\tau_1,\tau_2)$.

In equation (11), the noisy bispectrum $H_n = E[n(k)^3] = \gamma_n$ becomes zero because of skewness of noisy density function, which means the bispectrum suppress the white noisy portion and the extracted feature vectors have better noisy tolerance than the feature vectors from the power spectrum. The performance comparisons with noise for robust 2D shape classification between the power spectral and the bispectral features are shown in [4]. And trispectrum with cumulants order n=4 contains the noisy spectrum because of kurtosis of noisy density and the higher spectra with cumulant order more than n=4 have not widely used yet because of their computational complexity and the difficulty of feature extraction from n-dimensional spectrum space. Therefore the bispectrum is utilized for feature selection of 2D shape images in this paper.

The magnitude of bispectrum derived in a forth step, $|H_3(\omega_1, \omega_2)|$, is unchanged even after the sequence $d(k)$ is circular shifted because the magnitude of Fourier transform, $|F(\omega)|$, is not changed[9]. Thus $|H_3(\omega_1, \omega_2)|$ is invariant to the rotation of an image. Finally, the two dimensional bispectral magnitude(256 by 256) is projected to vertical axis($\omega_1$) by taking mean value of each column for feature extraction. It is shown in equation(12).

$$h(k) = \text{mean of } k\text{th column of } |H_3(\omega_1,\omega_2)| \quad (12)$$

where k=1,2,....256.

The first column and the row in the magnitude of bispectrum contain all zero value because the normalized contour sequence has a zero mean. It means $h(1)$ is always zero. And the projected bispectral components exceed to the sixteenth have very small values (near zero). Thus, for fast classification process with reliable accuracy, the projected bispectral components from the second to the sixteenth ($h(2)$, $h(3)$,.....$h(16)$) are chosen to be used as feature vectors to represent each image shape, which are fed into a proposed fuzzy classifier for classification process. These feature vectors have the desired format for planar image classification system, which means they are invariant to translation, rotation and scaling of the shape and highly tolerant to the noise. Figure 1, 2 and 3 show the fifteen projected bispectral feature vectors of rotated image, two different shapes and 10dB noisy image.

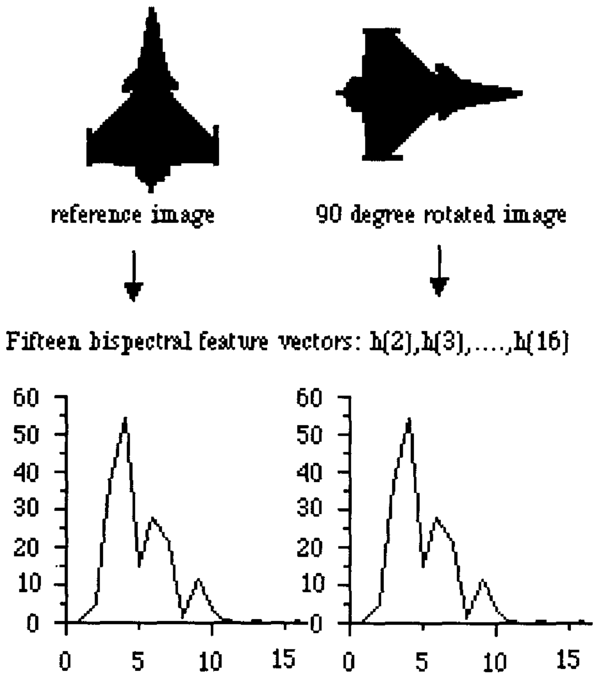


*Figure 1 - Bispectral feature vectors extracted from reference and rotated images*

## 3. The Proposed Feature Selection Method and the Weighted Fuzzy Mean Classifier

The proposed feature selection method is based on characteristics of variances of the eight feature values of each feature vector. In the experimental process, when the variance of specific feature values are small, it is found that the discrimination of image shapes is to be weak. So we propose that small variances are not considered as a weight. In other words, the feature values having small variances are not used in the proposed weighted fuzzy mean classifier.

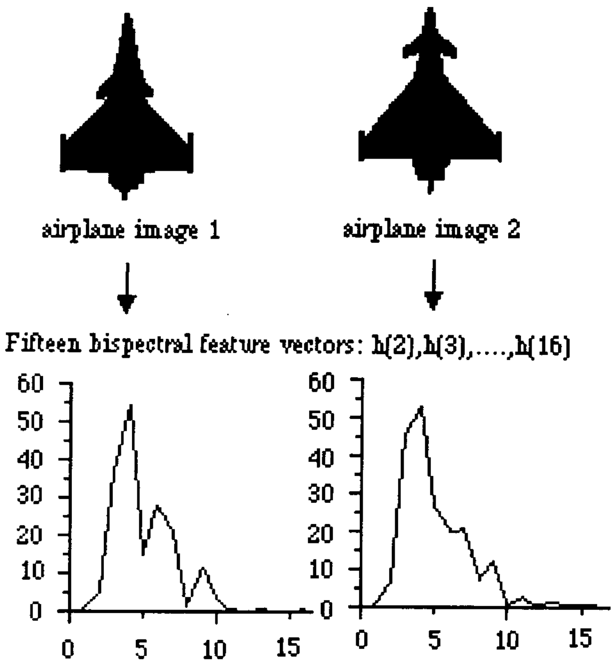


*Figure 2 - Bispectral feature vectors extracted from two different images*

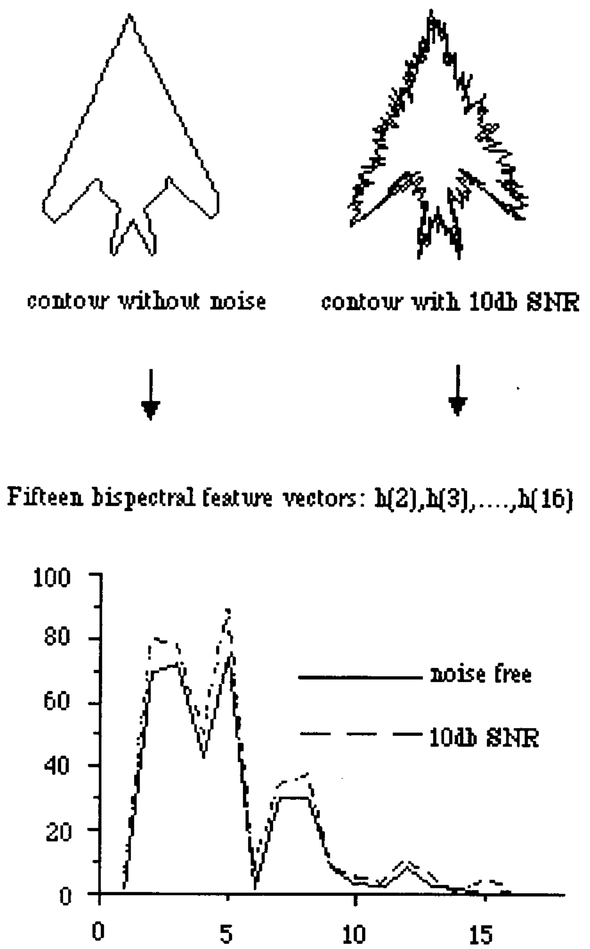


*Figure 3 - Bispectral feature vectors extracted from the contour images without noise and with 10dB SNR*

In this paper, the triangular type of fuzzy membership function and the weighted fuzzy mean method as in equation (15) whose variance is utilized for weights, are used for the experiment.

$$h_w(\mu_1(x), \mu_2(x), \Lambda\ , \mu_n(x); w_1, w_2, \Lambda\ \Lambda\ , w_n)$$

$$= \sum_{i=1}^{n} \mu_i(x) \cdot w_i \quad (\sum_{i=1}^{n} w_i = 1) \tag{13}$$

where $\mu_i$ is an $i$th membership grade, $w_i$ is an $i$th weight and $n$ is the number of fuzzy membership functions.

The triangular type of fuzzy membership functions is useful where the only one reference feature set is available as in this paper. The one advantage of the weighted fuzzy mean classifier is the use of a variance as an weight. In general, it is hard for the neural classifiers to improve the performance results because they are highly depend on the architectures, learning algorithm and training order[3][10]. However, the improvement of classification results for the proposed method is easily achieved with the new information extracted from analyzing the characteristics of the bispectral feature vectors. That is shown in next section. Therefore the triangular fuzzy membership function and the weighted fuzzy mean method using variance are utilized as a proposed classification method.

## 4. Experimental Results and Performance Assessment

The methodology presented in this paper, for the classification of closed planar shape, was evaluated with eight different shapes of aircraft. They are shown in figure 4.

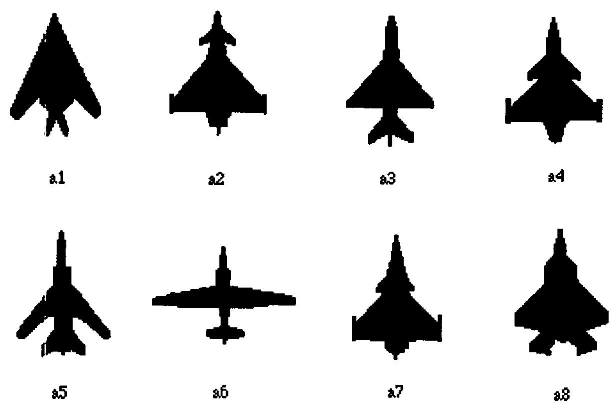


*Figure 4 - Eight different shapes of reference aircraft images*

From each reference shape of aircraft, 36 noisy-free patterns were generated by rotating the original image with 30 degree increment and scaling with three factor (1, 0.8 and 0,6). And forty noisy corrupted patterns were made by adding four different level of random Gaussian noise (25dB, 20dB, 15dB, 10dB SNR : ten noisy patterns for each SNR) to 36 noisy-free patterns.

Thus the data set for each reference aircraft image has 36 noisy-free patterns and 1440 (40×36) noisy corrupted patterns. The number of total test patterns becomes 11808 (1476×8 reference image). The sample contour images for a4 and a7 with noise-free and with 10dB SNR are shown in figure 5.

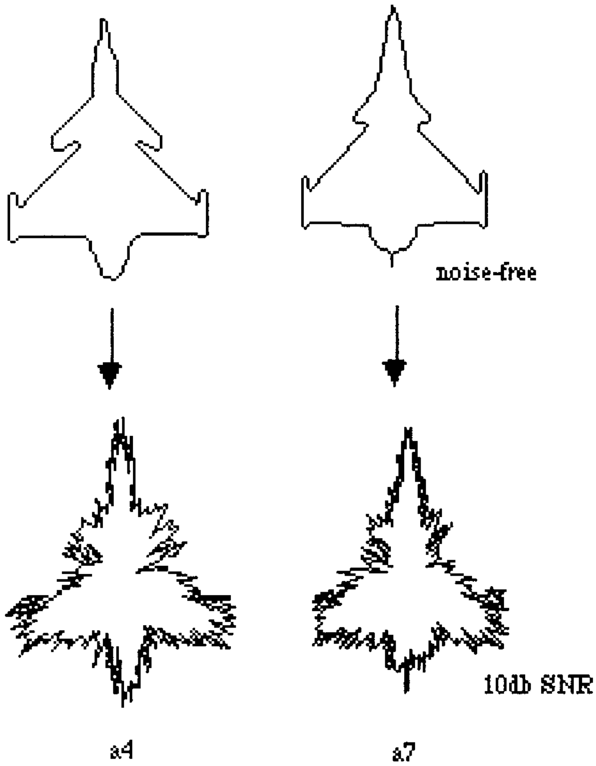


*Figure 5 - The sample contour images for a4 and a7 with noise-free and with 10dB SNR*

The construction of a proposed method and the classification process are done by as follows. First, the fifteen fuzzy membership functions for each reference aircraft image are established by using each of the fifteen dimensional bispectral feature values. The fuzzy membership functions are defined by equation (14).

$$\mu_{ij}(x_i) = 0.01(x_i - f_{ij}) + 1 \quad \text{if } x_i < f_{ij}$$
$$\mu_{ij}(x_i) = -0.01(x_i - f_{ij}) + 1 \quad \text{if } x_i \geq f_{ij} \qquad (14)$$
$$\mu_{ij}(x_i) = \begin{matrix} \mu_{ij}(x_i) & \text{if } x_i \geq f_{ij} \\ 0 & \text{if } x_i < f_{ij} \end{matrix}$$

where 0.01 is the selected slope of a fuzzy membership function, $x_i$ is an *i*th feature value of input aircraft image, $f_{ij}$ is an *i*th feature value of reference feature set for an image $a_j$, and $\mu_{ij}(x_i)$ is a membership grade for $x_i$.

The number of total fuzzy membership functions becomes 120 for the eight different type of aircraft images. In a second step, the variances for each of the normalized fifteen dimensional feature values with eight reference aircraft images are derived by equation (15) and (16). The normalized feature values from reference aircraft a1 to a8 are shown in figure 6 and the normalized variances of the feature values are shown in table 1.

$$m_j = \frac{1}{8}\sum_{i=1}^{8} x_{ij} \quad (j = 1,...,15) \qquad (15)$$

where $m_j$ is a mean of jth feature values for the eight different aircraft images and $x_{ij}$ is a *j*th feature value for aircraft image a*i*.

$$vr_j = \frac{1}{8}\sum_{i=1}^{8} (x_{ij} - m_j)^2 \quad (j = 1,...,15) \qquad (16)$$

where $vr_j$ is a variance of jth feature values for the eight different aircraft images(a1,a2,...,a8).

*Table 1 - Variances of the feature values*

| order of feature values | normalized variances of feature values | ranking of variances |
|---|---|---|
| 1 | 0.009189 | 6 |
| 2 | 0.007923 | 9 |
| 3 | 0.002842 | 15 |
| 4 | 0.011366 | 5 |
| 5 | 0.008234 | 8 |
| 6 | 0.005576 | 12 |
| 7 | 0.015466 | 4 |
| 8 | 0.003174 | 14 |
| 9 | 0.025460 | 1 |
| 10 | 0.018002 | 2 |
| 11 | 0.009161 | 7 |
| 12 | 0.007518 | 10 |
| 13 | 0.006190 | 11 |
| 14 | 0.016345 | 3 |
| 15 | 0.004392 | 13 |

In a third, the fifteen bispectral feature values of the incoming test aircraft image are applied to the corresponding fuzzy membership function for each of eight reference shapes and the membership grades are computed by equation (14). The fifteen membership grades for any one of eight reference shapes present the degree of similarity with that reference shape.

In a Fourth, the weighted mean values of the membership grades for each of eight reference aircraft shape are computed by equation (17) and (18), and a reference shape

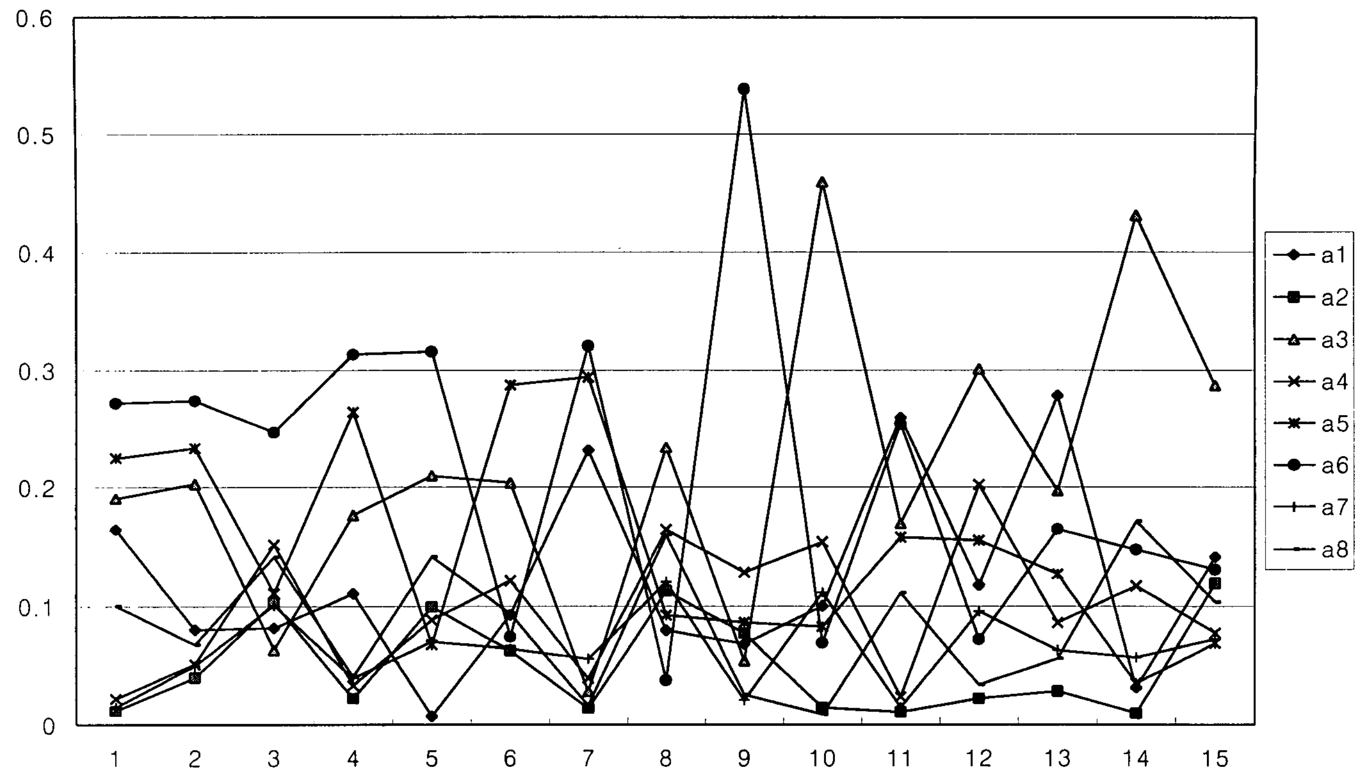


*Figure 6 - Normalized amplitudes of bispectral features*

of aircraft image having the largest weighted mean value (the largest $h_i$)is chosen as a classification result.

$$w_j = vr_j \quad (j = 1,...,15) \tag{17}$$

where $w_j$ is an weight for the $j$th feature value.

$$h_i(\mu_{i1}(x), \mu_{i2}(x), \Lambda\ , \mu_{i15}(x); w_1, w_2, \Lambda\ \Lambda\ , w_{15})$$

$$= \sum_{j=1}^{15} \mu_{ij}(x_j) \cdot w_j \quad (i = 1,...,8) \tag{18}$$

where $\mu_{ij}$ is an membership grade for the $j$th feature value of aircraft image a$i$ computed by equation (14), and $h_i$ is an weighted fuzzy mean value for each of eight reference aircraft images.

Finally, the least meaningful variance is set to zero, and the fourth step is repeated. Then the second least meaningful variance is also set to zero, and the fourth step is repeated. This step is repeated until the tenth least meaningful variance is set to zero. The experimental process was performed under three different experimental environments. These are as follows.

Experiment 1.

*Classifier algorithm*: the weighted fuzzy mean using variances.

*Reference data set for membership function*: only the 8 reference aircraft images.

Experiment 2.

*Classifier algorithm*: same as 1.

*Reference data set for membership function*: average of 8 reference patterns + 32 noisy patterns (4 noisy patterns with 25dB SNR generated from each of 8 reference images).

Experiment 3.

*Classifier algorithm*: same as 1.

*Reference data set for membership function*: average of 8 reference patterns + 32 noisy patterns (4 noisy patterns with each of 25dB, 20dB, 15dB and 10dB SNR generated from each of 8 reference images).

Under each of three different experimental environments, 11808 of total test patterns (1476 patterns for each reference image) were evaluated. The overall classification results of experiments 1-3 are summarized in table 2, 3 and 4. In table 2, 3 and 4, the best results of a proposed weighted fuzzy mean classifier are presented. In experiment of 2 for a weighted fuzzy mean classifier, the membership functions for each of eight reference shapes are constructed with a noise-free pattern and four of randomly selected noisy patterns. It means the classification results slightly depend on the selection of noisy patterns. Therefore the ten independent experiments with different styles of noisy patterns keeping the same SNR were evaluated and the best results are presented.

*Table 2 - The number of misclassified shapes of the experiment 1.*

| data sets | the number of used feature values | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 |
| Noise-free | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25dB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20dB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15dB | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 10dB | 74 | 66 | 58 | 59 | 133 | 131 | 140 | 98 | 243 | 233 | 276 |

*Table 3 - The number of misclassified shapes of the experiment 2.*

| data sets | the number of used feature values | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 |
| Noise-free | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25dB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20dB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15dB | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 10dB | 58 | 54 | 50 | 50 | 118 | 117 | 96 | 93 | 208 | 204 | 237 |

*Table 4 - The number of misclassified shapes of the experiment 3.*

| data sets | the number of used feature values | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 |
| Noise-free | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25dB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20dB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15dB | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10dB | 16 | 13 | 12 | 12 | 17 | 22 | 47 | 43 | 43 | 44 | 89 |

The classification results with the weighted fuzzy mean classifier can be increased by adding some noisy patterns to training process and to construction of membership function, respectively. Moreover the classification results can be more increased by selecting some appropriate feature values. These are shown in the results of the experiments. And table 2, 3 and 4 show that when only 13 set of feature values are used for classification, the best results are obtained. It means that the appropriate number of feature values, not all the feature values, are used for classification, the classification performances can be increased.

## 5. Conclusion

The classification results higher than former researches' are obtained when the most significant 13 sets of feature values selected from overall 15 sets of features values are used. The method for extracting feature vectors and for classifying image shapes are same as former research[11]. In this paper, the experimental results show that appropriate selection of feature values based on feature characteristics can increase the overall performance. In a near future, more realistic data such as the satellite images or the biomedical images will be tested and investigated for the practical applications.

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