Stabilization of Input-Delayed TS Fuzzy Systems

Ho Jae, Lee, Jin Bae Park, Dae Beum Cha*, and Young Hoon Joo*

Dept. of Electrical and Electronic Eng., Yonsei Univ. Seoul, Korea
*School of Electronic and Information Eng., Kunsan National Univ. Kunsan, Chonbuk, Korea

Abstract

In this paper, a control problem of the Takagi-Sugeno (TS) fuzzy system with time-varying input delay is considered. It is well known that the delay is one of the major sources responsible for the instability of the controlled system. A systematic design technique is developed based on the Lyapunov-Razumikhin stability theorem. A sufficient condition for the global asymptotic stability of the TS fuzzy systems is formulated in terms of linear matrix inequalities (LMIs). The derived condition can deal with any time-varying input delay within the admissible bound. The effectiveness of the proposed controller design technique is demonstrated by a numerical simulation.

1. Introduction

As the communication systems has been more reliable, some attempts have been tried to remotely control via communication networks such as the Internet. Examples include the virtual laboratory (VL) [2, 5]. Since the control loops of the remote-control system are closed over communication networks or field buses, time delay phenomena inevitably occur. The stability and performance of the controlled system are definitely dependent on the transmission performance of the communication networks. It is well known that the existence of time delay makes the closed-loop stabilization more difficult. Therefore, it is clear that, as the remote-control system is generally utilized, it will be more and more important to take the delays into account in the analysis and the design of the control systems.

Recent attentions in Takagi-Sugeno (TS) fuzzy-model-based control are focused on the time-delay, yet few research results about time delay are available [3, 4]. Moreover, all previous works did not consider the input delay and are based on the Lyapunov-Krasovskii's stability theorem [1]. Thus it need to take into account some supplementary requirements on the time-derivative of time delay $d(t)$-$\dot{d}(t)$ should be smaller than 1.

This paper aims at studying the control problem for a class of TS fuzzy systems in the presence of time-varying input delay. The input delays specially often occur in the remote-control system and critically influence on the stability and performance of the closed-loop system. This issue must also be carefully handled in TS fuzzy systems for safety and improved operational performance of the nonlinear remote-control systems such as VL.

The organization of this paper is as follows: Section 2 reviews the input delayed TS fuzzy systems. The main results of this paper are discussed and explained in Section 3. In Section 4, we include a simple example to verify and visualize the theory and method proposed in this paper. To the end, Section 5 concludes this paper with some remarks.

2. Input-Delayed TS Fuzzy Systems

Consider the TS fuzzy system described by the following fuzzy rules:

Plant Rules

$$\begin{aligned} R^i: \ & \text{If } x_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } \Gamma_n^i, \\ & \text{Then } \dot{x}(t) = A_i x(t) + B_i u(t-d(t)), \end{aligned} \quad (1)$$

where $\Gamma_j^i (i = 1, \ldots, q, j = 1, \ldots, n.)$ is the fuzzy set, $x(t) \in \mathbb{R}^n$ is the state, $u(t-d(t)) \in \mathbb{R}^m$ is the delayed control input, and $d(t)$ represents the time-lag satisfying $0 \le d(t) \le \tau$.

Using the center-average defuzzification, product inference, and singletone fuzzifier, the global dynamics of this TS fuzzy system (1) is described by

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i u(t-d(t))), \quad (2)$$

in which

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma_j^i(x_j(t)), \quad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q} \omega_i(x(t))},$$

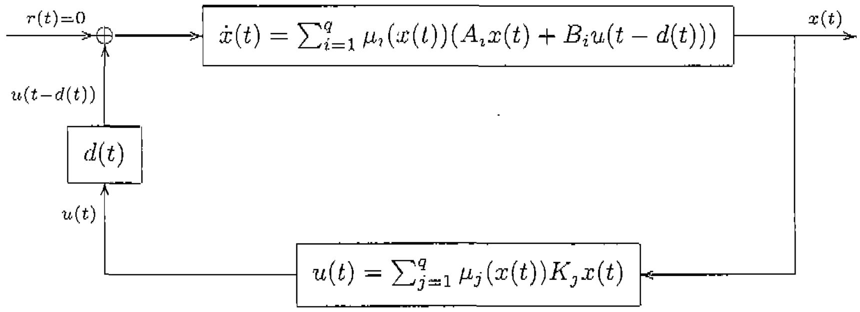

Figure 1: Block diagram of the fuzzy-model-based control system in the presence of time delay and uncertainties.

and $\Gamma_j^i(x_j(t))$ is the membership value of $x_j(t)$ in Γ_j^i. Some basic properties of $\mu_i(t)$ are:

$$\mu_i(x(t)) \geq 0\,, \quad \sum_{i=1}^{q} \mu_i(x(t)) = 1\,. \tag{3}$$

Throughout this paper, a state feedback TS fuzzy-model-based control law is utilized for the stabilization of the TS fuzzy system (2).

Controller Rules

$$R^i: \text{ If } x_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } \Gamma_n^i\,,$$
$$\text{Then } u(t) = K_i x(t)\,. \tag{4}$$

The defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^{q} \mu_i(x(t)) K_j x(t)\,. \tag{5}$$

The closed-loop system with (2) and (5) of retarded type is represented as

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(x(t))\mu_j(x(t-d(t)))(A_i x(t) \\ &\qquad + B_i K_j x(t-d(t)))\,, \\ x(t) &= \phi(t)\,, \quad t \in [-\tau,\ \ 0]\,,\end{aligned} \tag{6}$$

where $\phi(t)$ is a smooth vector-valued function defined in the Banach space $\mathbb{C}[-\tau,\ \ 0]$. The block diagram of the considered system is shown in Fig. 1.

3. Main Results

This section presents some sufficient conditions that guarantee the global asymptotic stability of the controlled TS fuzzy system (6).

Theorem 1 *If there exist a symmetric positive definite matrix P, and matrices K_j, and positive scalars α_1, α_2 such that the following LMIs are satisfied:*

$$\begin{bmatrix} \frac{1}{\tau}\Upsilon_{ij} + (\alpha_1+\alpha_2)Q & * \\ M_j^T B_i^T & -\frac{1}{2}Q \end{bmatrix} < 0\,, \quad i = 1,2,\ldots,q\,. \tag{7}$$

$$\begin{bmatrix} -\alpha_1 Q & * \\ A_i^T Q & -Q \end{bmatrix} < 0\,, \quad i = 1,2,\ldots,q\,. \tag{8}$$

$$\begin{bmatrix} -\alpha_2 Q & * \\ M_j^T B_i^T & -Q \end{bmatrix} < 0\,, \quad i,j = 1,2,\ldots,q\,. \tag{9}$$

then the TS fuzzy system (2) *is robustly globally asymptotically stabilizable by employing controller* (5) *with the time delay $d(t)$ satisfying*

$$0 \leq d(t) \leq \tau\,,$$

where

$$\Upsilon_{ij} = QA_i^T + A_i Q + M_j^T B_i^T + B_i M_j$$

with $Q = P^{-1}, M_j = K_j P^{-1}$, and $*$ denotes the transposed elements in the symmetric positions.

Proof: The proof is omitted due to lack of space. ■

In order to find the maximum delay τ, the following convex optimization algorithm is proposed.

Step 1: Find a positive definite matrix Q and matrices M_j such that the following LMIs are satisfied:

$$QA_i^T + A_i Q + M_j^T B_i^T + B_i M_j < 0\,,$$
$$i,j = 1,2,\ldots,q\,.$$

Step 2: For Q given in the previous step, find α_1, α_2 and M_j such that the following generalized eigenvalue problem (GEVP) $\mathcal{P}(\tau)$ has solutions,

$$\mathcal{P}(\tau) \quad \max_{M_j,\alpha_1,\alpha_2} \ \tau \ \text{ subject to}$$
$$\begin{bmatrix} \frac{1}{\tau}\Upsilon_{ij} + (\alpha_1+\alpha_2)Q & * \\ M_j^T B_i^T & -\frac{1}{2}Q \end{bmatrix} < 0,$$
$$\begin{bmatrix} -\alpha_1 Q & * \\ A_i^T Q & -Q \end{bmatrix} < 0,$$
$$\begin{bmatrix} -\alpha_2 Q & * \\ M_j^T B_i^T & -Q \end{bmatrix} < 0, \quad i,j = 1,2,\ldots,q.$$

Step 3: For M_j and α_1, α_2 given in the previous step, find Q such that $\mathcal{P}(\tau)$ has solutions.

Step 4: Return to Step 2 until the convergence of τ is obtained with a desired precision.

4. An Example

In this section, a numerical example is presented for illustrating the controller design technique proposed in Section 3.. Consider the following TS fuzzy systems.

Plant Rules

R^1: If $x_1(t)$ is about Γ_1,
THEN $\dot{x}(t) = A_1 x(t) + B_1 u(t - d(t))$,

R^2: If $x_1(t)$ is about Γ_2,
THEN $\dot{x}(t) = A_2 x(t) + B_2 u(t - d(t))$,

where

$$A_1 = \begin{bmatrix} -0.5 & 0.1 \\ 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0.1 \\ 1 & 0 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The membership functions are

$$\Gamma_1(x_1(t)) = \begin{cases} 0, & x_1(t) < -\Omega, \\ 1 - \frac{x_1^2(t)}{\Omega^2}, & -\Omega \le x_1(t) \le \Omega, \\ 0, & \Omega < x_1(t), \end{cases}$$
$$\Gamma_2(x_1(t)) = 1 - \Gamma_1(x_1(t)),$$

where $\Omega = 0.8165$. From Theorem 1 and the proposed iterative optimization method, we get

$$P = \begin{bmatrix} 0.0216 & 0.0095 \\ 0.0095 & 0.0072 \end{bmatrix}, \quad \alpha_1 = 0.4496, \quad \alpha_2 = 0.7997,$$
$$K_1 = \begin{bmatrix} -0.6564 & -0.4485 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.7519 & -0.8947 \end{bmatrix},$$

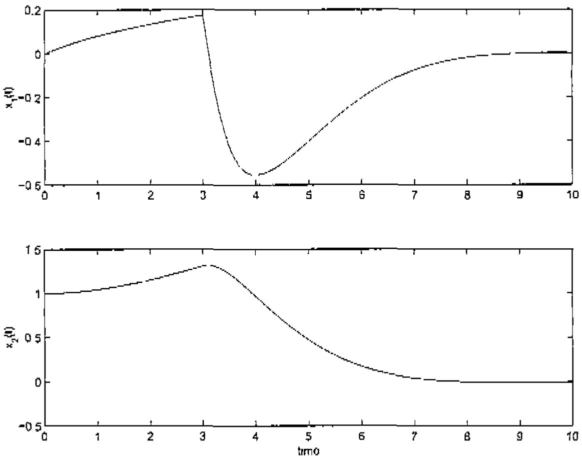

Figure 2: The controlled response of the time-varying input-delayed TS fuzzy system.

and $\tau = 0.3714$, which means that the designed TS fuzzy-model-based controller can robustly stabilize the TS fuzzy system against any time-varying input delay $d(t) \le \tau = 0.3714$.

The initial value is $x(0) = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. During the simulation process, the time-varying delay in the control law is assumed as $d(t) = \frac{\tau}{2}(\sin(100t) + 1)$, where the assumed time delay does not exceed the upper bound, $\tau = 0.3714$ and its maximal time-derivative is $50\tau \le 18.5690$. To the authors' knowledge, the stability analysis based on the Lyapunov-Krasovskii functional approach demands the restriction of the time-derivative of $d(t)$, i.e. $\dot{d}(t) < 1$. On the other hand, the proposed method do not need it. It also implies that the Lyapunov-Razumikhin stability theory is much suitable for the remote-control system based on network channels such as the Internet, because the time delay due to the traffic jam over the Internet may randomly varied.

The simulation result is shown in Fig. 2. For the purpose of comparison, the control input is activated at $t = 3$ sec.. Before the control input is activated, the trajectories of the system do not go to the equilibrium of the system. However, after $t = 3$ sec., the trajectories of the controlled system are quickly guided to the origin. From the simulation result, one can see that the designed controller can stabilize the TS fuzzy system with the input delay.

5. Conclusions

In this paper, the problem of stabilization of the TS fuzzy system with time-varying input delay has been addressed. In order to design the fuzzy-model-based controller, the Lyapunov-Razumikhin stability theorem has been applied. The sufficient condition for the stabilization of the closed-loop system has been given in terms of the linear matrix inequalities (LMIs). The maximal bound of the input delay preserving the asymptotic stability has been found by using the iterative convex optimization technique. From the numerical example has shown us the potential of the proposed method for the industrial applications with the delay phenomena.

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