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유전 알고리듬을 이용한 디지탈 퍼지 모델 기반 제어기의 설계

Design of digital fuzzy-model-based controllers by using genetic algorithms

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Abstracts This paper presents a new global state-matching intelligent digital redesign method for nonlinear systems by using genetic algorithms (GAs) . The proposed method results in global matching of the states of the analogously controlled system with those of the digitally controlled system while the conventional intelligent digital redesign method does not. The proposed method provides a new approach for the digital redesign of a class of fuzzy-model-based controllers.

1. Introduction

Digital redesign is to design an analog controller first and then convert the obtained analog controller to the equivalent digital controller maintaining the properties of the analogously controlled system, by which the benefits of both continuous-time controllers and the advanced digital technology can be achieved.

Many analytic digital redesign methods have been studied for linear systems [1–3]. However, there are relatively less research works on the digital redesign of nonlinear systems [4, 5] since it is very difficult, if not impossible, to derive an analytical solution of the digital redesign of complex nonlinear systems. Although the digital redesign technique is successfully applied to some nonlinear systems, it is still needed to develop the digital redesign method for the more general nonlinear systems. Joo *et al.* [5] developed an intelligent digital redesign technique for the digital control of the chaotic Chua's circuit with the TS fuzzy model and a fuzzy-model-based controller where each digital control rule is determined by applying the conventional digital redesign technique to each local closed-loop system. This approach is very appealing to the digital redesign of nonlinear systems since many nonlinear systems can be represented by the TS fuzzy models [5, 6]. Although this allows the designer to take advantage of the classical digital redesign techniques for nonlinear systems, it may lead to undesirable and/or inaccurate results since the digital redesign is considered for each sub-closed-loop system and does not guarantee the global equivalence between the analogously controlled system and the digitally controlled counterpart. However, the analytic global digital redesign of fuzzy-model-based controllers is difficult due to their nonlinear behaviours.

In the present paper, we propose a GA-based intelligent digital redesign method for the digital control of complex continuous-time nonlinear systems with the TS fuzzy models and fuzzy-model-based controllers.

2. Two representative TS fuzzy model structures

We use the following TS fuzzy model to represent a complex, multi-input multi-output, nonlinear system:

$$\begin{aligned}&\text{Rule } i\text{: IF } z_1(t) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } F_n^i,\\ &\text{THEN } \dot{\mathbf{x}}_c(t) = \mathbf{A}_i\mathbf{x}_c(t) + \mathbf{B}_i\mathbf{u}_c(t), \quad (i = 1, 2, \cdots q)\end{aligned} \tag{1}$$

where F_j^i $(j = 1, 2, \cdots, n)$ is the fuzzy set of the jth premise variable, q is the number of fuzzy rules, $\mathbf{A}_i \in \mathbb{R}^{n\times n}$, $\mathbf{B}_i \in \mathbb{R}^{n\times m}$, and $z_1(t), \cdots, z_n(t)$ are the premise variables. Using the center-average defuzzification, product inference, and singletone fuzzifier, the global dynamics of the system (1) can be described by

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}(\mu(z(t)))\mathbf{x}_c(t) + \mathbf{B}(\mu(z(t)))\mathbf{u}_c(t) \tag{2}$$

where $F_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in F_j^i. We use the fuzzy-model-based controller as shown below.

$$\begin{aligned}&\text{Rule } i\text{: IF } z_1(t) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } F_n^i,\\ &\text{THEN } \mathbf{u}_c(t) = -\mathbf{K}_c^i\mathbf{x}_c(t), \quad (i = 1, 2, \cdots, q)\end{aligned} \tag{3}$$

where $\mathbf{K}_c^i$ is the feedback gain in the ith fuzzy subspace, which is to be pre-designed. The defuzzified output of the fuzzy-model-based controller is given by

$$\mathbf{u}_c(t) = -\mathbf{K}_c(\mu(z(t)))\mathbf{x}_c(t) \tag{4}$$

where

$$\mathbf{K}_c(\mu(z(t))) = \sum_{i=1}^{q} \mu_i(z(t))\mathbf{K}_c^i$$

The rule structure of the corresponding discrete-time TS fuzzy model is

$$\begin{aligned}&\text{Rule } i\text{: IF } z_1(kT) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(kT) \text{ is } F_n^i,\\ &\text{THEN } \mathbf{x}_d(kT+T) = \mathbf{G}_i\mathbf{x}_d(kT) + \mathbf{H}_i\mathbf{u}_d(kT)\\ &(i = 1, 2, \cdots q)\end{aligned} \tag{5}$$

where $\mathbf{G}_i = \exp(\mathbf{A}_iT) \in \mathbb{R}^{n\times n}$, $\mathbf{H}_i = \int_0^T \exp(\mathbf{A}_i\tau)\mathbf{B}_i\tau d\tau \in \mathbb{R}^{n\times m}$, and $z_1(t), \cdots, z_n(t)$ are the premise variables, and T is the sampling period.

The global dynamics of system (5) can be described by

$$\mathbf{x}_d(kT+T) = \mathbf{G}(\mu(z(kT)))\mathbf{x}_d(kT) + \mathbf{H}(\mu(z(kT)))\mathbf{u}_d(kT) \tag{6}$$

The corresponding discrete-time fuzzy-model-based controller has the following structure:

$$\begin{aligned}&\text{IF } z_1(kT) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(kT) \text{ is } F_n^i,\\ &\text{THEN } \mathbf{u}_d(t) = \mathbf{u}_d(kT) = -\mathbf{K}_d^i\mathbf{x}_d(kT), \quad (i = 1, 2, \cdots, q)\\ &\text{for } kT \le t < kT + T\end{aligned} \tag{7}$$

where $\mathbf{K}_d^i$ is the feedback gain to be digitally redesigned in the ith fuzzy subspace. The defuzzified output of the fuzzy-model-based controller is given by

$$\mathbf{u}_d(kT) = -\mathbf{K}_d(\mu(z(kT)))\mathbf{x}_d(kT) \tag{8}$$

where

$$\mathbf{K}_d(\mu(z(kT))) = \sum_{i=1}^{q} \mu_i(z(kT))\mathbf{K}_d^i$$

3. Design of fuzzy-model-based controller by using genetic algorithms and digital redesign technique

GAs are iterative adaptive general purpose search strategies based on the principles of natural population genetics and natural selection, which are not fundamentally limited by the restrictive assumptions about the given problem such as continuity, existence of derivatives, etc.. In this paper, we employ GAs to achieve the intelligent digital redesign for complex nonlinear systems represented by the TS fuzzy models.

3.1 Problem description

The intelligent digital redesign problem can be stated as follows: Given the continuous-time TS fuzzy system, the continuous-time fuzzy-model-based controller (3) and the sampling time T, design a discrete-time fuzzy-model-based controller (7) to minimize the cost function defined by

$$\begin{aligned}J &= \sum_{i=1}^{n}\int_0^{t_f} |x_{ci}(t) - x_{di}(t)|dt\\ &\cong \sum_{i=1}^{n}\sum_{j=1}^{N} |x_{ci}(jT_f) - x_{di}(jT_f)|T_f\end{aligned} \tag{9}$$

where t_f is the final time of interest, $x_{ci}(t)$ and $x_{di}(t)$ are the i th state variables of the state vectors $\mathbf{x}_c(t)$ and $\mathbf{x}_d(t)$, respectively, $T_f = t_f/N$ with a sufficiently large integer N, $x_{ci}(jT_f)$ and $x_{di}(jT_f)$ are the i th state variables of the state vectors $\mathbf{x}_c(t)$ and $\mathbf{x}_d(t)$ evaluated at $t = jT_f$, respectively. The inter-sampling behaviour of the analogously controlled system can also be considered by the objective function (9). We also note that the above objective function considers the global state-matching between the analog and the digital system. The given intelligent digital redesign problem is stated as following optimization problem:

$$\text{Find } \mathbf{K}_d^i \text{ to minimize } J \tag{10}$$

where $\mathbf{K}_d^i$ is the digital gain found by the proposed GA-based intelligent digital redesign method.

3.2 Structure of the chromosome

GAs represent the parameters for the given problem by the chromosome S which may contain one or more substring(s). Each chromosome, therefore, contains a possible solution to the problem.

In the proposed GA-based intelligent digital redesign, the variables of interest are $\mathbf{K}_d^i$ and each chromosome must contain the information about the gains of the discrete-time fuzzy-model-based controller. The convenient way to represent the information into the chromosome is concatenating the jth rows of ith gain, which is shown below:

$$\begin{aligned}&S_i = \{k_{d11}^i, \cdots, k_{d1m}^i, \cdots, k_{dn1}^i, \cdots, k_{dnm}^i\}\\ &S = S_1 \cdots S_q\end{aligned} \tag{11}$$

where S_i is the binary coded substrings encoding the ith digital gain of the fuzzy-model-based controller and k_{djk}^i is the j, k entry of $\mathbf{K}_d^i$.

3.3 Initial population

The initial population is randomly generated within the given search space. However, the above developed interval method might give conservative results due to the use of interval arithmetic operations. Therefore, the performance of GAs may be degraded due to the insufficient reduction of the search space. To avoid this problem, we use the following scaled interval matrix in generating the initial population.

$$\mathbf{K}_{dr}^I = \alpha\mathbf{K}_{db}^I \tag{12}$$

where α is the scaling factor.

In addition, we also utilize another domain knowledge available, that is, the digital gains by the local intelligent digital redesign method [5] are first constructed and inserted into the population for the fast evolution of GAs. For the readers' convenience, we rewrite the result of the local intelligent digital redesign in the following equation.

$$\mathbf{K}_{dl}^i = \frac{1}{2}(\mathbf{I}_m + \frac{1}{2}\mathbf{K}_c^i\mathbf{H}_c^i)\mathbf{K}_c^i(\mathbf{I}_n + \mathbf{G}_i) \tag{13}$$

where $\mathbf{K}_{dl}^i$ is the digital gain by the local intelligent digital redesign method.

In summary, the initial population is randomly generated within the interval given in (12), and the local optimal solution is also inserted. Note that the reduced search space (12) is used only for the initialization of the population.

3.4 Evaluation of individual fitness

Since the GAs guide the optimal solution to the direction of maximizing the fitness function value, it is necessary to map the objective function (9) to the fitness function form by

$$f'(J_c) = \frac{1}{J_c + \lambda} \tag{14}$$

where λ is the coefficient to prevent the large raw fitness function value.

The raw fitness function in (14), however, may cause the early convergence of the solution due to the individual with the very high fitness value generated by the local intelligent digital redesign method. To avoid this problem, the individuals are sorted according to their raw fitness values, and the new fitness values are determined by their ranks as follows:

$$f_i = 1 - (i-1)\frac{1}{P_s - 1} \tag{15}$$

where i is the ranking of the individual and $P_s \geq 2$ is the population size.

3.5 Algorithm description

The GA-based intelligent digital redesign algorithm can be summarized as follows:

Step 1 Set the values of the parameters for GA: maximum number of generation (G_n), population size (P_s), crossover rate (P_c), and mutation rate (P_m).

Step 2 Construct initial population $P(0)$ with randomly generated individuals. In this paper, we use binary coded chromosomes to represent the solution parameters and each chromosome is generated to put the parameters of a discrete-time fuzzy-model-based controller within the interval given by (12).

Step 3 Decode the chromosome of each individual in the population and determine the discrete-time fuzzy-model-based controller. Evaluate the determined discrete-time fuzzy-model-based controller by (9) and give a fitness value to each individual in the population by (15).

Step 4 Evolve the new population $P(i+1)$ by reproduction, crossover and mutation.

Step 5 Increase the generation number by replacing old generation with new generation $P(i) = P(i+1)$. During the replacement, preserve an individual which has the maximum fitness value by the elitist reproduction.

Step 6 Repeat Step 3 Step 5 until one of the following conditions is satisfied.

(a) the satisfactory population appears.

(b) the generation number is reached to G_n.

(c) the fitness function is not increased for the prescribed number of generations.

4. An example

Consider the following TS fuzzy system [7]

$$\text{Rule } i\text{: IF } x_1 \text{ is } F_1^i, \text{ THEN } \dot{\mathbf{x}}_c(t) = \mathbf{A}_i\mathbf{x}_c(t) + \mathbf{B}_i\mathbf{u}_c(t), \quad (i = 1, 2) \tag{16}$$

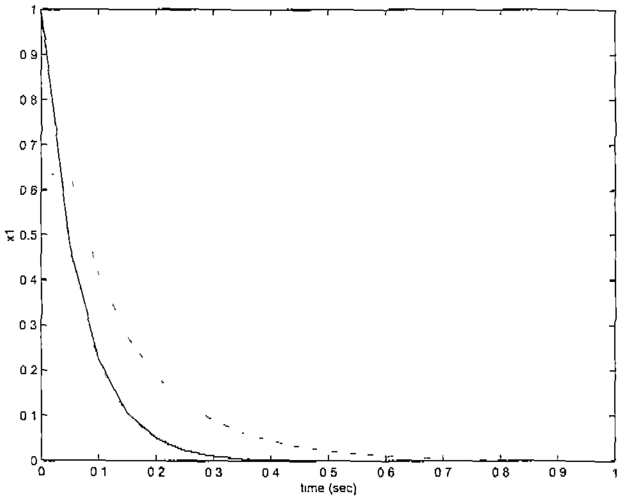

Figure 1: Responses of the state x_1 of the controlled Chens' chaotic attractor by the proposed method (solid line), the local intelligent digital redesign (dash-dotted line), and the analog controller (dashed line), with initial condition $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 1$

where $\mathbf{x}_c = [\ x_1 \quad x_2 \quad x_3\ ]^T$,

$$\mathbf{A}_1 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -x_{1\,\min} \\ 0 & x_{1\,\min} & -b \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -x_{1\,\max} \\ 0 & x_{1\,\max} & -b \end{bmatrix},$$

$$\mathbf{B}_i = \mathbf{I}_3, \quad (i = 1, 2)$$

and where $a = 35$, $b = 3$, and $c = 28$.

The rule structure of a continuous-time fuzzy-model-based controller is

$$\text{Rule } i\text{: IF } x_1 \text{ is } F_1^i, \text{ THEN } \mathbf{u}_c(t) = -\mathbf{K}_c^i\mathbf{x}_c(t), \quad (i = 1, 2) \tag{17}$$

Choosing the closed loop eigenvalues [-15,-15,-15], we get

$$\mathbf{K}_c^1 = \begin{bmatrix} -20 & 35 & 0 \\ -7 & 43 & 30 \\ 0 & -30 & 12 \end{bmatrix}, \quad \mathbf{K}_c^2 = \begin{bmatrix} -20 & 35 & 0 \\ -7 & 43 & -30 \\ 0 & 30 & 12 \end{bmatrix}$$

Figures 1 – 3 show the simulation results with the initial condition $(x_1(0), x_2(0), x_3(0)) = (1, 1, 1)$.

5. Conclusion

A new approach to controlling nonlinear systems via the intelligent digital redesign of a fuzzy-model-based controller has been developed with the aid of GAs. The proposed method combines GAs with the conventional digital redesign algorithms to enhance the previously developed local intelligent digital redesign method. The intelligent digital

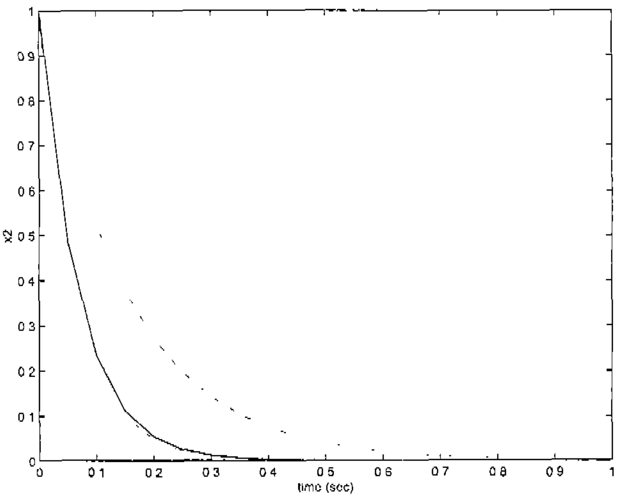

Figure 2: Responses of the state x_2 of the controlled Chens' chaotic attractor by the proposed method (solid line), the local intelligent digital redesign (dash-dotted line), and the analog controller (dashed line), with initial condition $x_1(0) = 1$, $x_2(0) = 1$. $x_3(0) = 1$

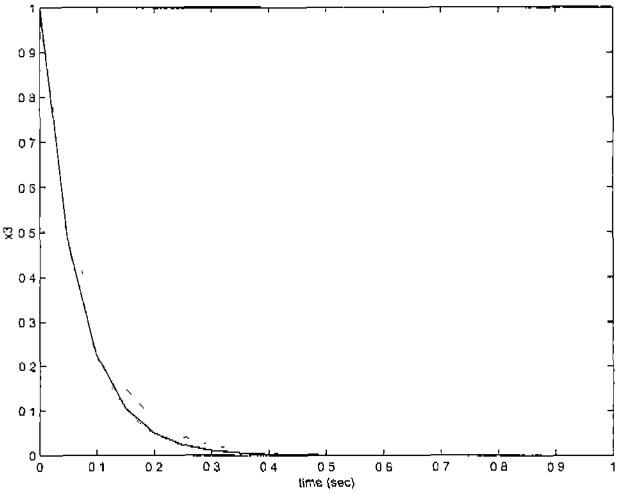

Figure 3: Responses of the state x_3 of the controlled Chens' chaotic attractor by the proposed method (solid line), the local intelligent digital redesign (dash-dotted line), and the analog controller (dashed line), with initial condition $x_1(0) = 1$, $x_2(0) = 1$. $x_3(0) = 1$

redesign problem is converted to the equivalent optimization problem taking into account the inter-sampling behaviour and global state-matching. We also developed a new method to determine the search space of GAs by using the interval arithmetic operations, which is used for the initialization and evolution of the population. To improve the performance of GAs, some other problem-specific information, such as the reduction of the search space and the insertion of the local optimal solution, is also utilized. Using the newly developed method, the resulting states of the digitally controlled sampled-data nonlinear systems are able to match closely those of the original analogously controlled continuous-time nonlinear systems.

6. Acknowledgement

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