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# TSK-type 시간 지연 퍼지 제어기의 강인한 안정성

## Robust Stability of TSK-type Time-Delay FLC

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**Abstract**

A stable TSK-type FLC can be designed by the method of Parallel Distributed Compensation (PDC), but in this case, solving the LMI problem is not a trivial task. To overcome such a difficulty, a Time-Delay based FLC (TDFLC) is proposed. TSK-type TDFLC consists of Time-Delay Control (TDC) and Sliding Mode Control (SMC) schemes, which result in a robust controller basaed upon an integral sliding surface.

**Key Words** : fuzzy logic control, robust stability, time-delay

### 1. Introduction

The TSK (Tagaki-Sugeno-Kang) fuzzy model [1] has been widely used for a stable fuzzy control system [2]-[5], because the linear control theory can be applied in the design of the stable TSK-type FLC. One of the notable design methods to stabilize a TSK fuzzy system is to apply the Parallel Distributed Compensation (PDC) suggested by Wang et. al [2]. In the system described by the TSK fuzzy model combined with PDC scheme, however, it is often difficult to find a positive-definite matrix that satisfies a set of linear matrix inequalities simultaneously. As another approach, the robust control theory [6][7] has been presented for the stability of TSK-fuzzy systems. In this method, the nonlinear time-varying fuzzy system is considered as a linear time-invariant system with a norm-bounded model uncertainty; in this case, the LMI problem embedded in the PDC becomes a problem of finding a positive definite solution of an algebraic Riccati equation that stems from the defined Lyapunov function. It is remarked that, since robustness is desirable in consideration of the model uncertainty, various efforts have been given for robust stabilization of a TSK fuzzy model [6]. There is no assurance, however, that the algebraic Riccati equation is always solvable. If unsolvable, the related matrices including the control gains in all the rules must be changed, which affects the norm-bounds of uncertainties. As a result, the LMI problem is indirectly embedded in solving the algebraic Riccati equation. Moreover, no analytical method is known to construct the solvable algebraic Riccati equation. For uncertain systems, a TDC (Time-Delay Control) technique is also known to be successfully applied for controller design [8][9]. The TDC algorithm is simple and requires little priori knowledge of the system dynamics. In this paper, we shall use the

TDC scheme for a new approach, called a TSK-type TDFLC (Time-Delay FLC), to avoid such difficulties of designing a stable TSK-type FLC: the LMI problem or the problem of selecting appropriate interdependent matrices in the algebraic Riccati equation. The proposed TSK-type TDFLC can be designed to overcome disadvantages of the TDC while ensuring the stability by means of a modified sufficient condition for stability.

## 2. Time-Delay Control [8]: Brief Review

Consider a class of nonlinear plants described by

$$x^{(n)}(t) = f(\underline{x},t) + b(\underline{x},t)u(t),\ y(t) = x_1(t),$$

or

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots 0 \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \underline{x}(t) + \underline{f}(\underline{x},t) + \underline{b}(\underline{x},t)u(t),$$

$$y(t) = x_1(t), \tag{1}$$

where $\underline{x} \in R^n$, $u \in R$, $\underline{f}(\underline{x},t) = [0,\cdots,0,f(\underline{x},t)] \in R^n$ with $f(\underline{x},t): R^n \to R \in C^\infty$, and $\underline{b}(\underline{x},t) = [0,\cdots 0,b(\underline{x},t)]^T \in R^n$ with $b(\underline{x},t): R^n \to R \in C^\infty$. The desired performance is defined by the response of a given stable linear time-invariant reference model:

$$\dot{\underline{x}}_m(t) = A_m \underline{x}_m(t) + \underline{b}_m r_m, \tag{2}$$

where $\underline{x}_m \in R^n, r_m \in R$ (a reference input), $A_m \in R^{n\times n}$ (a canonical Hurwitz matrix), and $\underline{b}_m = [0,\cdots,0,b_m] \in R^n$. Let $\underline{e}(t) = \underline{x}_m(t) - \underline{x}(t)$. Then, from (1) and (2), the error dynamics is written as

$$\dot{\underline{e}}(t) = A_m \underline{e}(t) + [-\underline{\hat{b}}\underline{\hat{b}}^+ \underline{\hat{f}}(\underline{x},t) + \underline{\hat{b}}\underline{b}^+ A_m \underline{x}(t) + \underline{\hat{b}}\underline{\hat{b}}^+ \underline{b}_m r_m - \underline{\hat{b}}u(t)], \tag{3}$$

where

$$\underline{\hat{f}}(\underline{x},t) = \underline{f}(\underline{x},t) + [\underline{b}(\underline{x},t) - \underline{\hat{b}}]u(t) \in R^n, \tag{4}$$

and $\underline{\hat{b}} = [0,\cdots,0,\hat{b}]^T \in R^n$ and $\underline{b}^+ = (\underline{b}^T \underline{b})^{-1} \underline{b}^T \in R^{1\times n}$ is a pseudoinverse of $\underline{b}$. Applying the control input

$$u_d^*(t) = \underline{\hat{b}}^+ [-\underline{\hat{f}}(\underline{x},t) + A_m \underline{x}(t) + \underline{b}_m r_m], \tag{5}$$

to (3) sets the bracket in (3) to be zero. Here $\underline{\hat{b}}^+ = (\underline{\hat{b}}^T \underline{\hat{b}})^{-1} \underline{\hat{b}}^T \in R^{1\times n}$ is a pseudoinverse of $\underline{\hat{b}}$. Under the assumption that $\underline{\hat{f}}(\underline{x},t)$ is a continuous function of its arguments, we can write

$$\underline{\hat{f}}(\underline{x},t) = \underline{\hat{f}}(\underline{x},t-L) + \varepsilon_{error}, \tag{6}$$

and, for small $L$, we have

$$\underline{\hat{f}}(\underline{x},t) \cong \underline{\hat{f}}(\underline{x},t-L). \tag{7}$$

By using (1) and (4), (7) is rewritten as

$$\underline{\hat{f}}(\underline{x},t) = \dot{\underline{x}}(t) - \underline{\hat{b}}u(t) \cong \dot{\underline{x}}(t-L) - \underline{\hat{b}}u(t-L). \tag{8}$$

Accordingly, combination of (5) and (8) results in the TDC control input

$$u_d(t) = \underline{\hat{b}}^+ [-\dot{\underline{x}}(t-L) + \underline{\hat{b}}u_d(t-L) + A_m \underline{x}(t-L) + \underline{b}_m r_m]$$

$$= u_d(t-L) + \underline{\hat{b}}^+ [-\dot{\underline{x}}(t-L) + A_m \underline{x}(t-L) + \underline{b}_m r_m], \tag{9}$$

For stability of TDC-based control system, recall the following Lemma [8].

***Lemma 1*** [8]: Given the system in (1), assume that there exists a positive number N such that

$$|b\hat{b}^{-1} - I_1| < 1 \text{ for } t > N, \tag{10}$$

where $I_1 \in R$ is an identity matrix. Then the TDC in (9) guarantees that $y(t) \to x_{m1}(t)$ for sufficiently small time-delay $L$ and sufficiently large for $t$.

The MIMO case ($B \in R^{n\times r}$ and $\hat{B} \in R^{n\times r}$ for $b$ and $\hat{b}$, respectively) can be proved by the result of [8], without loss of generality, if the number of the inputs is identical to that of the outputs.

***Theorem 1:*** Assume that there exists a positive number N such that

$$|\sum_{i=1}^{r}\sum_{j=1}^{r} h_{ci}(\underline{x}(t)) h_{mj}(\underline{x}(t)) \tilde{b}_i \hat{b}_j^{-1} - 1| < 1$$

$$\text{for } t > N, \tag{11}$$

and let the sliding surface be defined as

$$s(t) = \underline{i}_n^T [\underline{e}(t) - A_m \int_0^t \underline{e}(\tau) d\tau]. \tag{12}$$

Then, the TSK-type TDFLC input

$$u_t(t) = \sum_{i=1}^{r} h_{ci}(\underline{x}(t)) [u_{ti}(t-L) + \underline{\hat{b}}_i^+ (-\dot{\underline{x}}(t-L)$$

$$+ A_m \underline{x}(t-L) + \underline{b}_m r_m) + \frac{K_t}{\Phi} s(t)], \tag{13}$$

guarantees that $s(t) \to 0$ for sufficiently small time-delay $L$ and sufficiently large $t$.

***Proof:*** The TSK-type TDFLC input is rewritten as

$$u_t(t)=\sum_{i=1}^{r}h_{ci}(\underline{x}(t))[u_{ti}(t-L)$$
$$+\hat{\underline{b}}_i^{+}(A_m\underline{x}_m(t-L)+\underline{b}_m r_m-\dot{\underline{x}}(t-L)$$
$$+A_m(-\underline{x}_m(t-L)+\underline{x}(t-L))+\frac{K_i}{\Phi}s(t)\hat{\underline{b}}_i)]$$
$$=\sum_{i=1}^{r}h_{ci}(\underline{x}(t))[u_{ti}(t-L)+\hat{b}_i^{-1}(\dot{s}(t-L)+\frac{\hat{b}_i K_i}{\Phi}s(t))], \quad (14)$$

We have from (12) and (13) that

$$\dot{s}(t)=-\underline{i}_n^T\sum_{i=1}^{r}h_{mi}(\underline{x}(t))\widetilde{A}_i\underline{x}(t)+\underline{i}_n^T A_m\underline{x}(t)+\underline{i}_n^T\underline{b}_m r_m$$
$$-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1}\dot{s}(t-L)$$
$$-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1}\frac{\hat{b}_j K_j}{\Phi}s(t)$$
$$-\sum_{i=1}^{r}h_{mi}(\underline{x}(t))\widetilde{b}_i u_t(t-L), \quad (15)$$

Let $a(\underline{x}(t))=-\underline{i}_n^T\sum_{i=1}^{r}h_{mi}(\underline{x}(t))\widetilde{A}_i\underline{x}(t)+\underline{i}_n^T A_m\underline{x}(t)+\underline{i}_n^T\underline{b}_m r_m$, then we have

$$\dot{s}(t-L)=a(\underline{x}(t-L))-\sum_{i=1}^{r}h_{mi}(\underline{x}(t-L))\widetilde{b}_i u_t(t-L). \quad (16)$$

Applying (16) to (15) results in

$$\dot{s}(t)=(1-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1})\dot{s}(t-L)$$
$$-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1}\frac{\hat{b}_j K_j}{\Phi}s(t)$$
$$+a(\underline{x}(t))-a(\underline{x}(t-L))+\varepsilon_1, \quad (17)$$

where

$$\varepsilon_1=(\sum_{i=1}^{r}h_{mi}(\underline{x}(t-L))\widetilde{b}_i-\sum_{i=1}^{r}h_{mi}(\underline{x}(t))\widetilde{b}_i)u_t(t-L). \quad (18)$$

In the same manner as (17), we get

$$\dot{s}(t-L)=(1-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1})\dot{s}(t-2L)$$
$$-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1}\frac{\hat{b}_j K_j}{\Phi}s(t-L)$$
$$+a(\underline{x}(t-L))-a(\underline{x}(t-2L))+\varepsilon_2. \quad (19)$$

By subtracting (19) from (17), we obtain that

$$\dot{s}(t)-\dot{s}(t-L)=(1-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1})(\dot{s}(t-L)$$
$$-\dot{s}(t-2L))+\varepsilon_3, \quad (20)$$

where $|\varepsilon_3|\le\beta_1 L$ and $\beta_1>0$. Taking the norm on both sides of the above equation leads to

$$|\dot{s}(t)-\dot{s}(t-L)|\le|1-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1}||\dot{s}(t-L)$$
$$-\dot{s}(t-2L)|+|\varepsilon_3|$$
$$\le\alpha|\dot{s}(t-L)-\dot{s}(t-2L)|+|\varepsilon_3|$$
$$\le\alpha^{t-N}|\dot{s}(N)-\dot{s}(N-L)|+|\varepsilon_4|, \quad (21)$$

where $|\varepsilon_4|\le\beta_2 L+\beta_3\alpha^{t-N}$. Accordingly, we can say from $\alpha<1$ that

$$|\dot{s}(t)-\dot{s}(t-L)|\to 0 \text{ as } t\to\infty \text{ and } L\to 0, \quad (22)$$

or

$$\dot{s}(t-L)\to\dot{s}(t) \text{ as } t\to\infty \text{ and } L\to 0. \quad (23)$$

By applying the above result to

$$\dot{s}(t)-\dot{s}(t-L)=-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1})\dot{s}(t-L)$$
$$-\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1}\frac{\hat{b}_j K_j}{\Phi}s(t)$$
$$+a(\underline{x}(t))-a(\underline{x}(t-L))+\varepsilon_1, \quad (24)$$

we conclude that

$$\dot{s}(t-L)\to-\frac{\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i K_j}{\Phi\sum_{i=1}^{r}\sum_{j=1}^{r}h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\widetilde{b}_i\hat{b}_j^{-1}}s(t)$$
$$\text{as } t\to\infty \text{ and } L\to 0, \quad (25)$$

or

$$s(t)\to 0 \text{ as } t\to\infty \text{ and } L\to 0. \quad (26)$$

***Q.E.D***

To guarantee a stable TSK-type fuzzy system, the LMI problem should be solved, which is to find a common matrix $P$ satisfying the Lyapunov equations caused by the rules of the fuzzy system. In addition, the LMI problem is not easily solvable analytically, which makes the matter worse for the normal TSK-type fuzzy system. Also, it is remarked that the LMI problem becomes very complicated as the number of rules increases. But, compared with the normal case, the proposed general robust TSK-type TDFLC shows its stability depending on the related assumptions, without considering a common matrix $P$. Moreover, the proposed method can also overcome disadvantages which the normal TDC has: heavy dependence on a time-delay and a value of $\widetilde{b}(\underline{x},t)\hat{b}^{-1}$ for performance. Meanwhile, the replacement of $\mathrm{sgn}(s(t))$ with $\mathrm{sat}(s(t))$ can solve a chattering problem which the general robust TSK-type TDFLC causes.

## 3. Concluding Remarks

The approach for designing FLC using time-delay are presented is given. Since TDC has characteristics to require little knowledge of the system dynamics, the proposed TSK-type TDFLC overcomes disadvantages that a normal FLC has, such as LMI problem and the robustness. To consider the more general case, weakening the sufficient conditions and constraints for the proposed method needs to be studied.

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