CONVERTER DESIGN AND CONTROL OF PIEZOELECTRIC ACTUATORS IN SLIDING MODE OPERATION

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Abstract

Piezoelectric actuators are characterized by non-linear dynamics and high frequency oscillations of the piezocrystal. Both properties have to be taken into consideration when optimizing real time systems. Taking benefit of the almost linear behaviour between charge and strain, current source fed piezoelectric actuators are given preference for high dynamic applications. Here special emphasis is put on current sources for multi-actuator systems and the controller design for optimal system integration of the actuator. It is shown that sliding mode operation of the converter system offers good possibilities to guaranty high accuracy and dynamics of the actuators system. The presented multi-actuator system is used for positioning and vibration damping in flexible mechanical systems.

INTRODUCTION

Beside established electromechanical actuator systems piezoactuators (PA) have become more and more important. They are characterized by high forces, tiny displacements, high energy density and short reaction times. Due to direct energy conversion they have a high efficiency. For these reasons piezoelectric actuators have found an increasing field of application in the last years. However, when designing high performance actuator systems non-linear dynamics of the piezoeffect has to be taken into account. Whereas the conventional system behaviour of electric drives and actuators is well known and already investigated there is a lack of technically applicable mathematical models of unconventional actuators. The dynamics of the piezoactuators can be of decisive influence on the whole system behaviour when utilising the actuator under time critical conditions. Damping of high frequency mechanical vibrations (some hundreds of Hertz and more) by using piezoelectric actuators is one of the examples where real time operation of the control system is required and controller design has to take into consideration both sampling time of the digital control system and dynamics of the piezoactuator.

MODELLING OF PIEZOACTUATORS

As known the piezoelectric effect can be described by the basic piezoelectric equations:

$$D = dT + \varepsilon E \qquad (1)$$

$$S = sT + dE \qquad (2)$$

It is obvious that these equations only consider the steady-state behaviour of a piezoelectric element. To describe its dynamical performance the piezoelectric actuator is interpreted as electro-mechanical system, characterized by weight-loaded continua and electric fields. The governing equations are derived by Maxwell-theory coupled with Hamilton's principle [1-3]. The consequent application of this point of view leads to field equations and complex models with lots of parameters. The determination of all these parameters is only possible with several assumptions and on the other hand it is difficult to install this complex model into control structures. For that reason a simplified, physically explainable model was developed which parameters can be determined experimentally.

The following assumptions have been made for the used piezoactuator model shown in Fig. 1:

- The piezoactuator has a preference direction x. The piezoeffect in the other directions can be neglected (one-dimensional effect).
- The grid structure of the piezocrystal is composed regularly, so that homogeneous field distribution can be assumed when applying a voltage U to the actuator.
- The actuator pressed by a force F is considered as piece-wise concentrated mass, which corresponds to the piezo-discs. The equivalent mass m of the actuator and the stiffness k characterize its swinging behaviour.
- The friction in the crystal is taken into account as dissipative term with the damping coefficient b.
- The dielectric coefficient ε and the piezoelectric coefficient e are material parameters of the actuator.

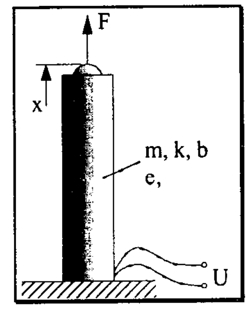

Fig. 1: Model of the actuator

For deriving the governing equations of the piezoactuator the electric charge q and the strain Δx of the actuator are defined as generalized coordinates and the voltage U and the loading F as generalized forces. Starting with the energy-balance the governing equations are obtained using the Lagrangian formalism [4]:

$$\frac{x_0}{\varepsilon A} q = \frac{e}{\varepsilon}(x - x_0) + U \qquad (3)$$

$$\tfrac{1}{3} m\ddot{x} + b\dot{x} + k(x - x_0) = F + \frac{e}{\varepsilon} q - \frac{e^2}{\varepsilon}\frac{A}{x_0}(x - x_0) \qquad (4)$$

These equations consider the dynamics of the actuator and correlate to the basic piezoelectric equations (1,2) when neglecting the derivatives. The block diagram is shown in fig. 2.

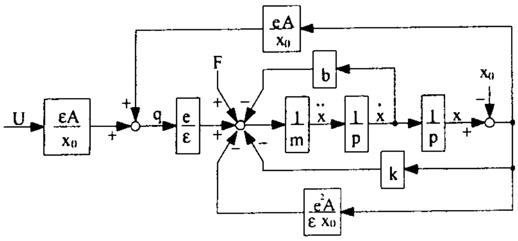

Fig. 2: Voltage fed piezoactuator

Practical experiments proof that the well-known hysteresis between strain and voltage is found between charge and voltage as well, whereas the behaviour between charge and strain is almost linear. So the dielectric constant ε and the piezoelectric constant e are non-linear and have hysteresis character. This feature is very important when designing the power converter of PA i. e. voltage fed actuators are influenced by hysteresis behaviour whereas current fed actuators represent practically linear systems.

As known, PA can be electrically considered as capacitive charge. The electric equivalent scheme is shown in fig. 3. Here, the capacity C_2 represents the electric capacity of the piezocrystal and the impedance C_1-L_1-R_1 stands for the mechanical oscillator. It is obvious that due to the capacity of the piezoactuator the system behaviour is mainly determined by the current limitation of the converter even in the case of voltage feeding. Consequently, dynamics of voltage fed actuators practically no differ from dynamics of current fed actuators. Because of the mentioned particularities in the following preference is given to the current fed actuator.

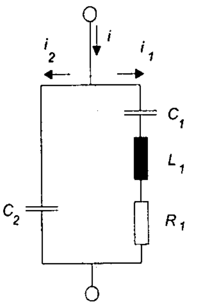

Fig.: 3 Electric equivalent scheme

CONVERTER DESIGN

Power converters with current source properties can be technically realised by using several topologies, like Buck, Cúk or Sheppard-Taylor topology [5-10]. Fig. 4 shows the classical Buck topology for mono-actuator feeding used in the proposed paper. Current source behaviour can be obtained when „charging" the inductance L_0 with a sufficiently high current. For this, the path T_1-T_2 or T_3-T_4 is short-circuited. When the desired value is reached the current is switched to one of the diagonal paths (T_1-T_4 or T_2-T_3) and the actuator is charged or discharged. Current source properties are given if the current through the inductance L_0 is kept nearly constant. Therefore L_0 must be sufficiently high and the switching time sufficiently small. In normal operation T_0 is opened and the actuator is charged by the constant inductance current or discharged via D_0 returning the energy to the inductance. Consequently,

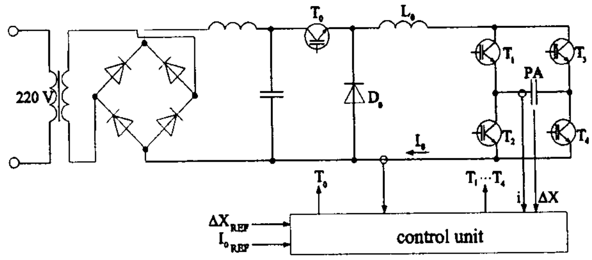

Fig. 4: Electric scheme of the current fed piezoactuator

the system is operating with full energy recovery. Losses can be covered by introducing switching cycles with closed T_0 and short-circuited paths T_1-T_2 or T_3-T_4. It is obvious that following the described strategy the actuator voltage can reach negative values which must be avoided. As the actuator voltage results from the imposed current the former can be easily controlled and kept to positive values by using appropriate control strategies. Current control can be realized by applying PWM technique or sliding mode operation. Switching control strategies have the advantage of small transistor losses. Furthermore, high dynamic properties can be guarantied with respect to current limitation and electric charge is varying in a strongly linear way.

The scheme in figure 4 can be easily modified for multi-actor feeding when connecting several current sources in series using a common inductance as shown in figure 5. The

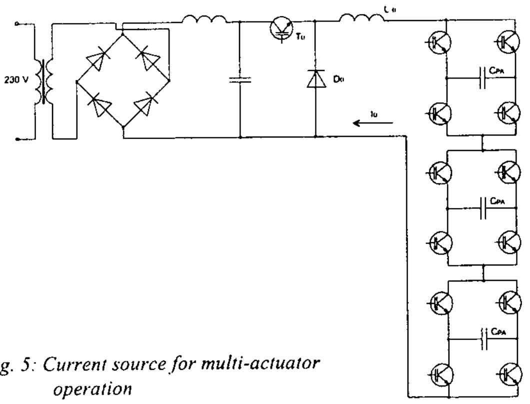

Fig. 5: Current source for multi-actuator operation

main advantage of this topology is that electrical energy can be exchanged among the PA in dependence on the direction of current through the PA. As the PA represents a capacity

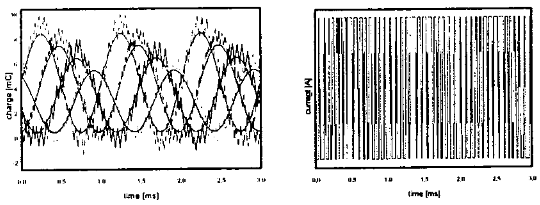

Fig. 6.: Simulation results of charge control in multi-actuator

current direction can be imposed without commutation problems by the control law. Figure 6. shows simulation results for 4 different PA fed by a common current source. Here, a constant commutation frequency of 100 kHz is chosen.

CONVERTER CONTROL

Control system design of piezoactuators has to take into consideration that piezoactuators are used when high dynamic position control and small displacements are required. Consequently, maximum current values must be utilized when charging and discharging the piezocapacity. As shown in the previous paragraph, operation with maximum current demands the modelling of the system behaviour in accordance with the current fed actuator model.

With respect to the above mentioned special features the charge control system represented in fig. 7 is proposed. To

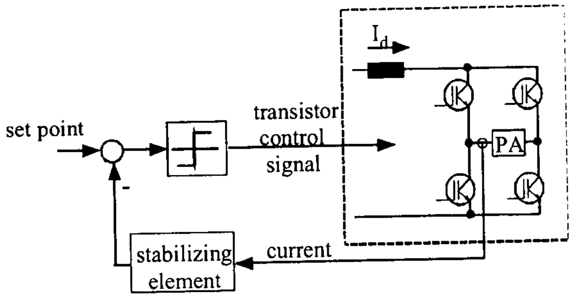

Fig. 7: Charge control system in sliding mode operation

avoid the influence of non-linear parameters of the actuator sliding mode operation of the current source is used. For satisfying the condition of sliding mode operation a stabilizing element is used. Fig. 8 shows the corresponding block diagram. As known, in sliding mode operation the motion of the system can be separated in a fast and a slow component. Both components are independent each other and can be analyzed separately. The high frequency „chattering" of the fast system is described by the element represented in fig. 8. The current source can be formally considered as simple transfer-factor when neglecting the influence of the switching time of the transistors on the system behaviour. The switching characteristic in the scheme represents the opening and closing of the corresponding transistor paths. When applying the method of the harmonic linearization to the non-linear system the switching characteristic can be simplified by an amplitude depending on the transfer-factor K_∞. Consequently, when considering harmonic operation the chattering element can be described by the transfer function:

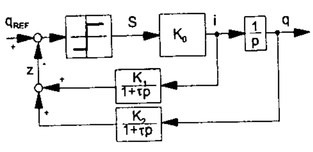

Fig. 8: Charge control in sliding mode operation (fast motion)

$$F(p) = \frac{q(p)}{q_{REF}(p)} = \frac{1}{K_2} \cdot \frac{1+\tau p}{1 + \frac{1+K_1 K_\infty}{K_2 K_\infty} \cdot p + \frac{\tau}{K_2 K_\infty} \cdot p^2} \quad (6)$$

In sliding mode operation we can suppose $K_\infty \rightarrow \infty$ and the transfer-function becomes

$$F(p) = \frac{q(p)}{q_{REF}(p)} \rightarrow \frac{1}{K_2} \cdot \frac{1+\tau p}{1 + \frac{K_1}{K_2} p} \quad (7)$$

In consequence, when choosing

$$\frac{K_1}{K_2} = \tau \quad (8)$$

the injection of the charge into the actuator can be realized without any time delay. Furthermore, when choosing

$$\frac{K_1}{K_2} < \tau \quad (9)$$

even an advance phase behaviour can be obtained. The

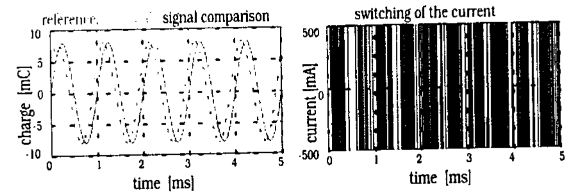

Fig. 9: Charge of the chattering element with phase advance

response of the charge is illustrated in fig. 9 when a sinusoidal signal is given to the input of the „chattering element". However, it must be mentioned that the shown inertialess behaviour of the charge is only realizable when the system energy resources (i.e. level of current limitation) are high enough to bring the system in sliding mode operation.

The system is operating in sliding mode when the parameters K_1 and K_2 are properly adjusted. The control variable S is either 1 or -1. The parameter K_0 represents the current limitation of the current source. The tranfer function relating the control variable S to the feed-back variable z is given as:

$$F(p) = \frac{z(p)}{S(p)} = K_0 K_2 \cdot \frac{1 + \frac{K_1}{K_2} p}{p(1+\tau p)} \quad (10)$$

The corresponding state equations are:

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = -\frac{z_2}{\tau} + \left(b_1 + b_2 \frac{d}{dt}\right) S \quad (11)$$

with $b_1 = \frac{K_0 K_2}{\tau}; b_2 = \frac{K_0 K_1}{\tau}$

For representing these equations in the phase space they must be transformed with

$$z_1 = b_1 \gamma_1 + b_2 \dot{\gamma}_1$$
$$z_2 = b_1 \gamma_2 + b_2 \dot{\gamma}_2 \quad (12)$$

into the following form:

$$\dot{\gamma}_1 = \gamma_2$$
$$\dot{\gamma}_2 = -\frac{\gamma_2}{\tau} + S \quad (13)$$

The phase space equation can be written as:

$$\frac{d\gamma_1}{d\gamma_2}=\frac{\tau\gamma_2}{\tau S-\gamma_2} \qquad (14)$$

The trajectories are obtained as followed:

$$\gamma_1=\begin{cases} C-\tau\gamma_2-\tau^2\ln|\tau-\gamma_2|; S=+1 \\ C-\tau\gamma_2+\tau^2\ln|\tau+\gamma_2|; S=-1 \end{cases} \qquad (15)$$

The switching line at the γ-plane is given as:

$$0=b_1\gamma_1+b_2\gamma_2 \qquad (16)$$

The switching line has to be placed in such a way that the trajectories can follow into the origin. This is given within

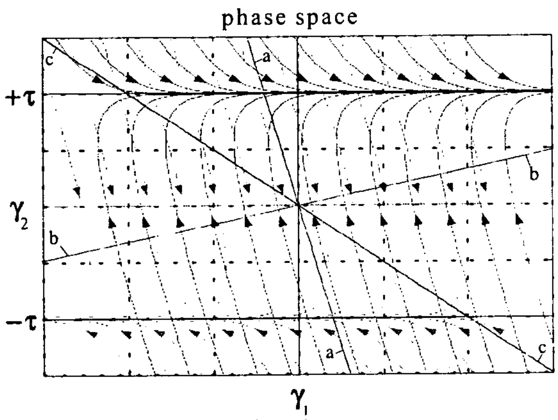

Fig. 10: Graphical diagram of the phase space

the lines (a) and (b) (fig. 10), where the trajectories are parallel or perpendicular to the switching line. The switching line (c) can be considered as optimal location. Depending on the placement of the switching line a certain area of the ratio of the K's is given where sliding mode appears and the system is stable.

SYSTEM INTEGRATION OF THE CHARGE CONTROLLED PIEZOACTUATOR

Positioning systems

For designing high performance positioning systems both hysteresis, non-linear and uncertain system parameters, limited energy resources of the converter and high frequency mechanical oscillations of the piezocrystal must

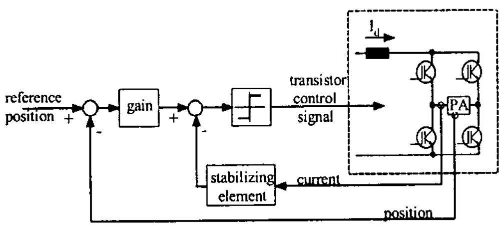

Fig. 11: Positioning system with charge controllod PA

be taken into consideration. When utilizing the current fed piezoactuator model for control system design the influence of the hysteresis can be neglected and current limitation can be respected. Figure 11 shows the integration of the charge controlled PA into a positioning system.

When the charge response to the output signal of the position controller can be obtained without time delay or even with a slight derivative component the system is globally stable and the position controller gain can theoretically increased up to very high values without stability problems. Consequently, due to the high controller gain, the position error tends to zero. Fig. 12 (a) shows

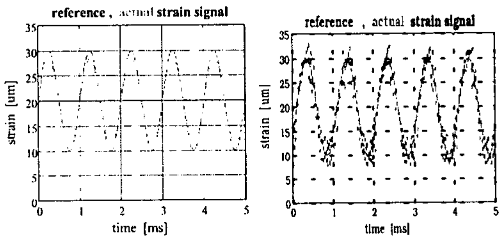

Fig. 12: (a) Simulation; (b) Experimental results

simulation results of the position controlled piezoactuator. It is obvious that the proposed sliding mode approach theoretically allows the design of systems with almost ideal dynamic properties. However, in practice the reachable dynamic performance will be limited. Existing small time constants in the system (mainly in the measurement and information system), limited switching frequencies and energy resources of the transistor converter affect the obtainable system dynamics. Fig. 12 (b) shows experimental results. It can be seen that the sinusoidal reference positions can be reproduced without considerable amplitude and phase error. The experiment was carried out with a DSP board based on the TMS320C40 processor with a sampling time of about 50 µs. Consequently, the reachable performance is additionally limited by the sampling time of the digital control unit. However, dynamics can be improved by using an analogue control system which is technically easily realisable because of the simplicity of the control algorithms and the high system robustness due to sliding mode operation.

Vibration damping system in rotating shafts

The piezoelectric vibration damping system consists of 3 torque blocks [11] mounted on a shaft (fig.: 13). The torque block consists of two piezoelectric stacks connected to an active pivot joint. These stacks are driven in opposite direction so that, at the top, linear motion is converted into a rotary motion. At the top of the torque blocks a disc ring is fixed. By applying a voltage to the torque drive a relative torsion between shaft and disc ring is obtained. The torque blocks with the mounted disc ring can be considered as spring mass system. For that reason the torque drive can work as passive absorber or even used for active vibration damping. The torque drive has a resonant frequency of 1,35 kHz which can be decreased by adding extra masses to the disc ring.

The control strategy of the damping system takes the absorber principle as basis. More details about controller design are given in [11]. Figures 14 and 15 show simulation

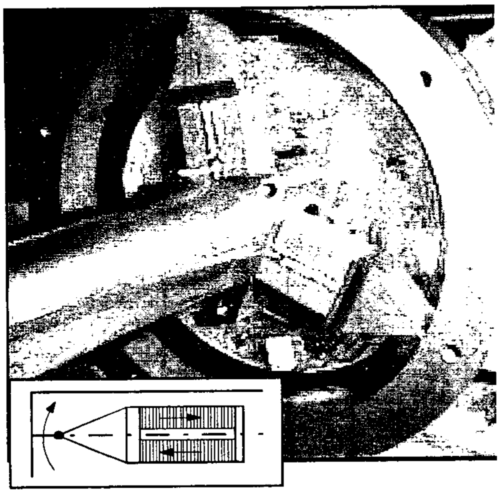

Fig. 13: Piezoelectric torque drive

and experimental results proving the efficiency of the proposed damping system.

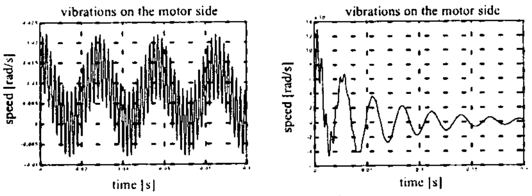

Fig. 14: Simulation results of vibration damping

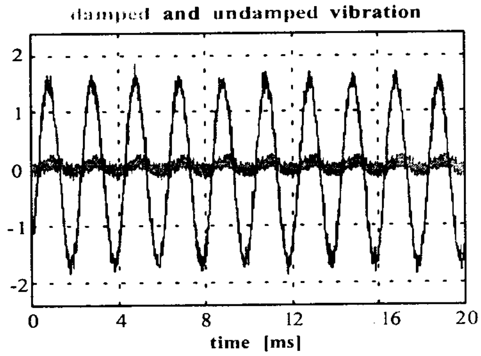

Fig. 15: Experimental results of vibration damping

CONCLUSIONS

The design of high performance position control systems of piezoelectric actuators requires special control techniques. Both hysteresis, non-linear and uncertain system parameters, limited energy resources of the converter and high frequency mechanical oscillations of the piezocrystal must be taken into consideration. When utilising current fed piezoactuators the influence of the hysteresis can be neglected and current limitation can be easily respected. The proposed current source topology allows multi-actuator feeding with energy recuperation. Sliding mode operation of the control unit and the converter guarantees high dynamics of the charge control system. However, in practice the system performance will be limited because of existing small time constants, limited switching frequencies in sliding mode operation, limited energy resources of the converter and the sampling time when digital control techniques are used. The efficiency of the presented converter and control system is illustrated for positioning and vibration damping systems.

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